Single-Carrier Space Time Transmission Diversity with Decision Feedback Equalization Over Frequency-Selective Fading Channels

Qifeng Zou[1,2]
1. School of Electronics and Information Engineering, Harbin Institute of Technology
2. Science and Technology on Information Transmission and Dissemination in Communication Networks Laboratory
Harbin 150080, China
E-mail:zouqifeng163@163.com

Xuezhi Tan[1,2] , Mei Liu[1] and Yonggang Chi[1,2]
1. School of Electronics and Information Engineering, Harbin Institute of Technology
2. Science and Technology on Information Transmission and Dissemination in Communication Networks Laboratory
Harbin 150080, China
E-mail: tanxz1957@hit.edu.cn

***Abstract*—In order to suppress inter-symbol interference (ISI), the Frequency domain decision feedback equalization (FD-DFE) algorithm is proposed in this paper for single carrier frequency domain equalization (SC-FDE) combined with Alamouti space time block coding (STBC) systems over frequency-selective channels. We use the cyclic prefix (CP) and two transmit antennas to achieve the significant diversity gain. The feedback equalization part is designed by Mean Square Error (MSE) criterion and the coefficients of filters are all in the frequency domain for the given channel frequency response. The outage and diversity analysis of system is also made to measure the equalization performance. Finally, the simulation results show that the proposed algorithm has better performance than the conventional liner equalization (LE) and hybrid structure decision feedback equalization (HDFE) schemes, and the computational complexity of our method is acceptable.**

***Index Terms*—decision feedback equalization (DFE); Single Carrier Frequency Domain Equalization (SC-FDE); space time block coding (STBC) ; MSE ; diversity**

I. Introduction

The broadband transmission in wireless environment often undergoes the severe inter-symbol interference (ISI). Single carrier frequency domain equalization (SC-FDE) has been shown to be an attractive scheme for broadband wireless channels which have long response memory [1, 2]. SC-FDE has similar performance and signal processing complexity compared with orthogonal frequency division multiplex (OFDM) [3]. Moreover, SC-FDE has lower peak-to-average power ratio (PAPR) and less sensitivity to carrier frequency errors than OFDM [4, 5]. Recently years, the SC-FDMA which is the multiple access style of SC-FDE has been adopted by LTE uplink. The key technologies of SC-FDE are the design of frequency domain equalizer [6, 7].

The traditional frequency domain linear equalization (FD-LE) such as zero forcing (ZF) and minimum mean square error (MMSE) has low complexity but the accuracy is poor [8]. In [9], D. Falconer proposed a hybrid structure decision feedback equalizer based on the minimum mean square error criterion (MMSE-HDFE), it has better performance however the computational complexity is very large because the feedback part is in time domain.

N. Benvenuto proposed the iterative block decision feedback equalization (IBDFE) algorithm in [10], the feedforward and feedback filters are all in frequency domain so it has low complexity when the number of iteration is small and has good performance, but the complexity will be increased quickly when the number of iteration is large. Moreover, the noise is amplified in the iteration part so that influence the equalization accuracy and make the overhead of system heavy [11].

Because the space time block coding (STBC) system has the advantage of facilitating maximum likelihood detection by easy processing, some researchers combined the SC-FDE with STBC to get the better performance improvement. Al-Dhahir gave a 2×1Alamouti STBC SC-FDE model in [12], and adopted the MMSE equalization in system. The two antenna scheme got the diversity gain more than one transmission antenna system, but the gain was limited for the equalizer performance. In [13], Zhou proposed a maximum likelihood equalizer based on Viterbi algorithm in single carrier STBC system, but the computational complexity is increased exponentially and can't get the full diversity gain. After that, Zhu proposed a hybrid structure equalizer in [14] based on the MMSE-HDFE scheme for the Alamouti STBC SC-FDE system. That method can achieve the full diversity and has a good performance, but the computational complexity is also very large when the feedback taps is increased. In order to

doi:10.4304/jsw.9.8.2232-2240

clear the performance gain by diversity, Tajer made analysis of the diversity order for the MMSE receiver in ISI channel [15]. In [16], Mehana extend the diversity analysis result in to more transmit antenna in CDD and Alamouti STBC SC-FDE system. However, nobody has give the diversity and outage probability performance analysis for the feedback structure equalizer in single carrier space time block coding diversity system.

In this paper, we use the signal processing property to propose a new equalization algorithm for single carrier STBC system. In section II we give the transmitter and receiver structure of Alamouti STBC SC-FDE system with CP, and introduce the system simulation model. Then we give the proposed algorithm theory analysis and parameters estimation in section III. After that, the computational complexity and simulation results are shown in section IV. Finally, the conclusions are given in section V.

II. System Model

We consider a single-carrier block transmission system, where the serial binary data streams are grouped into information data blocks having a length of N. The channel memory length is L, and the cyclic prefix with length $v \geq L$ is inserted at the beginning of each data-block, as shown in Fig. 1. For every (N+v) received symbols length, only N symbols length are processed by fast Fourier transform (FFT). Let us denote two continuous transmit block by $x_i(k)=[x_i(kN), x_i(kN+1),\ldots, x_i(kN+N-1)]^T$ i=1,2 at the time point k=0,2,4… . At the first time slot the $x_i(k)$ will be transmitted from ith antenna, and at the second time slot the $x_i(k+1)$ will be transmitted. Inspired by the Alamouti STBC system theory from [17], we can find that $x_1(k+1) = -x_2^*(k)$ and $x_2(k+1) = x_1^*(k)$. The received blocks at time k and k+1 are given by

$$\mathbf{y}(j) = \mathbf{H}_1(j)\mathbf{x}_1(j) + \mathbf{H}_2(j)\mathbf{x}_2(j) + \mathbf{w}(j) \quad (1)$$

where j=k, k+1. $\mathbf{H}_1(j)$ and $\mathbf{H}_2(j)$ are the circular channel matrices from the first and the second transmit antenna. We assume the channels are fixed over two consecutive block, so $\mathbf{H}_{1,2}(k) = \mathbf{H}_{1,2}(k+1) = \mathbf{H}_{1,2}$, and by [10] we have the FFT operations as $\mathbf{X}(p)$=$\mathbf{FX}$=$\sum_{n=0}^{N-1} x(n)e^{-j(2\pi/N)np}$, for p=0, 1…N-1.

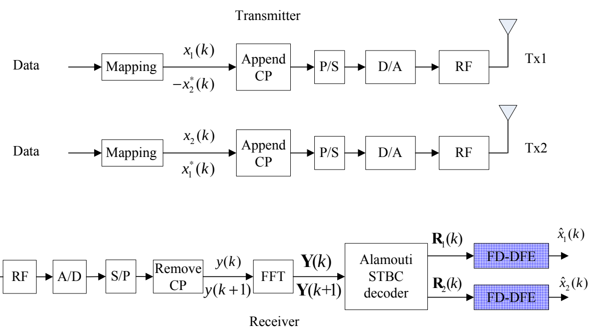

Figure 1. The single carrier STBC transceiver system model

Fig. 1.shows that: The SC-FDE system uses the single-carrier modulation technique at the transmitter. The cyclic prefix (CP) is inserted into the mapped signals. Then the signals are sent through the channel. After serial to parallel converters and remove the CP, the FFT technique is used to transform received signal from time domain into frequency domain at the receiver. Then the channel estimation algorithm in frequency domain provides the current channel state knowledge to the equalizer. The IFFT technique is used to transform the equalized signals from frequency domain back to time domain, and then the subsequent processing such as channel decoding are done. As a core part of this SC-FDE simulation system model, the proposed FD-DFE equalization method can suppressing the inter-symbol interference caused by multipath fading channels.

By the theory of diagonal matrix and the circular matrix in [13], the receive blocks at time k and k+1are given by

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y} \\ \mathbf{Y}_2^* \end{bmatrix} = \begin{bmatrix} \mathbf{\Lambda}_1 & \mathbf{\Lambda}_1 \\ \mathbf{\Lambda}_2^* & -\mathbf{\Lambda}_2^* \end{bmatrix} \begin{bmatrix} \mathbf{X}_1(k) \\ \mathbf{X}_2(k) \end{bmatrix} + \begin{bmatrix} \mathbf{W}_1 \\ \mathbf{W}_2^* \end{bmatrix} \\ = \mathbf{\Lambda X} + \mathbf{W} \quad (2)$$

where $(\cdot)^*$ denote the complex conjugate operation, and $\mathbf{\Lambda}_i = \mathbf{FH}_i\mathbf{F}^H$ (i=1,2) is an orthogonal diagonal matrix whose (k, k) entry is equal to the kth FFT coefficient of the channel impulse response (CIR). Frequency domain vector $\mathbf{W}_i$ (i=1, 2) is the additive white Gaussian noise (AWGN) term in the ith block period, with the time domain variance equal to σ_w^2.

We can multiply left side of (2) by a transposed matrix $\mathbf{\Lambda}^H$ in order to decouple the two transmitted signal $\mathbf{X}_1(k)$ and $\mathbf{X}_2(k)$ as

$$\tilde{\mathbf{Y}} = \mathbf{\Lambda}^H\mathbf{Y} = \begin{bmatrix} \tilde{\mathbf{Y}}_1(k) \\ \tilde{\mathbf{Y}}_2(k) \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{\Lambda}} & 0 \\ 0 & \tilde{\mathbf{\Lambda}} \end{bmatrix} \begin{bmatrix} \mathbf{X}_1(k) \\ \mathbf{X}_2(k) \end{bmatrix} + \tilde{\mathbf{W}} \quad (3)$$

where $\tilde{\mathbf{\Lambda}} = |\mathbf{\Lambda}_1|^2 + |\mathbf{\Lambda}_2|^2$ is an $N \times N$ matrix whose element is also equal to the sum of the squared FFT coefficients of the circular channel matrices from the first and the second transmit antenna. The noise vector is $\tilde{\mathbf{W}} = [\tilde{\mathbf{W}}_1, \tilde{\mathbf{W}}_2]^T$, and the two components are given by $\tilde{\mathbf{W}}_1 = \mathbf{\Lambda}_1^*\mathbf{W}_1 - \mathbf{\Lambda}_2\mathbf{W}_2^*$, $\tilde{\mathbf{W}}_2 = \mathbf{\Lambda}_2^*\mathbf{W}_1 + \mathbf{\Lambda}_1\mathbf{W}_2^*$.

Let us define $\mathbf{g} = \mathbf{\Lambda}_{12}^{1/2}$, and make the noise term still AWGN form with variance of σ_w^2, we can rewrite (3) as

$$\begin{bmatrix} \mathbf{R}_1(k) \\ \mathbf{R}_2(k) \end{bmatrix} = \begin{bmatrix} \mathbf{g} & 0 \\ 0 & \mathbf{g} \end{bmatrix} \begin{bmatrix} \mathbf{X}_1(k) \\ \mathbf{X}_2(k) \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{W}}_1 \\ \tilde{\mathbf{W}}_2 \end{bmatrix} \quad (4)$$

where $E\left(\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H\right) = \sigma_W^2 I_N$, $\sigma_W^2 = N\sigma_w^2$

Let us neglect the block number k and merge $\mathbf{R}_1(k)$ and $\mathbf{R}_2(k)$ as $\mathbf{R} = \mathbf{gX} + \tilde{\mathbf{W}}$

III. PROPOSED CHANNEL EQUALIZATION ALGORITHM

In this section, we use the MSE criterion to design the frequency domain decision feedback equalization (FD-DFE), and give the parameters estimation method for filters, we also give the outage and diversity analysis of our method and conventional FD-LE in given STBC system to compare the equalization performance.

A. Equalizer Architecture

The STBC SC-FDE simulation system receiver which performs the proposed channel equalization algorithm is established as shown in Fig. 1. The detail part of FD-DFE structure made in the system receiver is shown in Fig. 2. The results of time domain detected will be used for feedback equalization after by FFT converting process. We use the knowledge of the channel estimation and noise variance estimation to obtain instant signal to noise ratio (SNR) by the method of [18]. The frequency domain equalization part consists of feedforward and feedback filters, and the coefficients of filters is equal to C_n and B_n respectively, n=0, 1, 2, … , N-1.

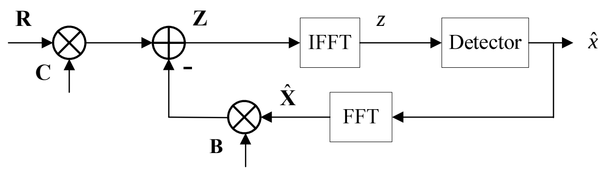

Figure 2. Proposed equalizer structure of FD-DFE

The output signal of decision feedback equalizer in frequency domain can be written as

$$\mathbf{Z} = \mathbf{CR} - \mathbf{B}\hat{\mathbf{X}} \tag{5}$$

where $\mathbf{C}$ and $\mathbf{B}$ are the diagonal matrix which diagonal elements composed by the coefficients of feedforward and feedback filters.

Making sure frequency domain correlation of the channel, we get the error vector of the output signal $\mathbf{Z}$ and input transmitted signal $\mathbf{X}$ is

$$\boldsymbol{\varepsilon} = \mathbf{Z} - \mathbf{X} = (\mathbf{Cg} - \mathbf{I}_N)\mathbf{X} - \mathbf{B}\hat{\mathbf{X}} + \mathbf{C}\tilde{\mathbf{W}} \tag{6}$$

By the orthogonality theory, we can get the mean square error (MSE) as

$$\text{MSE} = \text{tr}\left\{E\left\{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^H\right\}\right\} \tag{7}$$

where $\text{tr}\{\cdot\}$ is the trace of matrix, $E\{\cdot\}$ is the expectation.

We also find that the expectation $E\left\{\mathbf{X}\mathbf{X}^H\right\} = \sigma_x^2\mathbf{I}_N$, $E\left\{\mathbf{X}\hat{\mathbf{X}}^H\right\} \approx E\left\{\hat{\mathbf{X}}\hat{\mathbf{X}}^H\right\} \approx E\left\{\hat{\mathbf{X}}\mathbf{X}^H\right\} = \sigma_{\hat{x}}^2\mathbf{I}_N$.

The coefficients matrix of the feedforward filters is computed as

$$\mathbf{C} = (\sigma_{\hat{x}}^2 \cdot \mathbf{B} + \sigma_x^2\mathbf{I}_N)\mathbf{g}^H(\sigma_x^2 \cdot \mathbf{g}^H\mathbf{g} + \sigma_w^2\mathbf{I}_N)^{-1} \tag{8}$$

So we can get the Mean Square Error (MSE) of the decision part by coefficients form as

$$\begin{aligned} \text{MSE} = & \text{tr}\left\{\mathbf{C}(\sigma_x^2 \cdot \mathbf{g}^H\mathbf{g} + \sigma_w^2\mathbf{I}_N)\mathbf{C}^H + \sigma_x^2\mathbf{I}_N\right\} \\ & + \text{tr}\left\{(\sigma_{\hat{x}}^2(\mathbf{B} + \mathbf{B}^{\mathbf{H}}) - \sigma_x^2(\mathbf{Cg} + \mathbf{g}^H\mathbf{C}^H)\right\} \\ & + \text{tr}\left\{\sigma_{\hat{x}}^2 \cdot \mathbf{B}\mathbf{B}^{\mathbf{H}} - \sigma_{\hat{x}}^2(\mathbf{Cg}\mathbf{B}^{\mathbf{H}} + \mathbf{B}\mathbf{g}^H\mathbf{C}^H)\right\} \end{aligned} \tag{9}$$

Take notice that $\sum_{n=0}^{N-1} B_n = 0$ is the constrains, so the equalizer design problem can be converted to the optimization problem by

$$\min_{\mathbf{C},\mathbf{B}} \text{MSE} \quad \text{s.t.}\ \text{tr}\{\mathbf{B}\} = 0 \tag{10}$$

Let us use Lagrange optimization method compute gradient to minimize the MSE. So we define the Lagrange objective function as $f(\mathbf{B},\lambda) = \text{MSE} + \lambda\text{tr}\{\mathbf{B}\}$, and then we set the gradient of $f(\mathbf{B},\lambda)$ with respect to $\mathbf{B}$ and λ to zero, respectively, as follow

$$\frac{\partial f(\mathbf{B},\lambda)}{\partial \mathbf{B}} = \mathbf{Cg} - \mathbf{B} - (\frac{\lambda}{2\sigma_{\hat{x}}^2} + 1)\mathbf{I}_N = 0 \tag{11}$$

$$\frac{\partial f(\mathbf{B},\lambda)}{\partial \lambda} = \text{tr}\{\mathbf{B}\} = 0 \tag{12}$$

Therefore, we obtain the feedback filters coefficients as follow

$$\mathbf{B} = \sigma_w^2\mathbf{T}^{-1} - (\frac{\lambda}{2\sigma_{\hat{x}}^2})(\sigma_x^2 \cdot \mathbf{g}^H\mathbf{g} + \sigma_w^2\mathbf{I}_N) \cdot \mathbf{T}^{-1} \tag{13}$$

where $\mathbf{T} = (\sigma_x^2 - \sigma_{\hat{x}}^2)g^H g + \sigma_w^2\mathbf{I}_N$, we introduce

$$\alpha = \frac{1}{N}\text{tr}\left\{\mathbf{g}^H\mathbf{g}\mathbf{T}^{-1}\right\} \tag{14}$$

$$\beta = \frac{\lambda}{2\sigma_{\hat{x}}^2} + 1 \tag{15}$$

From (11) to (15), we get the coefficients of feedback filters as

$$\mathbf{B} = \mathbf{Cg} - \beta\mathbf{I}_N \tag{16}$$

The coefficients of feedforward filters can be rewritten as follow

$$\mathbf{C} = (\sigma_x^2 - \beta\sigma_{\hat{x}}^2)\ \mathbf{g}^H\mathbf{T}^{-1} \tag{17}$$

Notice that (15)-(17) only has the variance of detected symbol is unknown, so we just need estimate $\sigma_{\hat{x}}^2$ to calculate the coefficients of filters.

B. Parameter Estimation

We now analyze the parameters estimation of frequency domain decision feedback equalization. The modulation and the bit error rate (BER) decide the power of the decision errors. When we choose 2^R-PSK modulation (R=1, 2, 4…) and Gray-coded, we have

$$\sigma_{\hat{x}}^2 = \mu\sigma_x^2 \tag{18}$$

where $\mu = 1 - \tau P_e$ denotes the correlation between the decision symbols and the transmitted symbols in [10], and the value of τ is 2,2/5,2/21for BPSK, QAM, 16QAM. P_e is the pairwise error probability or the bit error rate.

C. Outage and Diversity Analysis

In this part, we will give the diversity analysis for STBC SC-FDE system.

For given SC-FDE system we default the value of signal to noise ratio as follow

$$\rho = \sigma_x^2 / \sigma_W^2 \tag{19}$$

The diversity gain describes the decay of average pairwise error probability with increase ρ .We give the definition of the diversity gain by

$$d = -\lim_{\rho \to \infty} \frac{\log P_e}{\log \rho} \tag{20}$$

For the equalizer influence, the effective mutual information between transmitted symbols *x* and the received symbols *z* after equalization is

$$I(x;z) = \frac{1}{N} \sum_{k=1}^{N} I\left(x_k ; z_k\right) \tag{21}$$

where $I\left(x_k ; z_k\right)$ is the mutual information of equalization symbols sub-steams.

If we have given the data transmission rate *R* (bit/s/Hz), we can achieve outage diversity and outage probability by

$$d_{\text{out}} = -\lim_{\rho \to \infty} \frac{\log P_{\text{out}}}{\log \rho} \tag{22}$$

$$P_{\text{out}} = P\left(I(x;z) < R\right) \tag{23}$$

For ZF receiver in given STBC SC-FDE system defined by (3), we can get the unbiased SINR γ_k^{ZF} for detecting symbol in the decision point as

$$\gamma_k^{\text{ZF}} = \frac{\rho}{\frac{1}{N}\text{tr}\left[\left(\mathbf{\Lambda}\mathbf{\Lambda}^H\right)^{-1}\right]} = \left[\frac{1}{N}\sum_{k=1}^{N}\left(\rho\left|\lambda_k\right|^2\right)^{-1}\right]^{-1} \tag{24}$$

where λ_k are the diagonal elements of $\mathbf{\Lambda}$,

The mutual information between *x* and *y* is given by

$$\begin{aligned} I_{\text{ZF}} &= \sum_{k=1}^{N} \log(1+\gamma_k^{\text{ZF}}) \\ &= -N \log\left[1+\left(\frac{1}{N}\sum_{k=1}^{N}\left(\rho\left|\lambda_k\right|^2\right)^{-1}\right)^{-1}\right] \end{aligned} \tag{25}$$

Let us define the α_k be the SNR exponent of the channel eigenvalues as

$$\alpha_k = -\frac{\log\left|\lambda_k\right|^2}{\log \rho} \tag{26}$$

We get the diversity for ZF method in two transmit antennas and one receive antenna STBC systems by

$$\begin{aligned} P_{\text{out}} &= P\left(\sum_{k=1}^{N}\frac{1}{\rho\left|\lambda_k\right|^2} > \frac{N}{2^R-1}\right) \\ &> P\left(\sum_{k=1}^{N}\frac{1}{\rho\left|\lambda_k\right|^2} > \frac{N}{2^R-1}\right) \\ &= P\left(\rho^{\alpha_k-1} > \frac{N}{2^R-1}\right) \\ &= \rho^{-1} \end{aligned} \tag{27}$$

For MMSE SC-FDE receiver in system defined by (3), we can also get the unbiased SINR γ_k^{MMSE} for detecting symbol in the decision point as

$$\begin{aligned} \gamma_k^{\text{MMSE}} &= \frac{\rho}{\frac{1}{N}\text{tr}\left(\rho^{-1}\text{I}+\tilde{\mathbf{\Lambda}}\right)^{-1}} \text{-1} \\ &= \left[\frac{1}{N}\text{tr}\left(\text{I}+\rho\tilde{\mathbf{\Lambda}}\right)^{-1}\right]^{-1} \text{-1} \end{aligned} \tag{28}$$

The mutual information of MMSE is given by

$$\begin{aligned} I_{\text{MMSE}} &= \sum_{k=1}^{N} \log(1+\gamma_k) \\ &= N \log\left[\left(\frac{1}{N}\text{tr}(\mathbf{I}_N + \rho\tilde{\mathbf{\Lambda}})^{-1}\right)^{-1}\right] \\ &= -N \log\left(\frac{1}{N}\sum_{k=1}^{N}\frac{1}{1+\rho\tilde{\lambda}_k}\right) \end{aligned} \tag{29}$$

where $\tilde{\lambda}_k$ are the diagonal elements of $\tilde{\mathbf{\Lambda}}$, and they can be defined as $\tilde{\lambda}_k = \left|\tilde{\lambda}_{1,k}\right|^2 + \left|\tilde{\lambda}_{2,k}\right|^2$, we notice that $\tilde{\lambda}_{i,k}$ are the eigenvalues of $\mathbf{H}_1(j)$ and $\mathbf{H}_2(j)$, *i*=1,2.

We define the $\tilde{\alpha}_k$ be the SNR exponent of the channel eigenvalues as

$$\tilde{\alpha}_k = -\frac{\log \tilde{\lambda}_k}{\log \rho} \tag{30}$$

So we can get outage probability of MMSE receiver by

$$\begin{aligned} P_{\text{out}} &= P\left(\frac{1}{N}I_{\text{MMSE}} < R\right) \\ &= P\left(\sum_{k=1}^{N}\frac{1}{1+\rho\tilde{\lambda}_k} > N2^{-R}\right) \\ &= P\left(\sum_{\tilde{\alpha}_k>1} 1 > N2^{-R}\right) \\ &= \rho^{-2(\lfloor N2^{-R}\rfloor+1)} \end{aligned} \tag{31}$$

where $\lfloor\cdot\rfloor$ is the floor function.

So we can get the diversity for MMSE method in two transmit antennas and one receive antenna STBC systems by follow equation

$$d_{\text{out}} = \begin{cases} 2(L+1) & \text{for } R \le \log\frac{N}{L} \\ 2\left(\left\lfloor 2^{-R} N \right\rfloor + 1\right) & \text{for } R > \log\frac{N}{L} \end{cases} \quad (32)$$

For our proposed frequency domain decision feedback equalization method, we can compute the unbiased SINR γ_{Proposed} for detecting symbol in the decision point by the asymptotic analysis theory from [19] as

$$\gamma = \rho \exp\left(\frac{1}{2\pi}\int_0^{2\pi} \ln\left(\left|H(\varphi)\right|^2 + \frac{1}{\rho}\right) d\varphi\right) - 1 \quad (33)$$

where $H(\varphi)$ is the DTFT of h_n ,n=0, …, N.

According to the Parseval's principle, we obtain:

$$\frac{1}{2\pi}\int_0^{2\pi} \left|H(\varphi)\right|^2 d\varphi = \sum_{k=0}^{N} \left|h_k\right|^2 \quad (34)$$

We define the function $\overline{f}(\varphi) = \left|H(\varphi)\right|^2 - \sum_{k=0}^{N}\left|h_k\right|^2$ so the (34) can be rewritten by

$$\frac{1}{2\pi}\int_0^{2\pi} \overline{f}(\varphi) d\varphi = 0 \quad (35)$$

Theorem: If $\overline{f}(\varphi)$ is an continuous function in $[0, 2\pi]$, there will be ξ and η that make $\overline{f}(\varphi) > 0$, for $\forall \varphi \in [\xi, \eta]$.

Proof: For $H(\varphi) = \sum_{k=0}^{N} h_k e^{-jl\varphi}, \quad N \ge 1$, the identity equation of $\left|H(\varphi)\right|^2 \equiv \sum_{k=0}^{N}\left|h_k\right|^2$ and $\overline{f}(\varphi) \equiv 0$ should not always be true, there must has $\varphi_0 \in [0, 2\pi]$ so that $\overline{f}(\varphi_0) > 0$, if not, $\overline{f}(\varphi)$ should not integral to 0. Since function $\overline{f}(\varphi)$ is continuous, we can notice there will be $\Delta\varphi > 0$ so that $\varphi_0 - \Delta\varphi \ge 0$ and $\varphi_0 + \Delta\varphi \le 2\pi$, the event $\overline{f}(\varphi) > 0$ will be true for $\varphi \in [\varphi_0 - \Delta\varphi, \varphi_0 + \Delta\varphi]$. Let us define $\xi = \varphi_0 - \Delta\varphi$ and $\eta = \varphi_0 + \Delta\varphi$, we get the result of theorem.

Let $p(\varphi) = \ln\left(\left|H(\varphi)\right|^2 + \frac{1}{\rho}\right)$, $q = \ln\left(\sum_{k=0}^{N}\left|h_k\right|^2 + \frac{1}{\rho}\right)$, by the theorem we get

$$\int_\xi^\eta p(\varphi) d\varphi > \int_\xi^\eta q d\varphi = (\eta - \xi) q \quad (36)$$

Moreover, we have

$$\begin{aligned} \int_0^{2\pi} p(\varphi) d\varphi &> \int_0^{\xi} p(\varphi) d\varphi + (\eta - \xi) q \\ &\quad + \int_\eta^{2\pi} p(\varphi) d\varphi \\ &\ge \left[2\pi - (\eta - \xi)\right] \ln\frac{1}{\rho} + (\eta - \xi) q \end{aligned} \quad (37)$$

Take notice that (37) will be holds because of $\left|H(\varphi)\right|^2 \ge 0$ and $p(\varphi) \ge \ln\frac{1}{\rho}$

So the (33) will be rewritten as

$$\frac{\gamma + 1}{\rho} \ge \rho^{-\frac{2\pi - (\xi - \eta)}{2\pi}} \left(\frac{1}{\rho} + \sum_{k=0}^{N}\left|h_k\right|^2\right)^{\frac{\xi - \eta}{2\pi}} \quad (38)$$

Then we get the lower bound of the SINR as

$$\gamma_{\text{Proposed}} \ge \left(1 + \rho\sum_{k=0}^{N}\left|h_k\right|^2\right)^{\frac{\xi - \eta}{2\pi}} - 1 \quad (39)$$

Finally, we get the outage probability of proposed equalization receiver as

$$\begin{aligned} P_{\text{out}} &\le P\left[\left(1 + \rho\sum_{k=0}^{N}\left|h_k\right|^2\right)^{\frac{\eta - \xi}{2\pi}} < 2^R\right] \\ &= P\left(\sum_{k=0}^{N}\left|h_k\right|^2 < \rho^{-1}\left(2^{\frac{2\pi R}{\eta - \xi}} - 1\right)\right) \\ &\approx \frac{\Omega_R}{(N+1)!\,\rho^{(N+1)}} \end{aligned} \quad (40)$$

Where Ω_R is a constant.

So we can get the diversity of two transmit antennas and one receive antenna STBC systems by

$$d_{\text{out, Proposed}} = -\lim_{\rho \to \infty} \frac{\log P_{\text{out}}}{\log \rho} = N + 1 \quad (41)$$

IV. Complexity Analysis and Simulation Results

In this section, we investigate the complexity of the proposed method with conventional FD-LE and FD-HDFE equalization methods at first.

The complexity of channel estimation is not considered because the different equalization schemes will estimate the channel using the same amount of computations.

The computational complexity of different algorithms is mainly measured by the numbers of complex multiplications. The traditional FD-HDFE in [9] and [14] has to estimate the complexity of equalizer and coefficients computation. We let N is the length of each block and B is the number of feedback taps in HDFE. We find that N point FFT or IFFT requires $(N/2)\log_2 N$ complex multiplications operations, When we get frequency output **Z** in (5), the equalizer make 2N complex multiplications. Note that coefficients of the feedforward and feedback filters got in (17) and (16) need make 3N complex multiplications operations. So our method will require (3N/2) $\log_2 N$ +7N multiplications. The proposed method has larger computational complexity than liner ZF and MMSE but it got better performance improvement than them also. With the number of feedback taps B increase, the computational complexity of HDFE is near the proposed equalization method. HDFE even has more computation than our FD-DFE method when B is large enough, but the performance is still poor than our method, that will be shown in next simulation.

The complexity computation of the equalizers in conventional algorithm and the proposed algorithm are given in Tab. I.

TABLE I.

COMPUTATIONAL COMPLEXITY OF THE EQUALIZATION ALGORITHMS

Algorithms	Number of complex multiplications
FD-ZF	$(N/2)\log_2 N+2N$
FD-MMSE	$N\log_2 N+N^2$
HDFE	$(N/2)\log_2 N+2B^2+(3B+2)N$
Proposed	$(3N/2)\log_2 N+7N$

Then we take simulation for our algorithm. The channel parameters of the ITU-EVA are given in Tab. II. ITU-EVA is a high delay model in wireless city environments. This channel model can simulate frequency selective fading channel satisfactorily. The maximum Doppler frequency shift f_D is 5Hz.

TABLE II.

CHANNEL MODEL OF ITU-EVA

Path Number	Tap delay (ns)	Power in each tap (dB)
0	0	0.0
1	30	-1.5
2	150	-1.4
3	310	-3.6
4	370	-0.6
5	710	-9.1
6	1090	-7.0
7	1730	-12.0
8	2510	-16.9

The simulation parameters of the Single-Carrier STBC system are given in Tab. III.

TABLE III.

SIMULATION PARAMETERS OF SC-STBC SYSTEM

Parameters	Value
Data modulation format	QPSK/16QAM
System bandwidth	5 MHz
Doppler maximal frequency	5 Hz
Sampling rate	7.68 MHz
Transmitter IFFT size	512
CP size	128/152
Channel Estimation	MMSE
Noise model	Independent AWGN
Channel coding	None/CC
Detection	Hard decision
Number of blocks	10^6

We use non-coded and convolution code (CC) style to test our proposed frequency decision feedback equalization method in SC-FDE system respectively. The BER-SNR performance is simulated in above given channel model shown as Fig. 3, Fig. 4 and Fig. 5 respectively. The modulation formats are QPSK or 16QAM in the simulations for different environments, and the CP size is 128 in the BER comparison.

As shown in Fig. 3, our proposed scheme has better BER performance than linear equalization and MMSE-HDFE in 2×1 uncoded QPSK modulation STBC system. At the BER level of 10^{-4}, the proposed method get about 5dB and 3.2dB gains respectively, compared with ZF and MMSE. It even achieve about 1.7dB, 1.1dB and 1dB improvement compared with the MMSE-HDFE when the number of HDFE feedback taps is 2, 6 and 10. From [14], we can see that the performance of HDFE is improve slightly when the number of feedback taps B>9. At this BER level, the proposed method performance loss is about 1dB compared with the four transmit antennas system. But the gain is 2.3dB compared with the one transmit antennas system. We can see that the more transmit antennas achieve more spatial diversity, and it gets better performance for the diversity order of the equivalent SISO channel is increased.

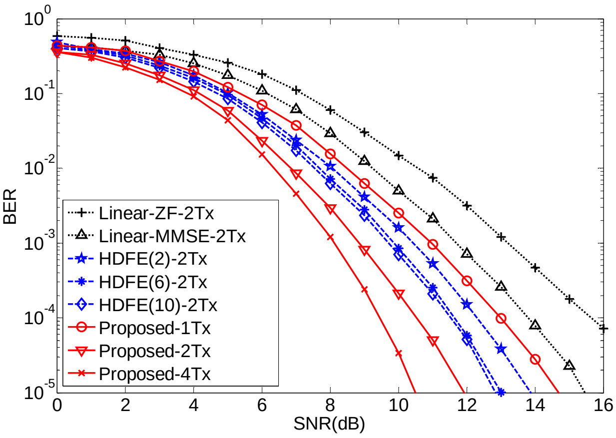

Figure 3. Average BER-SNR performance curves of different equalization methods in uncoded QPSK STBC system

From Fig. 4 we can see that the proposed method also has better BER performance than linear equalization in

2×1 QPSK convolution coded (CC) system. The performance of our algorithm in CC coded system is obviously better than that in non-coded system. At the BER level of 10^{-4}, the gains of the proposed method are about 3.4dB and 2.1dB respectively compared with ZF and MMSE. It even achieve about 1.3dB, 0.8dB and 0.7dB improvement over the conventional HDFE when the number of HDFE feedback taps is 2, 6 and 10. At this BER level, the proposed method performance loss is about 0.7dB compared with the four transmit antennas system. But the gain is 1.7dB compared with the one transmit antennas system.

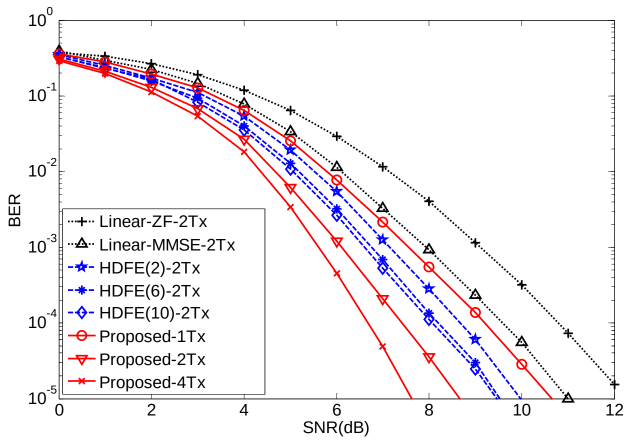

Figure 4. Average BER-SNR performance curves of different equalization methods in convolution coded QPSK system

According to Fig. 5, the proposed method has better BER performance than linear equalization in 16QAM uncoded system too. At the BER level of 10^{-4}, the gain of the proposed method in 2×1 system is about 9.3dB and 6.3dB over the conventional ZF and MMSE respectively. Compared with the HDFE equalization at BER=10^{-4}, the performance gain is about 4dB, 2.7dB and 2.2dB when the number of HDFE feedback taps is 2, 6 and 10. At this BER level, the proposed method performance loss is about 2.5dB compared with the four transmit antennas system. But the gain is 5dB compared with the one transmit antennas system. We can find that the performance of our algorithm in QPSK uncoded system is better than that in 16QAM system, and the performance gap of different equalization methods is narrowed in low-level modulation format or higher coding gain.

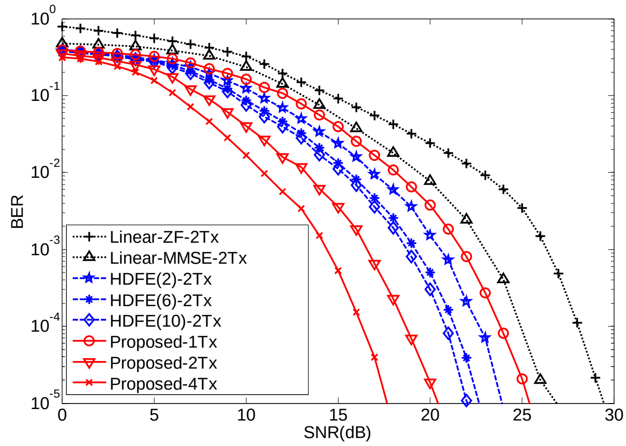

Figure 5. Average BER-SNR performance curves of different channel equalization methods in uncoded 16QAM STBC system

The Fig. 6 shows that the pairwise error probabilities of proposed algorithm in uncoded system. We assume the transmission block length N=512, channel memory length L=152, so the ratio of N/L is about 10:3. The rate is respectively equal to 1, 2, 3, 4 bit/s/Hz. Then the transmission symbols adopt 2^R-PSK modulation for R value.

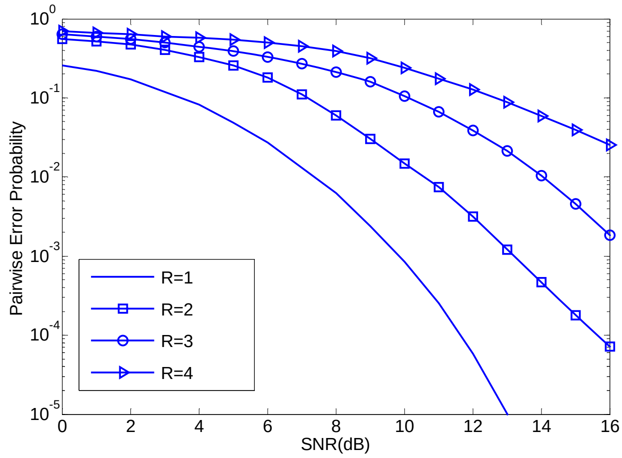

Figure 6. Achievable diversity oder for STBC transmission block with proposed channel equalization methods in uncode system

According to Fig. 7, the proposed method has better outage performance than linear equalization in 2×1 uncoded Alamouti transmission system. The diversity of ZF method is two for R in all rates. The full diversity of proposed method is achieved also regardless of the exact value of the rate. For $R \le \log\frac{N}{L}$, the diversity of MMSE is equal to two. When $R > \log\frac{N}{L}$, the diversity of MMSE is influence by rate value. For that case, the transmission block length N must less than the channel coherence time. So the practical performance of the equalizer is limited. At the outage probability level of 10^{-4} and the data rates R=1, 2, and 4 bit/s/Hz, the gain of the proposed method is about 1.2dB, 0.8dB and 0.4dB compared with the MMSE.

Compared with the ZF equalization, the performance gain is about 2.1 dB, 1.3dB and 0.5dB for the data rates R=1, 2, and 4 at the outage probability $P_{out}=10^{-4}$.

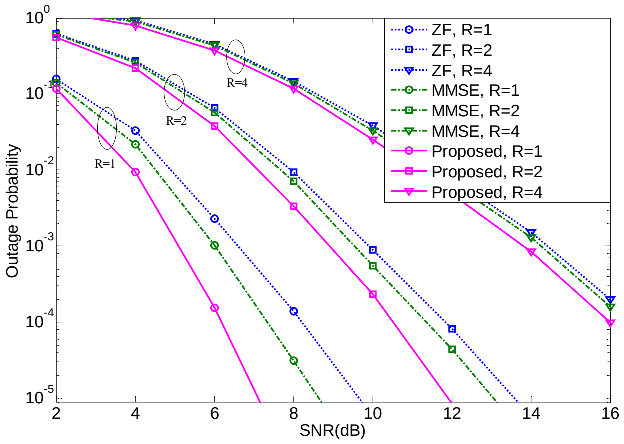

Figure 7. The outage probability versus SNR performance curves of different channel equalization methods in uncode system

V. Conclusion

In this paper, we give a novel channel equalization algorithm in order to defy frequency selective fading and improve the spectral efficiency in Alamouti scheme space time block coded SC-FDE systems. We design the filter structure of decision feedback equalizer in frequency domain and take parameters estimation. We also introduce outage and diversity analysis to measure the equalization precision and system performance. By use the mean square error criterion, the coefficients of FD-DFE is computed. The simulation results show that the performance of the proposed method is better than traditional linear equalization and hybrid structure equalization. Moreover, the complexity of the proposed algorithm is lower than traditional HDFE when the feedback taps of HDFE is large.

Acknowledgment

This paper is supported by National Natural Science Foundation and Civil Aviation Administration of China (Grant No. 61101122 and 61071104) and National Science and Technology Major Project (Grant No. 2011ZX03004-004), it is also supported by Technology on Information Transmission and Dissemination in Communication Networks Laboratory (Grant No. ITD-U12004/K1260010).

References

[1] F. Pancaldi, G. M. Vitetta, R. Kalbasi, N. Al-Dhahir, M. Uysal, and H. Mheidat, “Single-carrier frequency domain equalization,” IEEE Signal Process. Mag., vol. 25, no. 5, pp. 37–56, Sep. 2008.

[2] D. Kim, H.-M. Kim, and G.-H. Im, “Iterative channel estimation with frequency replacement for SC-FDMA systems,” IEEE Trans. Commun., vol. 60, no. 7, pp. 1877–1888, July 2012.

[3] J. Li, Y. Du, Y. Liu. “Comparison of Spectral Efficiency for OFDM and SC-FDE under IEEE 802.16 Scenarios,” Proceedings of the 11th IEEE Symposium on Computers and Communications.2006: 467 – 471.

[4] F. Hasegawa, A. Okazaki and H.Kubo, “Novel PAPR Reduction Scheme for SC-OFDM with Frequency Domain Multiplexed Pilots,” IEEE Commun. Let, Vol.16, pp. 1345-1348, Sep. 2012.

[5] J. Louveaux, L. Vandendorpe, and T. Sartenaer, “Cyclic prefixed single carrier and multicarrier transmission: bit rate comparison,” IEEE Commun. Letter., vol. 7, No. 4, pp. 180–182, Jul. 2003.

[6] J. Silva, R. Dinis and N. Souto , “Single-carrier frequency domain equalisation with hierarchical constellations: an efficient transmission technique for broadcast and multicast systems,” IEEE Commun. Iet , Vol.6, pp. 2065-2073, Jun. 2012.

[7] Y. Guo, L. Hu and T. Yuan, “Orthogonal wavelet transform dynamic weighted multi-modulus blind equalization algorithm based on dynamic particle swarm,” Journal of Computers (Finland), Vol.8, no. 8, pp. 2011-2017, Aug. 2013.

[8] R. Dinis., C.T. Lam, and D. Falconer, “Joint frequency-domain equalization and channel estimation using superimposed pilots,” IEEE Wireless Communications & Networking Conference, 2008: 447-452.

[9] D. Falconer, S. L. Ariyavisitakul, A. Benyamin-Seeyar, and B. Eidson, “Frequency domain equalization for single-carrier broadband wireless systems,” IEEE Commun. Mag., vol. 40, no. 4, pp. 58-66, April. 2002.

[10] N. Benvenuto and S. Tomasin, “Iterative design and detection of a DFE in the frequency domain,” IEEE Trans. Commun. vol.53, No.11, pp.1867–1875, Nov.2005.

[11] J. Chang, H. Wang and F. Ueng, “Iterative equalizer for OFDM-CDMA multiple access communication systems,” Journal of Software, vol. 5, no. 6, pp. 600–606, Jun. 2010.

[12] N. Al-Dhahir, “Single-carrier frequency-domain equalization for space time block-coded transmissions over frequency-selective fading channels,” IEEE Commun. Lett., vol. 5, no. 7, pp. 304–306, Jul. 2001.

[13] S. Zhou and G. Giannakis, “Single-carrier space-time block-coded transmissions over frequency-selective fading channels,” IEEE Trans. Inf. Theory, vol. 49, no. 1, pp. 164–179, Jan. 2003.

[14] Y. Zhu and K. B. Lataief, “Single-carrier frequency-domain equalization with decision-feedback processing for time-reversal space-time block coded systems,” IEEE Trans. Commun., vol. 53, no. 7, pp. 1127–1131, Jul. 2005.

[15] A. Tajer and A. Nosratinia, “Diversity order in ISI channels with singlecarrier frequency-domain equalizer,” IEEE Trans. Wireless Commun., vol. 9, no. 3, pp. 1022–1032, Mar. 2010.

[16] A.H Mehana. and A.Nosratinia, “Single-Carrier Frequency-Domain Equalizer with Multi-Antenna Transmit Diversity,” IEEE Trans. on Wireless Commun., Vol.12, pp. 388-397, Jan. 2013.

[17] S. M. Alamouti, “A simple transmit diversity technique for wireless communications,” IEEE J. Sel. Areas Commun., vol. 16, no. 8, pp. 1451–1458, Oct. 1998.

[18] D. R. Pauluzzi, N. C .Beaulieu, “A comparison of SNR estimation techniques for the AWGN channel,” IEEE Trans on Commun. vol. 48, pp. 1681–1691,Oct.2000.

[19] R. M. Gray, Toeplitz and circulant matrices: a review., available at http://ee.stanford.edu/~gray/toeplitz.pdf

Dr. Qifeng Zou received the B.S., M.S. degrees in communications engineering from the Heilongjiang University and Harbin University of Science and Technology in 2005, 2010, respectively. Now, he is currently working toward the Ph.D degree in communication research center, Harbin institute of technology (HIT). His major research interests are frequency domain channel equalization and channel estimation in single carrier communication system.

Prof. Xuezhi Tan received his ph.D. degree in 2005. He is a professor of electronic information engineering school in Harbin institute of technology. His main research interests include data communications, cognitive radio, mobile communications, trunking communication.

Prof. Mei Liu received her ph.D. degree in 2006. She is a professor of electronic information engineering school in Harbin institute of technology. Her main research interests include space communications, multi sensor data fusion, mobile communications, array signal processing.

Associate Prof. Yonggang Chi received his ph.D. degree in 2007. He is an associate professor of electronic information engineering school in Harbin institute of technology. His main research interests include spread spectrum communication, mobile communications, Satellite navigation technology.