Incorporating Transportation Cost into Joint Economic Lot Size For Single Vendor-Buyer

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***Abstract*— Considerable attention to coordinate the system between buyer and vendor has become an interesting issue to efficiently increase the performance of supply chain activities. Joint economic lot size model (JELS) has been introduced by many researchers as the spirit of coordinating the flow of material from the vendor to its downstream. As an inventory replenishment technique, JELS model is centered on reducing joint total cost of vendor and buyer by simultaneously deciding optimal delivery lot size, number of deliveries, and batch production lot. It is appropriate to take into account transportation costs as the function shipping weight and distance since delivery lot size has interrelated with shipping weight. Hence, this study constitutes an effort to develop the model of JELS by incorporating transportation cost. The solution procedure of the model is developed for solving two problems which are incapacitated and capacitated model. In addition, numerical examples were provided to illustrate the feasibility of the solution procedure in deriving optimal solution. The result presents central decision making which is useful for coordination and collaboration between vendor and buyer.**

***Index Terms*—Supply chain, Joint economic lot size, Transportation.**

I. Introduction

Supply chain management has been succeeded to divert the companies' attention, not to focus on internal organization only, but also should consider coordinating with their vendor. It is due to global market competitions which emphasize the companies to provide low cost strategy while still increasing customer service level [2]. The main element to cost reduction is an integration of supply chain stages such as procurement, production, and distribution that should be straightforward determined optimal solutions [3].

In the practice, most of procurement scenarios prove that vendor and buyer are treated independently for giving policy to order and production [4, 5, 6, 9]. It is important to acknowledge that if one party leads to an optimal solution to the decision, the other may get disadvantage. Unfortunately, if both parties still regulate separate decision, costly possibility will be occurred in inventory and distribution. Therefore, it is good to have a cooperative strategy in coordinating the flow of material from vendor to buyer.

One of the most well-known inventory techniques that represent coordination and collaboration between vendor and buyer is joint economic lot size (JELS) model. It was started by Goyal [1]. The objective of this model is to reduce the joint total cost between vendor and buyer. Rather than independent policy, joint policy resulting good agreement for inventory replenishment and considerable saving can be achieved. As planning tools, JELS models provide useful decisions in determining optimal policies which can be economically benefit for all parties involved in the supply chain. The important decision is usually pointing at delivery lot size, number of deliveries, and batch production lot.

Paying attention to delivery lot size, would be better to consider transportation as well. It is due to more than 50% of total annual logistic costs can be pointed toward transportation that known as freight cost [7]. Freight costs here are the function of shipping weight and distance [8]. Therefore, delivery lot size has a direct impact to the total freight cost of each shipment. In order to increase the degree of practical relevance as well, incorporating transportation cost into JELS model become very attracting to be developed and investigated, especially its effect in the determination of the optimal solution. In addition, it is important to note that the objective of the model is to reduce joint total cost between vendor and buyer.

As the remainder of this paper, the next section provides the literature related on joint economic lot size and transportation as a support for a new insight. Section III formulates the development of mathematical modeling of JELS by incorporating transportation cost and its solution procedures. Section IV presents numerical examples to illustrate and test the feasibility of the solution procedure of the model. The last section

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summarizes the results and recommendation for future research are given.

II. Literature Review

Traditionally, inventory replenishment decisions are treated independently from the viewpoints of vendor and buyer based on economic production quantity (EPQ) and economic order quantity (EOQ) respectively [4, 5, 6, 9]. In instance procurement cases, the optimal EOQ solution for the buyer was not acceptable to the vendor and otherwise as well. Instead of independent decision, the research studies about collaboration and coordination between vendor and buyer have been presented by useful inventory technique well known as joint economic lot size model.

The first model was initiated by Goyal [1]. The objective is to minimize joint total cost for both vendor and buyer problem by determining optimal lot size. Banerjee [9] generalized Goyal's model by assuming shipment policy based on the lot for lot basis under deterministic conditions. It means that deliveries can be carried out after vendor completing the production period. Unlike lot for lot policy, Banerjee and Kim [10] proposed better shipment policy by splitting the batch production lot size into sub-lot size so that the multiple-shipment can be executed during production period. It will be more economical for vendor by incurring a single setup cost for multiple shipment rather than lot for lot policy. This multiple-shipment policy is also more preferable in a manufacturing environment. Thereby, beside determination of lot sizes, the optimal number of shipments becomes important decisions as well to see how much batch production lot will be executed. Almost all the research regarding joint economic lot size has been extensively reviewed by [20].

As mentioned earlier that any determination of delivery lot size should then consider freight rate costs which are usually computed based on shipping weight and distance. Baumol and Vinod [11] research work was the first introduced inventory theoretic models as the integration of transportation and inventory costs. Whereas, the first incorporated freight rates function into lot sizing decision was Langley [12]. Actual shipping decision fall into three categories: 1) delivery which resulted in true truckload (TL) shipping quantities, 2) delivery which supposed to be over declare as TL, and 3) delivery are possibly shipped at less-than-truckload (LTL) [7]. Swenseth and Godfrey [13] have proposed freight rate function that emulates reality and better to represent actual freight rates such as freight rate functions such as proportional, adjusted inverse, constant, exponential, and inverse functions. Swenseth and Godfrey [7] hence studied the effect of joining freight function into an EOQ model for inventory replenishment decision. The study analyzed the combination of EOQ with inverse and adjusted inverse function. The result shows that inverse function is able to emulate freight rate for true TL and adjusted inverse function emulates LTL freight rates. Mendoza and Ventura [14] also have added quantity discount for EOQ model and transportation in which considering TL and LTL shipment. Mendoza and Ventura [15] studied estimation of freight rates in inventory replenishment decisions for vendor selection decision. Up to this explanation, the research concerned with incorporating transportation costs into inventory replenishment decision only focused on single stage model.

Now, consider incorporating transportation costs into JELS model. Nie, Xu, and Zhan [16] developed Kim and Ha [4]'s model by joining freight costs function of Swenseth and Godfrey [7] either inverse or adjusted inverse into JELS model. Chen and Sarker [17] considered a complex multi-stage of supply chain include multi- vendor and a single buyer. The study assumed that the milk run system was used to consolidate all the items from multi-vendor using a single truck under just-in-time (JIT) environments. Unlike previous freight cost function in [7], Chen and Sarker [17] revised the freight costs becomes the function of shipping weights and distances. It is due to the prior freight rates proposed by Swenseth and Godfrey [7] only focused upon the shipping weight, whereas distance assumed to be fixed. In the practical example, JIT system has characteristic on regulating frequent deliveries in small lot size, shorter lead time, and close vendor ties [8]. Thereby, it is better to take advantage on shipping distance computed in the freight rate as well since vendor should be closer to the main buyer. Since it is more appropriate to take into account of transportation as the function of shipping weight, and distance into inventory replenishment decision, this research proposes a development of JELS model. The objective is to minimize joint total cost between vendor and buyer by deciding optimal delivery lot size, number of deliveries, and batch production.

III. Problem Definitions

Suppose that a single vendor and single buyer have regulated long term JIT relationship. Taking the example in JIT practice, frequent delivery in smaller lot size becomes useful strategy to improve the system's performance of internal company where lower inventory cost constitutes its purpose. Nonetheless, the first problem is the imbalance lot sizing decision occurs between buyer and vendor. Since buyer leads to an optimal solution, the vendor should produce on required basis and certainly incurs high setup costs for each lot produced. On the other hand, high delivery cost will be incurred by the buyer because of conducting frequent delivery while reducing inventory cost. Coordination and collaboration between vendor and buyer should exist to overcome this problem. This study then proposed JELS model to reduce joint total cost of both parties by simultaneously optimizing delivery lot size, number of shipments, and batch production lots.

IV. Model Formulation

In this section, the mathematical models are elaborated to represent the coordination between buyer and vendor. This model consists of buyer and vendor cost functions.

Firstly, it will be described independently. Furthermore, JELS is presented as the composite of those cost functions. In addition, the solution procedures are also proposed to find the optimal solutions. There are two solution procedures to solve regarding transportation problems such as incapacitated and capacitated. Incapacitated problem discusses to actual shipping weight does not exceed capacity truckload. Otherwise, capacitated problem occurs if actual shipping weight exceeds the capacity of a truckload. Hence, the following assumptions and notations are introduced as:

A. Assumptions and Notations

To boundary the research, these assumptions are adopted from Kim and Ha [4] and its additional described as follows:

1) Demand, production, and delivery lead time assumed to be deterministic and constant.
2) No shortages and backordered allowed.
3) No quantity discount.
4) Decision variables are delivery lot size, number of shipments, and batch production lot.
5) The shipment policy can be carried out during production period.
6) The performance of the supply chain indicator is joint total cost.

Notations of the model are presented as below:

D	Demand rate of the buyer.
P	Production rate of the vendor $P > D$.
S	Setup cost per setup of the vendor.
A	Ordering cost per order of the buyer.
H_b	Holding costs per unit inventory per period of the buyer.
H_v	Holding costs per unit inventory per period of the vendor, $H_v < H_b$
F_o	Transportation costs per shipment.
w	Weight of a unit part (lbs).
d	Transportation distance (miles).
α	Discount factor for LTL shipments
F_x	The freight rate in dollar per pound for a given per mile for full truckload (FTL) product.
F_y	The freight rate in dollar per pound for a given per mile for the partial load.
W_x	Full truckload (FTL) shipping weight (lbs).
W_y	Actual shipping weight (lbs).
Q	Production lot size of parts of vendor (unit) $Q = qm$.
q	Delivery lot size of buyer (unit), $q = Q/m$.
m	Number of shipments.
$F(D,q,w,d)$	Total freight cost as the function of shipping weight and distance
TC_b	Total cost of the buyer.
TC_v	Total cost of the vendor.
JTC	Joint total cost of the systems.

B. Model Formulation

Previously, it is stated that JELS model composites of vendor and buyer cost functions. Let firstly discusses on vendor cost function. Vendor cost function consists of setup cost, fix transportation for preparing and receiving the material, and inventory cost respectively. In addition, the inventory cost is derived from Jogklekar [18]. The expression of vendor cost function is given as

$$TC_v(q,m) = \frac{D}{qm}S + \frac{D}{q}F_0 + \frac{q}{2}\left(m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right)H_v \quad (1)$$

Buyer cost function includes ordering cost, holding cost, and variable transportation cost or freight rates as the function of weight and distance. The first two terms are basically obtained from EOQ model. The third term, recall to JIT practice in which regulating frequent delivery in smaller lot size, emphasizing the buyer to use less-than truckload (LTL) shipment. One of freight rates function that able to emulate LTL shipment is adjusted inverse function [7, 13]. Nonetheless, the adjusted inverse function in previous research constitutes the function of shipping weight, then need to redefine it by adding the shipping distance parameter. Firstly needs to determine the freight rate for partial load F_y based on adjusted inverse function given as [9].

$$F_y = F_x + \alpha F_x\left(\frac{W_x - W_y}{W_y}\right) \quad (2)$$

Where α has interval from 0 to 1 that indicated as a discount factor for LTL shipments to increase the freight rate per pound over a given distance as W_y increase. Moreover, estimation of total freight cost per period as the function of shipping weight and distance for adjusted inverse yields

$$F(D,q,w,d) = \frac{D}{q}\alpha F_x W_x d + Ddw(1-\alpha)F_x \quad (3)$$

Hence, the buyer cost function can be formulated as:

$$TC_b(q) = \frac{D}{q}A + \frac{q}{2}H_b + F(D,q,w,d) \quad (4)$$

So far, both cost functions have been clearly introduced. Actually, either vendor or buyer cost functions have own optimal solution that can minimize his independent total cost. It is indicated that one party will get an advantage when one of them leads an optimal solution. Before moving toward JELS model, it is better to know the vendor and the buyer inventory status that shown in Fig. 1 which is adopted from [4]. It can be seen that the inventory status of vendor regulates multiple shipment that can be executed during production period. By combining Eqs. (1) and (4) then JELS model can be obtained.

$$JTC(q,m) = \frac{D}{qm}S + \frac{D}{q}(A+F_0) + \frac{q}{2}\left(\left(m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)H_v + H_b\right) + F(D,q,w,d) \quad (5)$$

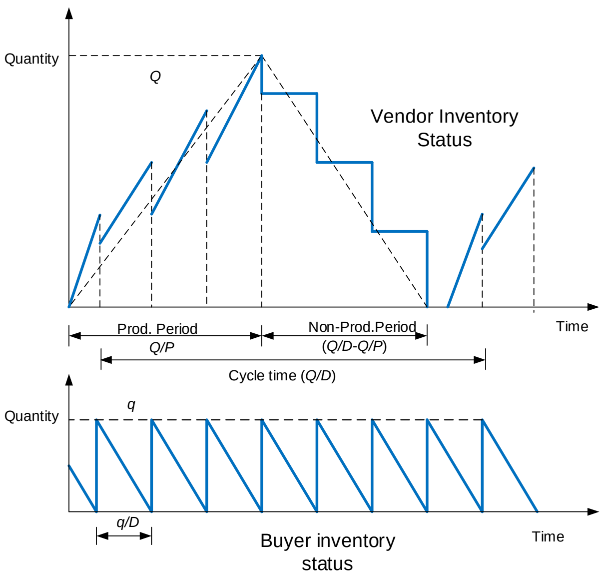

Figure 1. Vendor and buyer inventory status

Where $F(D,q,w,d)$ could be computed by querying actual freight rates to obtain total actual transportation cost. Meanwhile, if joint total cost is considered based on adjusted inverse function, Eq. (5) can be revised as

$$JTC(q,m) = \frac{D}{qm}S + \frac{D}{q}(A+F_0) + \frac{q}{2}\left(\left(m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)H_v + H_b\right) + \frac{D}{q}\alpha F_x W_x d + Ddw(1-\alpha)F_x \quad (6)$$

Subject to $(q)>0$, $(m)>0$, and integer value.

In order to obtain an optimal solution, take the first derivatives of Eq. (6) with respect q equal to zero.

$$\frac{\partial JTC(q,m)}{\partial q} = -\frac{D}{q^2}\left(A+\frac{S}{m}+F_0+\alpha F_x W_x d\right) + \frac{1}{2}\left(\left(m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)H_v + H_b\right) = 0 \quad (7)$$

Then resulting optimal q as,

$$q^* = \sqrt{\frac{2D\left(A+\frac{S}{m}+F_0+\alpha F_x W_x d\right)}{\left(m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)H_v + H_b}} \quad (8)$$

In order to test the convexity of joint total cost $JTC(q,m)$ respect to q for fixed value of m, hence taking second derivatives of Eq. (6).

$$\frac{\partial^2 JTC(q,m)}{\partial q^2} = \frac{2D}{q^3}\left(A+\frac{S}{m}+F_0+\alpha F_x W_x d\right) > 0 \quad (9)$$

Substitute Eq. (8) into Eq. (6) then obtained optimal joint total cost as the function of number deliveries is as follows:

$$JTC(m) = \left[2D\left(A+\frac{S}{m}+F_0+\alpha F_x W_x d\right) + \left(\left(m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)H_v + H_b\right)\right]^{\frac{1}{2}} \quad (10)$$

By taking first partial derivatives of Eq. (10) with respect to m equal to zero, then optimal number deliveries can be written as

$$m^* = \sqrt{\frac{S\left(\left(\frac{2D}{P}-1\right)H_v + H_b\right)}{\left(A+F_0+\alpha F_x W_x d\right)\left(1-\frac{D}{P}\right)}} \quad (11)$$

The number of deliveries m constitutes positive and integer value [6]. Therefore the value of (m) should meet the requirements likewise

$$m_1^* = \sqrt{\frac{S\left(\left(\frac{2D}{P}-1\right)H_v + H_b\right)}{\left(A+F_0+\alpha F_x W_x d\right)\left(1-\frac{D}{P}\right)}}, \text{for } JTC\left(m_1^*\right) \le JTC\left(m_1^*+1\right) \quad (12)$$

Or

$$m_2^* = \sqrt{\frac{S\left(\left(\frac{2D}{P}-1\right)H_v + H_b\right)}{\left(A+F_0+\alpha F_x W_x d\right)\left(1-\frac{D}{P}\right)}} + 1, \text{for } JTC\left(m_2^*\right) \le JTC\left(m_2^*-1\right) \quad (13)$$

In order to test the convexity of joint total cost $JTC(q,m)$ respect m to for fixed value of q, then taking second derivatives of Eq. (6).

$$\frac{\partial^2 JTC(q,m)}{\partial m^2} = \frac{2DS}{m^3 q} > 0 \quad (13)$$

Refer to practical relevance; it might be emerged two problems regarding shipment policy. The problems are incapacitated model and capacitated model that has already discussed above. Recall to incapacitated problem, it is condition where actual shipping weight is not larger than capacity truckload $W_y \le W_x$, otherwise capacitated problem occurs when actual shipping weight larger than capacity truckload $W_y > W_x$. Incapacitated are simply problem if the buyer provided larger truck. On the other hand, the Lagrangian relaxation approach is required to solve capacitated problem that has included in the solution procedures. Hence, in order to determine the optimal solution, the following solution procedures are introduced to solve two problems of shipment condition are described as:

Step 0: Determine the number of deliveries m^* using Eq. (11), and set it as an integer value.

Step 1: Set value of m^* using Eqs.(12) and (13), then substituting m_1^* and m_2^* into Eq. (10). Choose optimal number of deliveries m^* to get minimum $JTC(m)$.

Step 2: Determine optimal order quantity q^* using Eq. (8) For fixed m^*, and set it as an integer value.

Step 3: With q^* and m^* are optimal solutions. Then find minimum joint total cost without actual freight rates using Eq. (6)

Step 4: Evaluate actual shipping weight $W_y = qw$. If truckload restriction $W_y \le W_x$ is satisfied, then compute joint total cost using Eq. (5), where total transportation cost is computed by querying actual freight rates. Otherwise, do next step if truckload constraint is not satisfied $W_y > W_x$.

Step 5: Revised number of deliveries.

$$\Delta W = q^* w - W_x$$

$$\Delta m = \frac{m^* \Delta W}{W_x} \text{ and } m^{**} = m^* + \Delta m$$

Step 6: Revised delivery lo size.

$$q^{**} = W_x \frac{q^*}{q^* w} \text{ and } Wy^* = q^{**} w$$

The value of q^{**} is rounded to integer

Step 7: Recomputed joint total cost $JTC\left(q^{**}, m^{**}\right)$ using Eq. (5) where transportation cost is also computed using querying actual freight rates.

V. Numerical Example

In order to illustrate the effectiveness of the solution procedure, a numerical example is presented. The numerical example here includes actual freight rate schedule and parameter data. The actual freight rate schedule is adopted from Swenseth and Godfrey [7]. Since the previous freight rates are only the function of shipping weight and because of under study consider distance parameter, then it needs to redefine by considering the shipping distance. Table I presents the new freight rate schedule by considering shipping weight and distance. The essence of the prior data still exists such as a constant charge per pound and constant charge per shipment that signed by *. The data are redefined from the freight rate per pound to freight rate per pound per mile. For instance, assumed that distance can be delivered up to 600 miles and on the ranges 4,750-9,999 lb considered as a constant charge per shipment. So that freight rate per pound per mile is obtained by dividing freight rate per shipment with the highest break point and distance. Unlike the ranges of 10,000-18,000 lb are considered as a constant charge per pound. To change it into in freight rate per pound per mile can be computed by dividing freight rate per pound with distance. In addition, Swenseth and Godfrey [7] mentioned that some weights are exist and when multiplied by its corresponding freight rate will provide the same total cost as that for the next weight break. These concepts provide a choice to over-declared as TL or well-known by indifference points that used by shipper to obtain a lower total transportation cost. This is accomplished by artificially inflating the weight to a higher weight breakpoint resulting in a lower total cost [7].

TABLE I.
FREIGHT RATE SCHEDULE AS THE FUNCTION OF SHIPPING WEIGHT AND DISTANCE

Weight break	F_x /pound	F_x /pound/mile
1-227 lb*	$40	$0.000293685
228-420 lb	$0.176/lb	$0.000293333
421-499 lb*	$74	$0.000247161
500-932 lb	$0.148/lb	$0.000246666
933-999 lb*	$138	$0.000230230
1,000-1,855 lb	$0.138/lb	$0.000230000
1856-1,999 lb*	$256	$0.000213440
2,000-4,749 lb	$0.128/lb	$0.000213333
4,750-9,999 lb*	$608	$0.000101343
10,000-18,256 lb	$0.0608/lb	$0.000101333
18,257-46,000 lb*	$1110	$0.000040217

TABLE II
PARAMETER DATA

Parameter	Value
D	10,000 units
P	40,000 units
A	$30/order
S	$3,600/setup
H_b	$45/unit/year
H_v	$38/unit/year
F_0	$50/unit/year
α	0.11246
w	22 lb
d	600 miles
F_x	$0.0000402174/lb/miles
W_x	46,000 lb

In order to calculate total actual freight rate by considering shipping weight and distance for constant charge per pound, actual shipping weight is then just multiplied by freight rate per pound per mile and distance. Conversely, to calculate total actual freight cost for constant charge per shipment, then the highest weight breakpoints is multiplied by freight rate per pound per mile and distance. Furthermore, Table II also presents information data of vendor and buyer. The data were obtained from Swenseth and Godfrey [7], Kim and Ha [4], and Nie, Xu, and Zhan [16].

Let consider testing the proposed model using data provided. By taking the first example of incapacitated problem and applying the solution procedure, so obtained the result obtained is as follows:

Step 0: Number of deliveries m^* is 4

Step 1: $JTC(m_1 = 4)$=\$55,619.44

$JTC(m_2 = 5)$=\$55,827.18.

$JTC(m_2 = 5)$ - $JTC(m_1 = 4)$=\$207.74.

Step 2: Optimal delivery lot size q^*=397 units.

Step 3: Minimum joint total cost without actual freight rates $JTC(q^*, m^*)$= \$60,331.08.

Step 4: Actual shipping decision $W_y = q\,w$ = 8,734 lb, $W_y \le W_x$. Minimum joint total cost by considering actual freight rates $JTC(q^*, m^*)$= \$67,790. It is feasible solution and procedure stops.

Elaborating the result above shows that, optimal delivery lot size q^* = 397 units, the number of deliveries m^* = 4, batch production lot Q^* =1, 588 units. Actual shipping weights of 397 units were 8,734 lbs and fall into a corresponding freight rate of \$0.000101343 which yielding variable transportation cost per shipment \$0.000101343$\times$ 9,999 lb$\times$600 miles= \$608. This actual shipping quantity can be over-declared as truckload (10,000 lb). Shipping weight of 8,734 lbs resulting the same freight rate as the next weight break-point. The annual number of shipments was 10,000 units/ 397units= 25.188 and total actual variable transportation cost 25.188$\times$\$608= \$15,314.86. Based on optimal solutions, obtain minimum join total cost without considering actual freight rates is \$60,331.08 while the minimum joint cost by considering actual freight rates is \$67,790. A comparison between the joint total cost with and without actual freight rates are deviated around 11%.

Now try to solve the second problem of shipping condition that is capacitated problem. Having restriction of capacity truckload might be resulting actual shipping weight exceed the capacity of a truckload. To overcome this problem, the Lagrangian relaxation method is then required. This approach also aimed to try relaxing the solution in order to adjust it to the capacity of truckload while still reducing joint total cost. Assumed that truckload capacity W_x is revised as 5,000 lb, freight rate F_x changes to be \$0.000101343 and the other parameter data remain unchanged. Hence, the result of illustrating the solution procedures to find optimal solution can be seen as follows:

Step 0: Number of deliveries m^* is 4

Step 1: $JTC(m_1 = 5)$=\$53,021.

$JTC(m_2 = 6)$=\$53,046.

$JTC(m_2 = 6)$ - $JTC(m_1 = 5)$=\$25..

Step 2: Optimal delivery lot size q^*=315 units.

Step 3: Minimum joint total cost without actual freight rates $JTC(q^*, m^*)$= \$64,893.87..

Step 4: Actual shipping decision $W_y = q\,w$ =6,930 lb, $W_y > W_x$. It is not feasible solution and do the next step.

Step 5: The revised number of deliveries m^{**} =7

Step 6: Revised the delivery lot size q^{**} 227 units and shipping weight W_y =4,994, $W_y \le W_x$. The solution is feasible and approximate to the optimal solution.

Step 7: Recalculate joint total cost $JTC(q^{**}, m^{**})$ = \$78,558.37. Procedures stop.

It can be observed that delivery lot size decrease as the number of deliveries increase and the solution becomes feasible. Therefore, the Lagrangian relaxation method provides a good solution where truckload limitation can be solved. The joint total cost for capacitated problem increase 15.88%, compared to incapacitated problem.

A sensitivity analysis is carried out for this model to study how the delivery lot size, number of shipments and total cost of the system are affected due to the change parameters: S, H_b, F_x, α. The values of S, A, H_b, F_0, F_x, α will be varied from 25% to 100% to test the impact of parameters. And the other parameters remain unchanged. The system performance to be analyzed here use joint total cost without considering actual freight because it used to see the behavior of true model before connected to actual freight rate. The result of sensitivity analysis is presented in Table III.

The optimal order quantity, number of deliveries, and joint total cost are dependent on the various cost parameters. Also, as reminder, batch production lot has not analyzed its effect due to the change of parameter because it is just the function of delivery lot size and number of deliveries. How sensitive one parameter to the joint total cost is can be determined by calculating $\Delta JTC = JTC^* - JTC / JTC \times 100\%$ where JTC is the initial joint total cost. There are three categories to determine about how sensitive the joint total cost of the change of parameters such as slightly sensitive, moderately sensitive, and highly sensitive [19]. If the deviation ΔJTC on average less than 2%, between 2-3%, and above 3% so it is considered as slightly sensitive, moderately sensitive, and highly sensitive respectively. In addition, the rate of changes of parameters can be seen and observed whether the solution is directly related or inversely related to the parameter over feasible range.

TABLE III
SENSITIVITY ANALYSIS

Parameter	Changed Value	q^*	m^*	JTC ($)	ΔJTC (%)
S =3600	3600	397	4	60331.09	0.00
	4500	362	5	65730.28	8.95
	5400	391	5	70513.47	16.88
	6300	417	5	74971.69	24.27
	7200	378	6	79109.43	31.13
A =30	30	397	4	60331.09	0.00
	37.5	399	4	60519.56	0.31
	45	400	4	60707.37	0.62
	52.5	401	4	60894.59	0.93
	60	403	4	61081.17	1.24
H_b =45	45	397	4	60331.09	0.00
	56.25	321	5	62372.39	3.38
	67.5	311	5	64149.41	6.33
	78.75	264	6	65656.6	8.83
	90	231	7	67084.91	11.19
F_0 =50	50	397	4	60331.09	0.00
	62.5	400	4	60644.87	0.52
	75	402	4	60956.84	1.04
	87.5	404	4	61267.11	1.55
	100	406	4	61575.71	2.06
F_x =0.00004021	0.00004021	397	4	60331.09	0.00
	0.00005027	403	4	62289.04	3.25
	0.00006032	408	4	64236.37	6.47
	0.00007038	518	3	66051.49	9.48
	0.00008043	524	3	67828.23	12.43
α =0.1124	0.1124	397	4	60331.09	0.00
	0.1406	403	4	60961.88	1.05
	0.1687	408	4	61582.05	2.07
	0.1968	518	3	62070.01	2.88
	0.2249	524	3	62519.59	3.63

The rate of changing a parameter of the optimal solution can be observed by taking the partial derivatives of $\left(q^*\right)$ and $\left(m^*\right)$ with respect to S, A, H_b, H_v, F_0, F_x, α by mathematical expression are $\left\{\frac{\partial q^*}{\partial S}, \frac{\partial q^*}{\partial A}, \frac{\partial q^*}{\partial H_b}, \frac{\partial q^*}{\partial F_0}, \frac{\partial q^*}{\partial F_x}, \frac{\partial q^*}{\partial \alpha}\right\}$ and $\left\{\frac{\partial m^*}{\partial S}, \frac{\partial m^*}{\partial A}, \frac{\partial m^*}{\partial H_b}, \frac{\partial m^*}{\partial F_0}, \frac{\partial m^*}{\partial F_x}, \frac{\partial m^*}{\partial \alpha}\right\}$. All the rate of direction of change of solution can be seen in the figure placed on Appendix B. The result of Table III can be described as follows:

1) Effect of setup cost S to the solutions and joint total cost.
   It can be observed that the above feasible range from 25% to 100% of setup cost S, the deviation ΔJTC of the joint total cost is highly sensitive to setup cost. Moreover, the solution $\left(q^*, m^*\right)$ and joint total cost JTC are directly related to setup cost. It means that the value of optimal solution and joint total cost increase as setup cost increase. The rate of direction of change of solution due to parameter S can be seen by mathematical expression $\left\{\frac{\partial q^*}{\partial S}, \frac{\partial m^*}{\partial S}\right\}$.
2) Effect of ordering cost A to the solution and joint total cost.
   It can be noted that the deviation of ΔJTC of the joint total cost is slightly sensitive and directly related to ordering cost A. Based on the rate of direction of change of solution due to parameter A by mathematical expression $\left\{\frac{\partial q^*}{\partial A}, \frac{\partial m^*}{\partial A}\right\}$, it can be inferred that delivery lot size q^* is directly related to parameters A and number of deliveries m^* is inversely related to the parameter A. Inversely relationship means that number of deliveries decrease as ordering cost increase.
3) Effect of holding cost H_b to the solution and joint total cost.
   Based on the result in Table III shows that the deviation ΔJTC of the joint total cost is highly sensitive and directly related to ordering cost H_b. However, it can be summarized from the rate of direction of change of solution $\left\{\frac{\partial q^*}{\partial H_b}, \frac{\partial m^*}{\partial H_b}\right\}$ due to the parameter H_b that delivery lot size q^* is inversely related to parameters A and number of deliveries m^* is directly related to the parameter A.
4) Effect of fix transportation F_0 to the solution and joint total cost.
   Above feasible range from 25% to 100% of fix transportation shows that the deviation ΔJTC of the

joint total cost is slightly sensitive and directly related to fix transportation F_0 . Also, delivery lot size is directly related to fix transportation F_0 . Otherwise the number of deliveries is inversely related to fix transportation F_0 . The rate of direction of change of parameters can be seen by mathematical expression $\left\{\frac{\partial q^*}{\partial F_0}, \frac{\partial m^*}{\partial F_0}\right\}$.

5) Effect of freight rate cost F_x to the solution and joint total cost.
It can be observed that above a feasible range from 25% to 100% of freight rate shows that the deviation ΔJTC of the joint total cost is highly sensitive and directly related to freight cost. The rate of direction of change of solution can be seen by mathematical expression $\left\{\frac{\partial q^*}{\partial F_x}, \frac{\partial m^*}{\partial F_x}\right\}$ and inferred that delivery lot size is directly related to freight rate. Otherwise the number of deliveries is inversely related freight rate.

6) Effect of discount factor α to the solution and joint total cost.
The deviation ΔJTC of the joint total cost is moderately sensitive and directly related to discount factor α . Based on the rate of direction of change of solution $\left\{\frac{\partial q^*}{\partial \alpha}, \frac{\partial m^*}{\partial \alpha}\right\}$, can be observed that delivery lot size is directly related to α . Otherwise the number of deliveries is inversely related freight rate α .

VI. Conclusion

One of the important points in this study is synchronization of the production flow from vendor to buyer. Joint economic lot size (JELS) model has proved its ability to represent the coordination and collaboration for both parties as well as managing the flow of inventory replenishment by simultaneously determining optimal delivery lot size and number of deliveries. It has been emphasized refer to the importance of considering the transportation cost into inventory replenishment decisions in the supply chain system. Transportation cost has been included to determine optimal delivery lot size and number of deliveries. Thus, by incorporating transportation cost into JELS model, lower joint total cost as a composite of inventory cost, setup cost, ordering cost, fix transportation cost and freight rate cost can be achieved. The advantages from the impact of the total cost reduction, provide the economic benefit that can be shared by both parties through bargaining and negotiation process which significantly improve the relationship of vendor and buyer. It is also contributing a useful insight for improving the performance of the supply chain.

For further research, firstly, since the number of demand and production are assumed to be deterministic; the research may be extended by changing the demand and production to be dynamic. It requires some methods to face uncertainty in demand and production like artificial intelligence. Secondly, in the practice, consignment can be transported using TL or LTL shipment. In order to provide a complete range of shipping decision, this research also needs to consider TL shipment. Therefore, a company can decide which mode of transportations that efficiently meet the requirement of the company. Third, quality is always become important issues to increase customer service level. In the most practical issues in a manufacturing environment, items that shipped by a vendor do not qualified 100% accepted. So that, incorporating quality issues into inventory replenishment decision becomes more attracting to be developed. Lastly, to be more practical, then the research also can be extended by considering multiple actors, multiple items, and shortage items.

Appendix A Convexity Test of Joint Total Cost Using the Hessian Matrix

Proof. Taking the second derivatives of JTC with respect to q and m.

$$\frac{\partial^2 JTC(q,m)}{\partial q^2} = \frac{2D}{q^3}\left(A + \frac{S}{m} + F_0 + \alpha F_x W_x d\right) > 0 \quad \text{A.1}$$

$$\frac{\partial^2 JTC(q,m)}{\partial m^2} = \frac{2DS}{m^3 q} > 0 \quad \text{A.2}$$

$$\frac{\partial^2 JTC(q,m)}{\partial q \partial m} = \frac{DS}{q^2 m^2} + \frac{1}{2}\left(1 - \frac{D}{P}\right) H_v \text{ ,With } P > D \quad \text{A.3}$$

Thus,

$$\left[\frac{\partial^2 JTC(q,m)}{\partial q^2}\right]\left[\frac{\partial^2 JTC(q,m)}{\partial m^2}\right] - \left[\frac{\partial^2 JTC(q,m)}{\partial q \partial m}\right]^2 \quad \text{A.4}$$

$$= \frac{4D^2 AS}{m^3 q^4} + \frac{4D^2 F_0 S}{m^3 q^4} + \frac{4D^2 \alpha F_x W_x dS}{m^3 q^4} + \frac{4D^2 S^2}{m^4 q^4} - \frac{D^2 S^2}{m^4 q^4} - \frac{DS}{m^4 q^4}\left(1 - \frac{D}{P}\right) H_v + \frac{1}{4}\left(1 - \frac{D}{P}\right)^2 H_v{}^2$$

For condition $\lim_{D \to P} \frac{D}{P} = 1$, then obtained

$$= \frac{4D^2 AS}{m^3 q^4} + \frac{4D^2 F_0 S}{m^3 q^4} + \frac{4D^2 \alpha F_x W_x dS}{m^3 q^4} + \frac{3D^2 S^2}{m^4 q^4} > 0 \quad \text{A.5}$$

Since $\forall$ D , A , m , F_0 , F_x , W_x , d > 0 , therefore JTC is a convex function at point (q,m). The proof is satisfied.

Appendix B The Rate of Direction of Change of Solution to Parameter.

B1. The rate of direction of the solution q^* and m^* to the parameter setup cost S

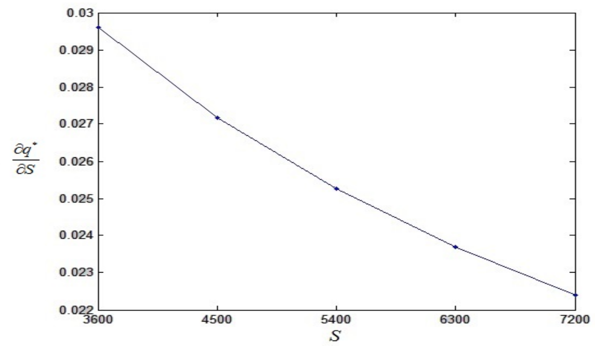

Figure 2. The rate of direction of the solution q^* to the S

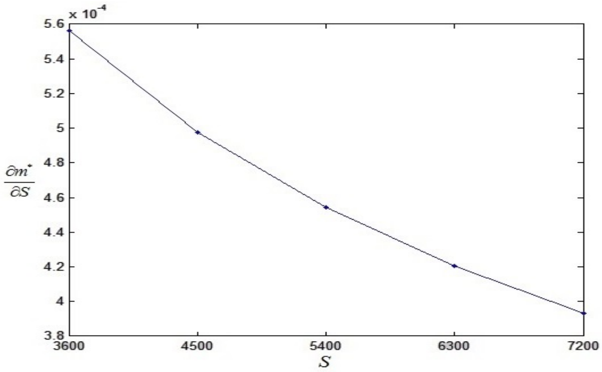

Figure 3. The rate of direction of the solution m^* to the S

B2. The rate of direction of the solution q^* and m^* to the ordering cost A

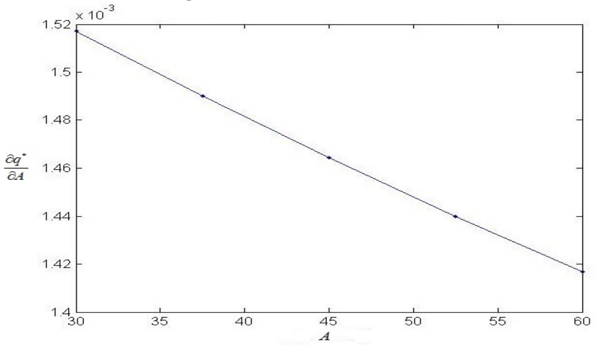

Figure 4. The rate of direction of the solution q^* to the A

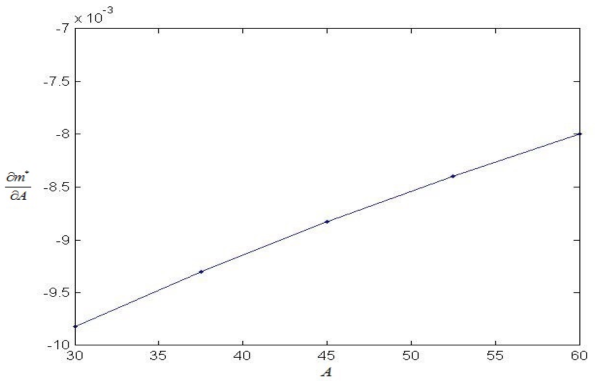

Figure 5. The rate of direction of the solution m^* to the A

B3. The rate of direction of the solution q^* and m^* to the holding cost H_b

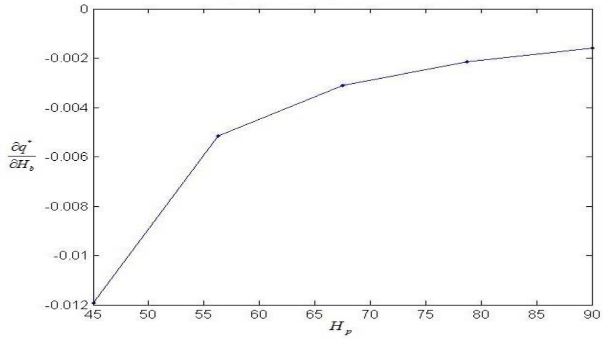

Figure 6. The rate of direction of the solution q^* to the H_b

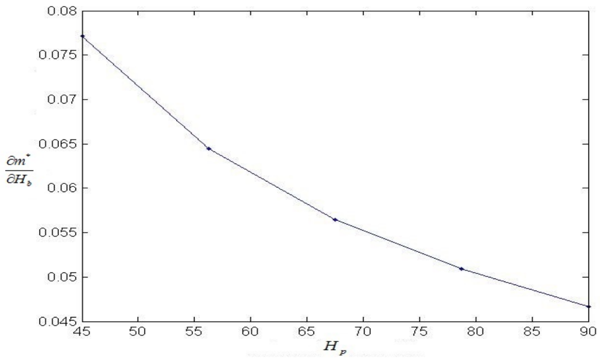

Figure 7. The rate of direction of the solution m^* to the holding cost

B4. The rate of direction of the solution q^* and m^* to the fix transportation cost F_0

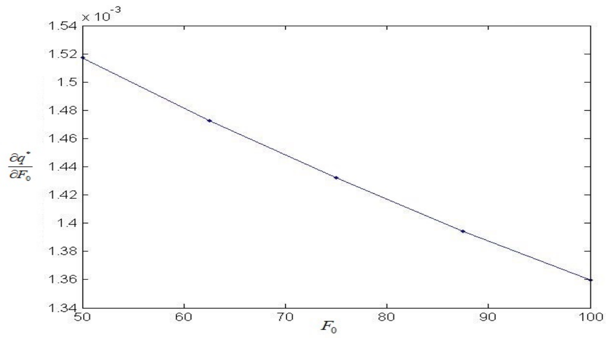

Figure 8. The rate of direction of the solution q^* to the F_0

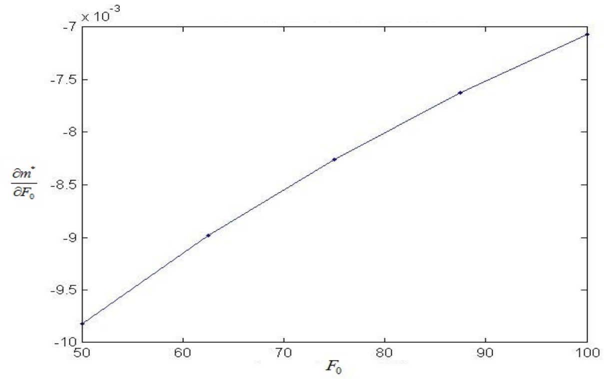

Figure 9. The rate of direction of the solution to m^* the F_0

B.5 The rate of direction of the solution q^* and m^* to the freight rate F_x

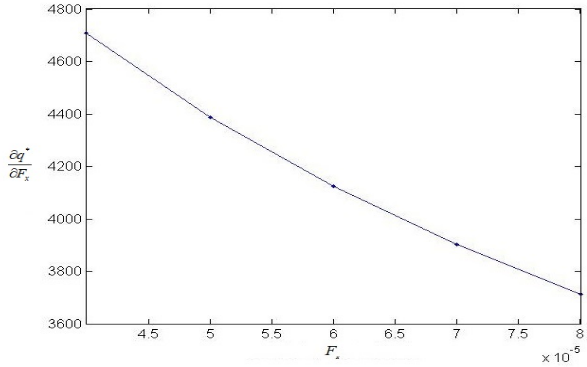

Figure 10. The rate of direction of the solution to q^* the F_x

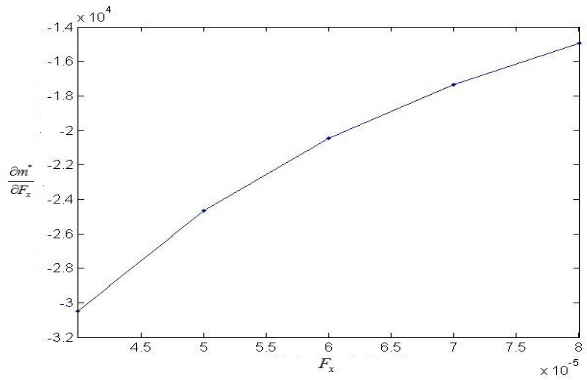

Figure 11. The rate of direction of the solution to m^* the F_x

B.6 The rate of direction of the solution q^* and m^* to the discount factor α

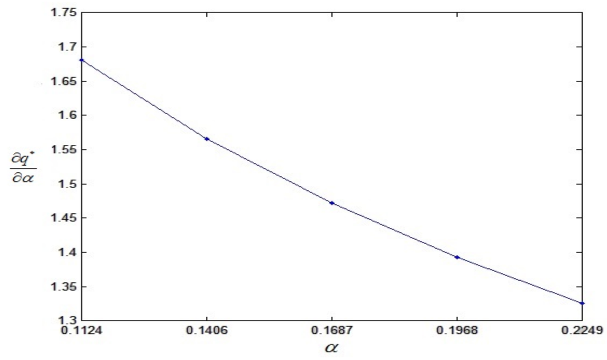

Figure 12. The rate of direction of the solution to q^* the α

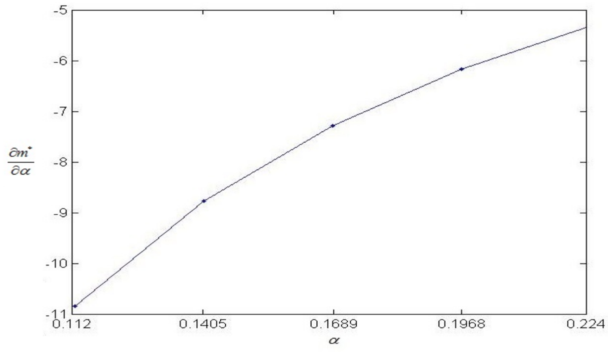

Figure 13. The rate of direction of the solution to m^* the α

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REFERENCES

[1] S. K. Goyal, "An integrated inventory model for a single vendor and single customer problem," *International Journal Production Research*, Vol. 15, pp. 107–111, 1977.

[2] D. J. Thomas and P. M. Griffin, "Coordinated supply chain management," *European Journal of Operational Research*, Vol. 94, pp. 1–15, 1996.

[3] H. Stadler, "Supply chain management and advanced planning-basics overview and challenges," *European Journal of Operational Research*, Vol. 163, pp. 575-588, 2005.

[4] S. L. Kim and D. Ha, "A JIT lot splitting model for supply chain management: Enhancing buyer-vendor linkage." *International Journal Production Economics,* Vol. 86, pp. 1-10, 2003.

[5] Yang, P.C., Wee, H.M., and Yang, H.J. "Global optimal policy for vendor-buyer integrated inventory system within just in time environment." *Journal Global Optimal,* Vol. 37, pp. 505-511, 2007.

[6] Sucky, E. "Inventory management in supply chains: A bargaining problem." *International Journal Production Economics,* Vol. 93, pp. 253-262, 2005.

[7] S. R. Swenseth and M. R. Godfrey, "Incorporating transportation costs into inventory replenishment decisions," *International Journal Production Economics*, Vol. 77, pp.113–130, 2002.

[8] S. R. Swenseth, and F. P. Buffa, "Just-in-Time: Some effects on the logistic function," *The International Journal of Logistic Management,* Vol. 1, pp. 25-34, 1990.

[9] A. Banerjee,"A joint economic lot size model for buyer and vendor," *Decision sciences*, Vol. 17, pp. 292–311, 1986.

[10] A. Banerjee and S. L. Kim, " An integrated JIT inventory model," *International Journal of Operation & Production Management*. Vol, 15, pp. 237-244, 1995.

[11] W. J. Boumul and H. D. Vinod,"An inventory theoretic model of freight transport demand." *Management Science*, Vol. 16, pp. 413-421, 1970.

[12] J. C. Langley,"The inclusion of transportation costs in inventory models: some consideration." *Journal of Business Logistic*, Vol. 2, pp. 106-125, 1980.

[13] S. R. Swenseth and M. R. Godfrey ,"The estimating freight rates for logistic decisions." *Journal of Business Logistic*, Vol. 17, pp. 213-231, 1996.

[14] A. Mendoza and J. A. Ventura," Incorporating quantity discounts to the EOQ model with transportation costs," International Journal Production Economics, Vol. 113, pp. 754-765, 2008.
[15] Mendoza, A., and Ventura, J.A. "Estimating freight rates in inventory replenishment and vendor selection decisions." *Logistic Research*, Vol. 1, pp. 185-196, 2009.
[16] L. Nie, X. Xu, and D. Zhan, "Incorporating transportation costs into JIT lot splitting decision for coordinated supply." *Journal of Advance Manufacturing Systems*, Vol. 5, pp. 111-121, 2006.
[17] Z. X. Chen and B. R. Sarker,"Multi-vendor integrated procurement-production system under shared transportation and Just-in-Time delivery system." *Journal of the Operation Research Society*, Vol. 61, pp. 1654-1666, 2010.
[18] P. N. Joglekar, "Comments on 'A quantity discount pricing model to increase vendor profits'." *Management Science*, Vol. 34, pp. 1391-1398, 1988.
[19] C. H. Su,"Optimal replenishment policy for integrated inventory system with defective items and allowable shortages under trade credit," *International Journal Production Economics*, Vol. 139, pp. 247-256, 2012.
[20] C. H. Glock,"The joint economic lot size problem: A review,"*International Journal Production Economic,* Vol. 135, pp.671-686, 2012.