An Approach to Multi-attribute Group Decision Making and Its Application to Project Risk Assessment

Zhiguang Zhang
Department of Mathematics, Dezhou University, Shandong 253023, China
zhiguangzhang@126.com

Abstract—**Multiple attribute decision making is an important part of modern decision science. It has been extensively applied to various problems. There are several methods for this decision making problem. Based on TOPSIS method and Gray relational analysis method, this paper presents a combined approach to the multi-attribute group decision making. Finally, an illustrative example of evaluating projects risk assessment is given to show the feasibility and effectiveness of the developed method.**

Index Terms—**multi-attribute group decision making, TOPSIS, GRA, project risk assessment**

I. INTRODUCTION

Multiple attribute decision making (MADM) problems are to select an optimal alternative from a finite number of feasible alternatives based on the features of each attributes with respect to every alternative. It has been extensively applied to various areas. Many researchers have presented several methods for MADM such as AHP, TOPSIS, VIKOR, GRA, etc [1-9]. TOPSIS (technique for order preference by similarity to an ideal solution) method was presented by Hwang and Yoon [2]. The basic principle is that the chosen alternative should have the shortest distance from the ideal solution and the farthest distance from the negative ideal solution. Gray relational analysis (GRA) method was originally developed by Deng and has been successfully applied in solving a variety of MADM problems [4][10]. The gray relational degree between ideal alternative and every comparability alternative is calculated. If a comparability alternative has the highest gray relational degree between the ideal alternative and itself, that alternative will be the best choice.

GRA method only considers the shape similarity of data sequence curve of alternative's attribute to that of ideal solution's [4]. However, TOPSIS method only considers the position approximation [11]. Combining TOPSIS and GRA method, we present a combined approach that can accurately reflect the relationship between alternative's data and ideal solution's.

With the development of modern society, the socioeconomic environment has been becoming more complex, more multi-attribute decision making processes are determined by group decision makers [12-13]. Problems in engineering project risk is ubiquitous and no methods to avoid, but can be prevented and controlled as long as taking appropriate measures. Under certain conditions, we can farthest reduce the risk of loss. Engineering project risk evaluation is the important content of the engineering project risk management. For the comprehensive evaluation, we firstly identify risk factors and risk events or the specific parts, and then employ the related industry experts evaluate the risk factors or risk event. Finally, we use the multi-attribute group decision making approach to determine the overall risk of the project.

The rest of this paper is structured as follows. In the next section, we present a brief introduction of TOPSIS method. In Section 3, Gray relational analysis (GRA) method is introduced. Combining TOPSIS and GRA method, we present a combined decision making approach in Section 4. For multi-attribute group decision making, an approach is given in Section 5. A numerical example given to illustrate the proposed method is shown in Section 6.

II. TOPSIS METHOD

Before we describe the procedure of TOPSIS method, let us assume a decision matrix, A , be defined as

$$A=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}.$$

where A is composed of n alternatives $X=\{x_1,x_2,\cdots,x_n\}$ and m attributes $U=\{u_1,u_2,\cdots,u_m\}$; a_{ij} denotes the evaluations of the i th alternative with respect to the j th attribute. We want to select an appropriate alternative from the n alternatives. The procedure of TOPSIS method can be expressed in the following six steps [2].

Step 1. Establish the normalized decision matrix.

For the purpose of comparability between attributes, the raw decision matrix has to be normalized or preprocessed. The normalized value of a_{ij} , r_{ij} , can be obtained by

doi:10.4304/jsw.9.2.404-408

$$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{n} a_{ij}^{2}}}, i = 1, 2, \cdots, n, j = 1, 2, \cdots, m. \quad (1)$$

Step 2. Establish the weighted normalized decision matrix $Z = (z_{ij})_{n \times m}$.

The weighted normalized value of r_{ij}, z_{ij}, can be calculated by

$$z_{ij} = w_j r_{ij}, i = 1, 2, \cdots, n, j = 1, 2, \cdots, m, \quad (2)$$

where w_j is the weight of the j th attribute, and $\sum_{j=1}^{m} w_j = 1$.

Step 3. Determine the positive ideal and negative ideal alternatives.

The ideal alternative is a hypothetical alternative in which all attribute values correspond to the best level. On the contrary, the anti-ideal alternative is also a hypothetical alternative in which all attribute values correspond to the worst level. Denote the ideal alternative, A^+, and the anti-ideal alternative, A^-, as

$$\begin{aligned} A^+ &= \left\{ (\max_i z_{ij} \,|\, j \in J), (\min_i z_{ij} \,|\, j \in J') \,\middle|\, i = 1, 2, \right. \\ &\qquad \left. \cdots, n \right\} = [z_1^+, z_2^+, \cdots, z_m^+], \\ A^- &= \left\{ (\min_i z_{ij} \,|\, j \in J), (\max_i z_{ij} \,|\, j \in J') \,\middle|\, i = 1, 2, \right. \\ &\qquad \left. \cdots, n \right\} = [z_1^-, z_2^-, \cdots, z_m^-], \end{aligned} \quad (3)$$

where J and J' are the attribute sets of the larger-the-better type (such as benefit) and the smaller- the-better type (such as cost), respectively.

Step 4. Calculate the separation measures, using the m-dimensional Euclidean distance.

The separation of each alternative from the ideal alternative, A^+, is given as

$$d_i^+ = \sqrt{\sum_{j=1}^{m} (z_{ij} - z_j^{+})^2}, i = 1, 2, \cdots, n. \quad (4)$$

Similarly, the separation of each alternative from the negative ideal alternative, A^-, is given as

$$d_i^- = \sqrt{\sum_{j=1}^{m} (z_{ij} - z_j^{-})^2}, i = 1, 2, \cdots, n. \quad (5)$$

Step 5. Calculate the relative closeness to the ideal alternative.

The relative closeness of the i th alternative, f_i, with respect to the ideal alternative is defined as

$$f_i = \frac{d_i^-}{d_i^+ + d_i^-}, i = 1, 2, \cdots, n. \quad (6)$$

Since $d_i^+ \geq 0$ and $d_i^- \geq 0$, then, obviously, $f_i \in [0,1]$.

Step 6. Rank the preference order.

A set of alternatives then can be preference ranked according to the descending order of f_i; in other words, larger f_i means better alternative.

III. GREY RELATION ANALYSIS (GRA)

Gray relational analysis (GRA) method was originally developed by Deng [4] and has been successfully applied in solving a variety of MADM problems. The main procedure of GRA is first translating the performance of all alternatives into a comparability sequence. This Step is called gray relational generating. According to these sequences, an ideal target sequence is defined. Then, the gray relational coefficient between all comparability sequences and ideal target sequence is calculated. Finally, base on these gray relational coefficients, the gray relational degree between ideal target sequence and every comparability sequences is calculated. If a comparability sequence translated from an alternative has the highest gray relational degree between the ideal target sequence and itself, that alternative will be the best choice.

In the following, we give the procedure of GRA method to solve MADM [4].

Step 1. Determine the positive ideal and negative ideal solution

We let $Z = (z_{ij})_{n \times m}$ denote the decision matrix, then using formula (1), we obtain the positive ideal solution A^+ and negative ideal solution A^-.

Step 2. Calculate the grey relational coefficients of each alternative from the positive ideal and negative ideal solution using the following equation, respectively.

The grey relational coefficients of each alternative from the positive ideal solution is given as

$$\gamma_{ij}^+ = \frac{\min_i \min_j \left| z_j^+ - z_{ij} \right| + \rho \max_i \max_j \left| z_j^+ - z_{ij} \right|}{\left| z_j^+ - z_{ij} \right| + \rho \max_i \max_j \left| z_j^+ - z_{ij} \right|},$$
$$i = 1, 2, \cdots, n, j = 1, 2, \cdots, m. \quad (7)$$

Similarly, the grey relational coefficients of each alternative from the negative ideal solution is given as

$$\gamma_{ij}^- = \frac{\min_i \min_j \left| z_j^- - z_{ij} \right| + \rho \max_i \max_j \left| z_j^- - z_{ij} \right|}{\left| z_j^- - z_{ij} \right| + \rho \max_i \max_j \left| z_j^- - z_{ij} \right|},$$
$$i = 1, 2, \cdots, n, j = 1, 2, \cdots, m, \quad (8)$$

where the identification coefficient $\rho = 0.5$.

Step 3. Calculate the degree of grey relational coefficients of each alternative from the positive ideal and negative ideal solution using the following equation, respectively.

$$\gamma_i^+ = \frac{1}{m} \sum_{j=1}^{m} \gamma_{ij}^+, \ \gamma_i^- = \frac{1}{m} \sum_{j=1}^{m} \gamma_{ij}^-, \ i = 1, 2, \cdots, n. \quad (9)$$

The basic principle of the GRA method is that the chosen alternative should have the "largest degree of grey relation" from the positive ideal solution and the "smallest degree of grey relation" from the negative ideal solution.

Step 4. Calculate the grey relational degree of each alternative from the positive ideal solution using the following equation

$$\gamma_i = \frac{\gamma_i^+}{\gamma_i^+ + \gamma_i^-},\ i = 1, 2, \cdots, n. \tag{10}$$

Step 5. Rank all the alternatives and select the best one in accordance with γ_i. If an alternative has the highest value, then it is the best alternative.

Ⅳ. A COMBINED APPROACH OF TOPSIS AND GRA METHOD

GRA method only considers the shape similarity of data sequence curve of alternative's attribute to that of ideal solution's. However, TOPSIS method only considers the position approximation. Combining TOPSIS and GRA method, we present a combined approach that can accurately reflect the relationship between alternative's data and ideal solution's.

In TOPSIS method, f_i denotes the relative closeness of the i th alternative to the ideal alternative. In GRA method, the grey relational degree of the i th alternative from the ideal solution is denoted by γ_i. Using γ_i and f_i, we create a new relation closeness as follows:

$$C_i = \alpha f_i + (1-\alpha)\gamma_i,\ i = 1, 2, \cdots, n, \tag{11}$$

where α expresses the decision maker's preference for TOPSIS and GRA method, and meets $\alpha \in [0,1]$.

Rank the alternatives, sorting by the values C_i, in decreasing order. The best alternative, ranked by C_i, is the one with the maximum value of C_i.

Ⅴ. AN APPROACH TO MULTI-ATTRIBUTE GROUP DECISION MAKING TOPSIS METHOD

The multi-attribute group decision making problem under study can be described as follows:

Let $X = \{x_1, x_2, \cdots, x_n\}$ be a set of n feasible alternatives, and let $U = \{u_1, u_2, \cdots, u_m\}$ be a set of attributes. Suppose that there are p decision makers $E_k (k = 1, 2, \cdots, p)$, whose weighted vector is $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_p)$, where $\lambda_k \ge 0, k = 1, 2, \cdots, p$, $\sum_{k=1}^{p} \lambda_k = 1$. The weighted vector of m attributes is $w = (w_1, w_2, \cdots, w_m)$, where w_j is the weight of the j th attribute, and $w_j \ge 0, j = 1, 2, \cdots, m, \sum_{j=1}^{m} w_j = 1$.

Let $A_k = \left(a_{ij}^{(k)}\right)_{n\times m}, k = 1, 2, \cdots, p,$ be an individual decision matrix that is provided by decision maker E_k (see Table 1), where $a_{ij}^{(k)}$ is an attribute value, which takes the form of positive real numbers, of the alternative $x_i \in X$ with respect to the attribute $u_j \in U$.

Based on our combining approach of TOPSIS and GRA methods, we develop an approach to the multi-attribute group decision making as follows:

Step 1. Utilize DWA operator [14]

$$a_{ij} = \sum_{k=1}^{p} \lambda_k a_{ij}^{(k)} \tag{12}$$

to aggregate all the individual decision matrices A_k by decision maker E_k into a collective decision matrix $A = (a_{ij})_{n\times m}$.

Step 2. Establish the normalized decision matrix $R = (r_{ij})_{n\times m}$.

$$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{n} a_{ij}^2}}, i = 1, 2, \cdots, n, j = 1, 2, \cdots, m.$$

Step 3. Establish the weighted normalized decision matrix $Z = (z_{ij})_{n\times m}$.

$$z_{ij} = w_j r_{ij}, i = 1, 2, \cdots, n, j = 1, 2, \cdots, m,$$

where w_j is the weight of the j th attribute, and $\sum_{j=1}^{m} w_j = 1$.

Step 4. Determine the positive ideal A^+ and negative ideal alternatives A^-

$$A^+ = \left\{(\max_i z_{ij} \mid j \in J), (\min_i z_{ij} \mid j \in J') \middle| i = 1, 2, \cdots, n\right\} = [z_1^+, z_2^+, \cdots, z_m^+],$$

$$A^- = \left\{(\min_i z_{ij} \mid j \in J), (\max_i z_{ij} \mid j \in J') \middle| i = 1, 2, \cdots, n\right\} = [z_1^-, z_2^-, \cdots, z_m^-],$$

where J and J' are the attribute sets of the larger-the-better type (such as benefit) and the smaller- the-better type (such as cost), respectively.

Step 5. Calculate the separation measures, using the m-dimensional Euclidean distance.

The separation of each alternative from the ideal alternative, A^+, is given as

$$d_i^+ = \sqrt{\sum_{j=1}^{m} (z_{ij} - z_j^+)^2}, i = 1, 2, \cdots, n.$$

Similarly, the separation of each alternative from the negative ideal alternative, A^-, is given as

$$d_i^- = \sqrt{\sum_{j=1}^{m} (z_{ij} - z_j^-)^2}, i = 1, 2, \cdots, n.$$

Step 6. Calculate the relative closeness f_i of the i th alternative to the ideal alternative.

$$f_i = \frac{d_i^-}{d_i^+ + d_i^-}, i = 1, 2, \cdots, n.$$

Step 7. Calculate the grey relational coefficients of each alternative from the positive ideal and negative ideal alternative using the following equation, respectively.

$$\gamma_{ij}^+ = \frac{\min_i \min_j \left|z_j^+ - z_{ij}\right| + \rho \max_i \max_j \left|z_j^+ - z_{ij}\right|}{\left|z_j^+ - z_{ij}\right| + \rho \max_i \max_j \left|z_j^+ - z_{ij}\right|},$$

$$\gamma_{ij}^- = \frac{\min_i \min_j \left|z_j^- - z_{ij}\right| + \rho \max_i \max_j \left|z_j^- - z_{ij}\right|}{\left|z_j^- - z_{ij}\right| + \rho \max_i \max_j \left|z_j^- - z_{ij}\right|},$$

$$i = 1, 2, \cdots, n, j = 1, 2, \cdots, m,$$

where the identification coefficient $\rho = 0.5$.

Step 8. Calculate the degree of grey relational coefficients of each alternative from the positive ideal and negative ideal alternative using the following equation, respectively.

$$\gamma_i^+ = \frac{1}{m} \sum_{j=1}^{m} \gamma_{ij}^+, \ \gamma_i^- = \frac{1}{m} \sum_{j=1}^{m} \gamma_{ij}^-, \ i = 1, 2, \cdots, n.$$

Step 9. Calculate the grey relational degree of each alternative from the positive ideal alternative using the following equation

$$\gamma_i = \frac{\gamma_i^+}{\gamma_i^+ + \gamma_i^-}, \ i = 1, 2, \cdots, n.$$

Step 10. Using γ_i and f_i, We calculate a new relation closeness as follows:

$$C_i = \alpha f_i + (1 - \alpha)\gamma_i, \ i = 1, 2, \cdots, n.$$

where α expresses the decision maker's preference for TOPSIS and GRA method, and $\alpha \in [0,1]$.

Rank the alternatives, sorting by the values C_i, in decreasing order. The best alternative, ranked by C_i, is the one with the maximum value of C_i.

VI. ILLUSTRATIVE EXAMPLE

In this section, we discuss a problem concerning project risk, and evaluate the overall level of risk on the projects. First, according to the main factors influencing the project management, we select five main risk evaluation indicators: cost risk u_1; time limit risk u_2; quality risk u_3; organization risk u_4; technical risks u_5. There are four experts whose weighted vector is $\lambda = (0.30, 0.24, 0.20, 0.26)$. The decision matrices A_1, A_2, A_3, A_4 (see Table Ⅰ), which are given by these experts, represent the performance of four construction projects $X = \{x_1, x_2, x_3, x_4\}$. Suppose the weight vectors of five attributes are $w = (0.25, 0.15, 0.30, 0.15, 0.15)$. Then, we utilize the developed approach to evaluate these four construction projects.

TABLE I.
DECISION MATRIXES

	A_1					A_2				
x_1	6	7	6	8	5	5	6	6	7	5
x_2	6	8	6	5	7	6	7	6	6	8
x_3	7	8	5	6	5	7	8	6	7	5
x_4	8	6	7	6	8	7	6	8	7	8
	A_3					A_4				
x_1	6	7	6	7	5	7	8	6	7	6
x_2	7	6	5	6	7	8	7	5	6	8
x_3	6	8	6	7	6	7	7	6	8	5
x_4	8	6	8	7	6	7	6	8	6	7

Now we utilize the proposed method to this problem. Using the formula (12), we aggregate all the individual decision matrices A_1, A_2, A_3, A_4 into a collective decision matrix A. Then utilize formula (1) (2), we obtain the normalized decision matrix R (see Table Ⅱ) and the weighted normalized decision matrix Z (see Table Ⅲ).

TABLE II.
DECISION MATRIX R

	u_1	u_2	u_3	u_4	u_5
x_1	0.4439	0.5019	0.4766	0.5507	0.4097
x_2	0.4956	0.5077	0.4401	0.4300	0.5842
x_3	0.5015	0.5534	0.4528	0.5251	0.4050
x_4	0.5531	0.4290	0.6117	0.4859	0.5717

TABLE III.
DECISION MATRIX Z

	u_1	u_2	u_3	u_4	u_5
x_1	0.1110	0.0753	0.1430	0.0826	0.0615
x_2	0.1239	0.0761	0.1320	0.0645	0.0876
x_3	0.1254	0.0830	0.1358	0.0788	0.0608
x_4	0.1383	0.0644	0.1835	0.0729	0.0858

Determine the positive ideal A^+ and negative ideal alternatives A^- as follows

$$A^+ = (0.1110, 0.0644, 0.1320, 0.0645, 0.0608),$$

$A^- = (0.1383, 0.0830, 0.1835, 0.0826, 0.0876)$.

Then using the formula (4) (5), we obtain the separation d_i^+, d_i^- of each alternative from the ideal alternative A^+ and the negative ideal alternatives A^-. We calculate the relative closeness f_i of the i th alternative to the ideal alternative by the formula (6). The grey relational coefficients $\gamma_{ij}^+, \gamma_{ij}^-$ of each alternative from the positive ideal and negative ideal alternative are calculated by using the formula (7)(8), respectively. Then we obtain γ_i^+, γ_i^- , the degree of grey relational coefficients of each alternative from the positive ideal and negative ideal alternative using the equation (9). We utilize the formula (10) to calculate the grey relational degree γ_i of each alternative from the positive ideal alternative. Using γ_i and f_i , we can get the relation closeness C_i by the following equation

$$C_i = \alpha f_i + (1-\alpha)\gamma_i,\ i = 1, 2, \cdots, n,$$

where $\alpha = 0.5$. The above parameters and the ranking results by these three methods are listed in the following Table.

TABLE IV.
PARAMETERS AND THE RANKING RESULTS BY THESE THREE METHODS

	d_i^+	d_i^-	f_i	Rank by TOPSIS
x_1	0.0238	0.0559	0.7011	1
x_2	0.0320	0.0569	0.6402	3
x_3	0.0278	0.0563	0.6697	2
x_4	0.0640	0.0211	0.2476	4
	γ_i^+	γ_i^-	γ_i	Rank by GRA
x_1	0.7927	0.6278	0.5581	1
x_2	0.7682	0.6703	0.5340	2
x_3	0.7471	0.6752	0.5253	3
x_4	0.6161	0.8475	0.4209	4

	C_i	Rank by the combined method
x_1	0.6296	1
x_2	0.5867	3
x_3	0.5975	2
x_4	0.3347	4

From Table Ⅳ, we can see the sorted result by the combined method is relatively close to GRA sorting and TOPSIS sorting, which fully demonstrate the new approach is effective. Using the combined approach of TOPSIS and GRA method for multiple attribute decision making to analyze problems, is more comprehensive and objective. This method has certain popularization and application value.

REFERENCES

[1] T.L. Saaty, "The Analytic Hierarchy Process", *McGraw-Hill, New York, USA,* 1980.
[2] C.L. Hwang, K. Yoon, "Multiple Attribute Decision Making: Methods and Applications", *Springer-Verlag, Berlin*, 1981.
[3] Houxing Tang. Multiple-Attribute Decision Making Based on Attribute Preference. *Journal of software*, vol.7, no.3, pp. 644–651, March 2012.
[4] Deng, J. L. "Introduction to grey system", *The Journal of Grey System (UK)*, vol. 1, no.1, pp.1–24, 1989.
[5] Z.S. Xu, "Uncertain Multiple Attribute Decision Making: Methods and Applications", *Tsinghua University Press, Beijing*, 2004.
[6] Zhiguang Zhang, "An Approach to Dynamic Multi-attribute Decision Making for Choosing Green Supplier", *Journal of Convergence Information Technology*, vol. 7, no. 21, pp. 261–269, 2012
[7] Opricovic S., & Tzeng, G.H., "Extended VIKOR Method in Comparison with Outranking Methods", *European Journal of Operational Research*, vol.178, pp.514–529, 2007.
[8] Bingbing Qiu, James Nga Kwok Liu, Wei-min Ma. novel methods for intuitionistic fuzzy multiple attribute decision making .*Journal of software*, vol. 7, no. 3, pp. 2553–2559, November 2012
[9] Xu Jing, "Method for Dynamic Multiple Attribute Decision Making under Interval Uncertainty and its Application to Supplier Selection", *Advances in Information Sciences and Service Sciences*, vol.5, no.1, pp. 246–253, 2013.
[10] Li, G. D., Yamaguchi, D., & Nagai, M. "A grey-based decision-making approach to the supplier selection problem", *Mathematical and Computer Modelling*, vol.46, pp. 573–581, 2007.
[11] Serafim Opricovic,Gwo-Hshiung Tzeng. "Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS", *European Journal Research*, vol.156, pp.445–455, 2004.
[12] Xu, Z. S. "Multi-period multi-attribute group decision-making under linguistic assessments", *International Journal of General Systems*, vol.38, no.8, pp. 823–850, 2009.
[13] Kahraman, C., Engin, O., Kabak, O., & Kayaet, I. "Information systems outsourcing decisions using a group decision-making approach", *Engineering Applications of Artificial Intelligence*, vol.22, no.6, pp.832–841, 2009.
[14] Xu, Z. S. "On multi-period multi-attribute decision making", *Knowledge-Based Systems*, vol.21, no.2, pp. 164–171, 2008.

Zhiguang Zhang, born in 1970, is an associate professor of Department of Mathematics, Dezhou University, China. His research interests include decision making analysis, mathematical modeling and its applications etc.