Eye Location Based on Hough Transform and Direct Least Square Ellipse Fitting

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***Abstract*—Feature extraction and location of eye are very important in face expression recognition, human-computer interaction, face detection and identification etc. This paper presents a new efficient method for eye location. Firstly, gradient, integral image, and Hough transform are used to obtain location and radius of the pupil. Secondly, CANNY operator and ellipse fitting of direct least square are used to obtain elliptic equation of the eyelid. Finally, the center of the ellipse is deemed to be the center of the eyelid, and the edge of fitting ellipse is deemed to be the edge of the eyelid. This method is extremely robust and accurate in the condition that two eyes are horizontal and inclined. The average error is less than 1.82 pixels.**

***Index Terms*—Gradient; integral image; Hough transform; CANNY operator; ellipse fitting of direct least square**

I.INTRODUCTION

Currently, face recognition, which is used as biological characteristics, has great application prospect. Face detection is a pretreatment of the face recognition. Its detection accuracy is very important to the accuracy of the face recognition [1-3]. Moreover, the eye is an important part of the facial feature. Thus, in facial expression, human computer interaction, identification, face detection, location and feature extraction of the eye are extremely important. Now, the state-of-the-art eye location mainly has two basic methods: algorithm based on template matching [4, 5] and algorithm based on feature points of the eye [6, 7]. The algorithm based on template matching can obtain not only the shape but also position of the eye. But its parameter selection is more complex, and calculation speed is very slow. The method based on feature point makes good use of feature point to construct the eye contour. Its advantage is that it is robust, and its calculation speed is very fast. Its drawback is that the accuracy of the calculation is relatively low.

In order to further improve the accuracy of the calculation with the method based on the feature point, a new method of eye location is proposed in this paper. Firstly, the approximate area of the eyelid is obtained by utilizing gradient information, integral image, etc. Then the exact location and size of the pupil are obtained by utilizing Hough transform. Thus, the approximate rectangular region of right eye and left eye may be obtained. Meanwhile, the length of the minor axis of fitting ellipse, which is the edge of the eyelid, can be obtained by using the radius of the pupil. At last, edge detection of the left and right eye region is executed by CANNY operator. By searching connected region of the maximum edge pixel, the edge image of eyelid is obtained. Then, elliptic equation of the edge of eyelid is obtained by utilizing the method of ellipse fitting. Finally, the center of the ellipse is seen as the center of the eye. Contour of the fitting ellipse is regarded as the edge of the eyelid.

The rest of the paper is organized as follows. In section Ⅱ, location and radius of the pupil are obtained by utilizing gradient, integral image, and Hough transform. Then, the eye region can be obtained. In section Ⅲ, Hough transform is used to extract pupil. In section Ⅳ,the new eye location method is proposed. Eyelid is obtained with ellipse fitting method, which is very robust when two eyes are horizontal and inclined. Performance evaluation and conclusions are presented in section Ⅴ and section Ⅵ.

II. THE PRETREATMENT OF THE EYE REGION

In order to determine the position of the eye, firstly, the method of face detection in literature [8] is adopted to extract face region. Secondly, brightness and chrominance information of the face, from which the background and hair information can be removed, are obtained by the method of skin color detection, corrosion and expansion of mathematical morphology. They are Y, Cb, and Cr image. In the experiments, it is found that the Cb of the eye region is larger than the other in Cb color space in Fig. 1. But the Cr of the eye region is smaller than the other in Cr color space. So Cr is transformed with Cr=255-Cr. After it is transformed, the Cr of the eye region is larger than the other in Cr color space in Fig. 2.

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Fig. 1 Cb image Fig. 2 Cr image being transformed

In order to reduce the influence on the detection accuracy of the eye due to the change of the luminance, the gradient information is calculated from the brightness of the image by the following formula.

$$\nabla f = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \partial f / \partial x \\ \partial f / \partial y \end{bmatrix} \quad (1)$$

The modulus of the gradient vector is

$$| G_A | = \sqrt{G_x^2(A) + G_y^2(A)} \quad (2)$$

In formula (1), $f(x, y)$ is luminance value of the point A in the image. $G_x(A)$ and $G_y(A)$ are the gradient component of the point A. In order to calculate the gradient conveniently, the gradient is normalized firstly, then it is adjusted to a range from 0 to 255.

The eye diagram information $\mathrm{EyeMapC(i, j)}$ can be calculated by chrominance and gradient information:

$$\mathrm{EyeMapC(i, j)} = 0.25 * (\mathrm{Cb(i, j)} + \mathrm{Cr(i, j)}) + 0.5 * \mathrm{G_A(i, j)} \quad (3)$$

After $\mathrm{EyeMapC(i, j)}$ is calculated with formula (3), luminance of the eye region is brighter than the other region'. See Fig. 3. During the experiments, the edge of the face has an effect on subsequent detection. So the facial edge is removed from the face image in experiments. The eye diagram can be obtained by processing with threshold in the Fig. 3. See Fig. 4.

Fig. 3 Eye diagram of the removed facial edge

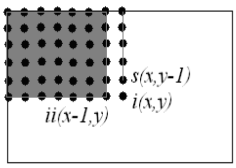
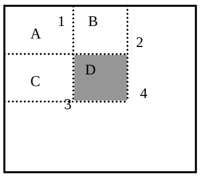


Fig. 4 Eye diagram being processed through threshold

Fig. 5 Calculation method of integral image

The approximate location of the eye can be found from integral image. Integral image is the sum of pixels within the rectangular area from the upper left point to the lower right point. For the point $i(x, y)$ in the image, its integral image value is:

$$ii(x, y) = \sum_{y' < y} \sum_{x' < x} i(x', y') \quad (4)$$



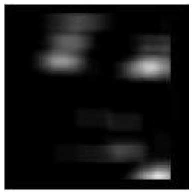


Fig. 6 Calculation method of rectangular integral image

Fig. 7 Image be processed with integral image

In Fig. 5, the shaded portion is the value of the integral image of the point i(x-1, y). The integral image value can be calculated iteratively by the following formula:

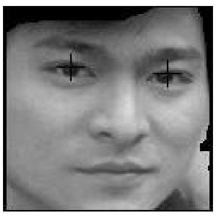
$$s(x, y) = s(x, y-1) + i(x, y) \quad (5)$$

$$ii(x, y) = ii(x-1, y) + s(x, y) \quad (6)$$

In formula (5), $s(x, y)$ is the sum of the gray value of pixels which column ordinate does not exceed column ordinate of the point $i(x, y)$. See Fig. 5. After integral image is calculated, the sum of pixel value within any rectangular area of the image can be obtained through the integral image. See Fig. 6, $\mathrm{RectSum(1)}$ is the sum of pixels value in the area A. $\mathrm{RectSum(2)}$ is the sum of pixels value in the area (A+B). $\mathrm{RectSum(3)}$ is the sum of pixels value in the area (A+C). $\mathrm{RectSum(4)}$ is the sum of pixels value in the area (A+B+C+D). So the sum of pixels value in the area D is:

$$\mathrm{RectSum(D)} = \mathrm{RectSum(4)} + \mathrm{RectSum(1)} - \mathrm{RectSum(2)} - \mathrm{RectSum(3)} \quad (7)$$

If the length and width of the eye are in the ratio 2:1, the image of the eye in Fig. 4 can be calculated by method of the integral image. See Fig. 7. In Fig. 7, the brightest location of the image is the area of eye. Although there are three relatively bright positions, the positions of the two eyes can be obtained according to the relationship of the x-axis and y-axis of the two eyes. See equation (8), (9). After the positions of the two eyes are found, it is marked in the original image. See Fig. 8. Then the region containing two eyes can be extracted according to the distance of the two eyes, and gradient information of two eyes can be obtained by CANNY operator. See Fig. 9.



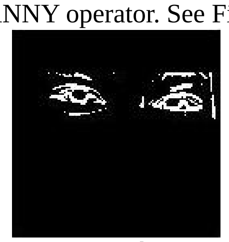


Fig. 8 Approximate location of eye area

Fig. 9 Gradient image of eye area

$$| x_{zuo} - x_{you} | \le \varepsilon_1 \quad \varepsilon_1 \text{ is threshold 1} \quad (8)$$

$$| y_{zuo} - y_{you} | \ge \varepsilon_2 \quad \varepsilon_2 \text{ is threshold 2} \quad (9)$$

III. Extract Pupil With Hough Transform

Hough transform is a mapping relationship from the image space to parameter space. It is to cluster pixels together by means which have a certain relationship in the image space. It is to look for cumulative corresponding point in parameter space, which can link these points by certain analytical form. It is very tolerant to fault and very robust when region boundary is intermittent, which is interfered by noise or covered by other object. This transform has desired effect in the parameter space that is less than two-dimension. But computation complexity will increase rapidly with parameter space dimension increasing [9].

The standard equation of the circle is: $(x-x_0)^2+(y-y_0)^2=r^2$. In order to reduce computation, r is set to change from R0 to Rmax in standard equation. So x_0 is set to change from xstart (xstart=x-r) to xend (xend=x+r). y_0 is set to change from ystart (ystart=y-r) to yend (yend=y+r). In practice, error of r is allowed. Therefore, equation (9) is used in the experiments.

$$(x-x_0)^2+(y-y_0)^2=(r\pm\varepsilon)^2 \qquad (9)$$

In equation (9), ε is used to control range of error. Step of the Hough transform of the circle is:

(1) Set up a three-dimensional array $B(x_0,y_0,r)$.

(2) For an arbitrary point which value is not zero in Fig.9. Firstly, r is set to certain value. Then x_0 and y_0 are changed. If it meets the equation (9), accumulator array $B(x_0,y_0,r)$ plus 1.

(3) The value of r is changed according to the step. Then x_0 and y_0 are changed. If it meets the equation (9), accumulator array $B(x_0,y_0,r)$ plus 1.

(4) Accumulator arrays are voted alternately. Each point which value is not zero is calculated. If the peak of the accumulator array is found, then the center and the radius of the circle are obtained according to equation (7), (8). The obtained center and the radius of the circle are the center and the radius of the pupil respectively. See Fig. 10. Then approximate range of minor axis and center of the fitting ellipse of eyelid are obtained.

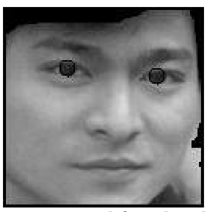


Fig. 10 Pupil found with Hough transform

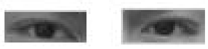


Fig. 11 Image of left and right eye

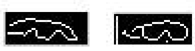


Fig. 12 Edge image of two eyes

Fig. 13 Maximum edge pixel connected region

After center and radius of left and right pupil are obtained, approximate region of the left and right eye can be got. See Fig. 11. It is processed with method of edge detection. See Fig. 12. In Fig. 12, not only edge of eyelid contour, but also edge of non-eyelid contour is included. So maximum edge pixel connected region must be found, which is considered to be the true contour of the eye. See Fig. 13.

IV. Determining Eyelid With Ellipse Fitting Method

There are two methods to find ellipse mainly. The first method is voting clustering. The second method is optimal algorithm. Hough transform belongs to the first class. The direct least square fitting method belongs to the second class. This algorithm is that the smallest squared sum of algebraic distance is used as fitting mathematical principle. The plane conic general equation of ellipse can be represented by formula (10) [10].

$$F(B,X)=B*X=ax^2+bxy+cy^2+dx+ey+f \qquad (10)$$

In formula (10), $B=[a,b,c,d,e,f]$, $X=[x^2,xy,y^2,x,y,1]^T$, $F(B,X_i)$ is algebraic distance from point $[x_i,y_i]$ to quadratic curve $F(B,X)$. If $4ac-b^2$ is subject to $4ac-b^2=1$, the fitting curve is ellipse [10,11].

Smallest squared sum of algebraic distance from point $[x_i,y_i]$ to quadratic curve $F(B,X)$ is to find minimal $\sum_{i=1}^{N}F(B;X_i)^2$. N is the number of fitting point. Supposing:

$$H_{N\times 6}=[X_1\ \ X_2\ \ ....X_N]^T,$$

$$C=\begin{bmatrix}0&0&2&0&0&0\\0&-1&0&0&0&0\\2&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix} \qquad (11)$$

The method of direct least square fitting can be transformed into finding coefficient $[a,b,c,d,e,f]$ to obtain minimal $\|HB\|^2$ under the condition of $B^TCB=1$. Then Lagrange operator is introduced, and the same solution equation is obtained by equation (12, 13):

$$H^THB-\lambda CB=0 \qquad (12)$$

$$B^TCB=1 \qquad (13)$$

The solution of equation (12) can be obtained with the generalized eigenvalue and generalized eigenvector. Its solution is (λ_i, u_i). For any $\alpha_i \in R^+$, $(\lambda_i, \alpha_i u_i)$ is also the solution of equation (12). So $(\lambda_i, \alpha_i u_i)$ must meet $\alpha_i^2 u_i^T C u_i = 1$.Then

$$\alpha_i = \sqrt{\frac{1}{u_i^T C u_i}} = \sqrt{\frac{\lambda_i}{u_i^T H^T H u_i}} \quad (14)$$

Usually, elements of $H^T H$ are positive for the image data. Thus, denominator $u_i^T H^T H u_i$ always are positive for all u_i. Thus, the existing condition of the square root in equation (14) is $\lambda_i > 0$. Therefore, the solution of the equation (12) must have a positive eigenvalue. Because the eigenvalue of the C is [-2,-1,2,0,0,0], there is only one unique generalized eigenvalue $\lambda_i > 0$ and generalized eigenvector u_i that is used as solution of ellipse fitting. Thus, the solution of fitting ellipse is $B_i = \alpha_i u_i$. After six parameters of the elliptic equation are obtained, equation (15) can be obtained when quadratic curve $ax^2 + bxy + cy^2 + dx + ey + f = 0$ is rotated and shifted.

$$\frac{(x^{'} - x0)^2}{M^2} + \frac{(y^{'} - y0)^2}{N^2} = 1 \quad (15)$$

The angle of inclination of the ellipse can be obtained:

$$\text{theta} = 0.5 * \text{atan}(b/(a - c)) \quad (16)$$

The center coordinate of the ellipse in rotating coordinate system is:

$$x0 = \text{round}(-(0.5*d*\cos(\text{theta}) + 0.5*e *\sin(\text{theta}))/(a + 0.5*b*\tan(\text{theta}))) \quad (17)$$

$$y0 = \text{round}(-(-0.5*d*\sin(\text{theta}) + 0.5*e *\cos(\text{theta}))/(c - 0.5*b*\tan(\text{theta}))) \quad (18)$$

The center coordinate of the ellipse in image is:

$$xc = x0*\cos(\text{theta}) - y0*\sin(\text{theta}) \quad (19)$$

$$yc = x0*\sin(\text{theta}) + y0*\cos(\text{theta}) \quad (20)$$

The length of the major axis and the minor axis of the ellipse are respectively:

$$M = \sqrt{| -C_1/(a + 0.5*b*\tan(\text{theta})) |} \quad (21)$$

$$N = \sqrt{| -C_1/(c - 0.5*b*\tan(\text{theta})) |} \quad (22)$$

In above formula, C_1 is the intermediate variable. Edge image in Fig. 13 is fitted to obtain fitting ellipse with direct least square method. See Fig. 14. In Fig. 14, the center of the ellipse is deemed to be the center of the eyelid, and the edge of the fitting ellipse is deemed to be the edge of the eyelid.

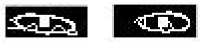


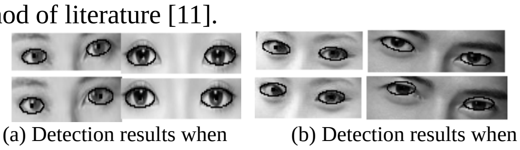
Fig. 14 Fitting ellipse of two eyes

V. Analysis of Experimental Results

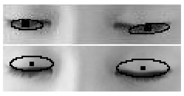
In experiments, PC is dual-core Pentium(R) 4 CPU 3.06G，3.07G，Memory is 2.25G，software is MATLAB7.11(R2010b). All procedures are programmed with MATLAB. Due to using the chrominance information in experiments, this method is only suitable for color images. All color images are obtained from the internet. The first group is the face image that two eyes are horizontal. The second group is the face image that two eyes are inclined. The third group is the face image that two eyes are closed. There are 400 images each group. There are 1200 images in all. The size of each image is 128 * 128.

The first experiment is that the eye location and ellipse fitting is executed to the first group, the second group, and the third group with the method proposed in this paper and the method in literature [11]. See Fig. 15. In Fig. 15, the top row of the (a), (b), (c) is the detecting result respectively to three groups of image with the proposed method. The bottom row is the detecting result respectively to three groups of image with the method of literature [11]. The detection accuracy and the detection time are shown in Table 1 and Table 2. As can be seen from Fig. 15, Table 1 and Table 2, the detection accuracy is much higher with the proposed method than with the method of literature [11]. Especially to the inclined ellipse, the detection accuracy is much higher. But the detection time is much longer with the proposed method than with the method of literature [11]. The reason is that ellipse is set to be horizontal and no inclined in literature [11], while ellipse is assumed to be inclined and rotary in this paper. The angle of inclination can be calculated accurately with the proposed method. So the calculated time will be longer.

The second experiment is to find the center of fitting ellipse, calculate the length of the major axis and the minor axis of the ellipse respectively, and obtain the error of the center of the eye position with the proposed method and the method of literature [11].Three groups of images are tested respectively. The results are shown in Table 3 and Table 4. As can be seen from Table 3 and Table 4, the error of coordinate of the eye center and the error of the length of the major axis and the minor axis measured with the proposed method are smaller than with the method of literature [11].



(a) Detection results when two eyes are horizontal (b) Detection results when two eyes are inclined



(c) Detection result when two eyes are closed

Fig. 15 Detection results with two methods

TABLE 1
THE DETECTION ACCURACY AND THE DETECTION TIME WITH THE PROPOSED METHOD

group	detection accuracy	detection time /s
First	97%	0.15
Send	92%	0.12
Third	75%	0.14

TABLE 2
THE DETECTION ACCURACY AND THE DETECTION TIME WITH THE METHOD OF LITERATURE [11]

group	detection accuracy	detection time /s
First	95%	0.11
Send	85%	0.10
Third	73%	0.12

TABLE 3
AVERAGE ERROR WITH THE PROPOSED METHOD (PIXEL)

	The proposed method		
group	ellipse center	major axis	minor axis
First	1.5	1.9	1.1
Second	1.6	2.3	1.2
Third	2.2	2.5	1.5

Table 4 Average error with the method of literature [11] (pixel)

group	ellipse center	major axis	minor axis
First	1.6	2.0	1.2
Second	2.3	2.8	1.7
Third	2.4	2.6	1.54

VI. CONCLUSIONS

In this paper, gradient information, integral image, Hough transform, CANNY operator edge detection, the method of maximum edge pixel connection region, the method of least squares ellipse fitting are used to obtain ellipse contour of the eyelid. Then the center of the ellipse is deemed to be the center of the eyelid. Fitting ellipse is deemed to be the edge of the eyelid. Thereby, eye is located. In the experiments, the proposed method is proved to be strongly robust and highly accurate. Its average error is 1.71 pixels. The inadequacy of the algorithm is that detection accuracy is low in the closed eye or nearly closed. The next step will be to consider how to improve it.

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