A Fast Global Minimization of Region-Scalable Fitting Model for Medical Image Segmentation

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***Abstract*—Active contour model (ACM) which has been extensively studied recently is one of the most successful methods in image segmentation. The present paper advances an improved hybrid model based on Region-Scalable Fitting Model by combining global convex segmentation method with edge detector operator. The proposed model not only inherits the ability of RSF model to deal with the images with intensity inhomogeneity, but also overcomes such a drawback: existence of local minima because of non-convexity that makes the segmentation result highly dependent of the initial position of the contour. In addition, the paper exploits two fast numerical implementation schemes to overcome a huge amount of level set methods. The duality projection method is implemented by introducing dual variables which lead to semi-implicit iterative scheme of dual variables as well as exact formulation of primal variables. The Split-Bregman method is implemented by introducing auxiliary variables which transform the relaxed convex model into solving simple poisson equations and exact soft thresholding formulation. Experimental results for synthetic and real medical images prove that the proposed model is featured by greater numerical accuracy and faster division speed.**

***Index Terms*—Region-Scalable fitting model, global convex segmentation, dual projection method, Split Bregman method, image inhomogeneity**

I. INTRODUCTION

Segmentation and object extraction is a crucial task in image processing and computer vision. It aims to find a partition of an image into a finite number of regions that belong to distinct objects. Image segmentation is especially required for better visualization, quantification of diseases, and plan of an intervention.

Over the last decade, variational methods and partial differential equations (PDE) base techniques have been introduced to image segmentation, and the well-known and successful active contour model (ACM) is a case in point. ACM has obtained great success because of intuitional realization idea and strong mathematical properties. Its core idea is to evolve the active contour under distinguished type of driving force towards the boundaries of objects to be segmented.

There are two classes about active contours models: edge-based ACM and region-based ACM. Classical active contour models such as Snake [1], Geometric active contour model [2], Geodesic active contour model [3] depend on the image gradient, thus they are edge-based active contour model. But they make the evolving curve pass through the edge of object when the boundary of interesting region is blurred or discrete. And they are prone to fall into local optimum, thus the quality of the segmentation result depends a lot on the choice of the initial contour, which means that a bad initial contour may lead to an unsatisfactory result.

Region-based ACM evolves the active curve by employing statistical information on the sides of curves. Compared with edge-based ACM, Region-based ACM has better performance for the images with weak object boundaries or strong noise[4].

One of most well-known region-based model is Chan-Vese model (Active Contours Without Edges, ACWE)[5] that restricts the Mumford and Shah (MS) model[6] to piecewise constant function. The model does not depend on gradient information and it segments the image according to region-segmentation principle. The model can choose initial contour flexibly and detect the inner contour of the object automatically. In general, the region-based ACM is more robust than the edge-based ACM.Recently,some works such as [7] which applied the region-based model in medical field have been proposed.

However, most of region-based active contour models typically rely on intensity homogeneity in each of the regions to be segmented. The famous Chan-Vese model makes the assumption that image intensities are constant (roughly statistically homogeneous) in foreground and background regions.

But it cannot often be satisfied in real images. In particular, intensity inhomogeneity often occurs in medical images because of hardware disturbance and artifacts[4]. The more sophisticated models such as piecewise smooth models[8-10] can overcome the limitation of piceewise contant models in segmenting images with intensity inhomogeneity by applying piecewise smooth function to approximate the image intensities of the regions. Although piecewise smooth models have exhibited certain capability of handling

Manuscript received January 14, 2013; revised July 15, 2013; accepted July 17, 2013.
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doi:10.4304/jsw.9.2.280-286

intensity inhomogeneity, they are computationally expensive. For example, Vese-Chan model[7], as a typical PS model, not only solves the PDE(Partial differential Equation) of the main function ϕ by a sequence of iterations but also updates the piecewise smooth functions u^+ and u^- by solving two elliptic PDEs. So the involved computation is expensive.

Recently, Li et al. proposes an effective local region-based ACM, known as Region-Scalable fitting (RSF) energy model[4]. The model computes local intensity fitting energy on the two sides of active contour due to Gassian kernel function, thus the local energy drives the active contour toward object boundaries. The model can therefore address intensity inhomogeneity.

But the model has a non-convex problem and local minima, thus the quality of the segmentation result depends a lot on the choice of the initial contour. In literature [11]，Chan, Esedo□lu and Nikolova proposed a new approach to dealing with global minimum and overcoming the limitation of local minima according to Strang's method. Inspired by their work, the present paper incorporates the global convex segmentation (GCS) method into the RSF energy model. Consequently, a global convex optimization energy model is established. Then the paper manipulates new numerical schemes to perform the contour evolution in an efficient and fast way. We solve contour propagation problem with a dual formulation of the total variation (TV) norm (introduced and developed in[12-14]) and split Bregman method that have been developed recently in papers[15][16]. These implementation schemes prove to be efficient, easy and fast in implementation of the proposed convex model.

The remainder of the paper is as follows. In the next section, we introduce region-scalable fitting energy functional and analyze its limitations. Section 3 presents the global minimization model of region-scalable fitting energy functional. The implementation method is given in Section 4. We verify our model by various experimental results on synthetic and real medical images in Section 5. Finally, the paper is concluded in Section 6.

II. Introduction of Segmentation Model

Region-Scalable fitting model improves the Chan-Vese model by substituting two Region-Scalable fitting terms for the two average intensities of global fitting energy term in Chan-Vese model.

A. Chan-Vese Model

D. Mumford and J. Shah proposed a seminal model in 1989, namely Mumford-Shah Model(M-S model). The model takes advantage of global information of homogeneous region, thus it can obtain good segmentation results when the boundary of object region is blurred or discontinuous in the image. But it is difficult to minimize the M-S model due to the unknown contour of lower dimension. Chan and Vese consider a restriction of M-S model to piecewise constant function. A level set implementation of this functional known as the Chan-Vese model was proposed in literature [4].

For an image $I(x):\Omega \to \mathbb{R}$ on the image domain Ω, define the evolving curve C in Ω as the boundary of an open subse ω (i.e. $\omega \subset \Omega$, and $C = \partial\omega$). $inside(C)$ denotes the region ω , and $outside(C)$ denotes the region $\Omega \setminus \bar{\omega}$. The energy functional is defined by

$$F_{CV}(C,c_1,c_2) = \mu L(C) + \lambda_1 \int_{inside(c)} \left|I(x)-c_1\right|^2 dx + \lambda_2 \int_{outside(c)} \left|I(x)-c_2\right|^2 dx \quad (1)$$

This energy functional can be converted to a level set formulation by embedding the dynamic contour C as the zero level set of level set function(LSF) ϕ. The level set formulation of $F(C,c_1,c_2)$ can be written as follows:

$$F_{CV}(\phi,c_1,c_2) = \mu\int_\Omega \delta(\phi)\left|\nabla\phi\right| dx + \lambda_1\int_\Omega \left|I(x)-c_1\right|^2 H(\phi)dx + \lambda_2\int_\Omega \left|I(x)-c_2\right|^2 (1-H(\phi))dx \quad (2)$$

where μ,λ_1,λ_2 are fixed positive parameters, The first term in the formulation is level set representation of length term , the following two terms are the global fitting energy term. c_1 and c_2 are two average image intensity in ω and $\Omega \setminus \bar{\omega}$ respectively.

The optimal constants c_1 and c_2 can be far away from the original data when the intensities within ω and $\Omega \setminus \bar{\omega}$ are not homogeneous especially in medical image. As a consequence, it causes the failure of image segmentation. c_1 and c_2 do not contain any local intensity information[17-18], which is crucial for segmentation of image with intensity inhomogeneity[4].

B. Region-Scalable fitting Model

To relax the above problem of Chan-Vese model, Li proposed a new region-based ACM by introducing local average intensity. The energy functional can be written as:

$$F_{RSF}(C, f_1(x), f_2(x)) = \sum_{i=1}^{2} \lambda_i \int\int_{\Omega_i} K(x-y)\left|I(y)-f_i(x)\right|^2 dydx + \nu\,|C| \quad (3)$$

Where C is closed contour, which separates image domain Ω into two regions. $f_1(x)$ and $f_2(x)$ are two functions that approximate local image intensities in Ω_1 and Ω_2 respectively. K is Kernel function which controls the size of local region centered at point x and usually is chosen as a Gaussian kernel.

$K_\sigma(u) = \frac{1}{(2\pi)^{1/2}\sigma} e^{-|u|^2/2\sigma^2}$ with a scale parameter $\sigma > 0$

To handle topological changes, we convert the above formulation to a level set formulation. The energy functional in (3) is then written as follows:

$$F_{RSF}^{LSF}(\phi, f_1(x), f_2(x)) = \sum_{i=1}^{2} \lambda_i \int \left[\int_{\Omega_i} K_\sigma(x-y) \left| I(y) - f_i(x) \right|^2 M_i^\epsilon(\phi(y)) dy \right] dx + \nu \int |\nabla H_\epsilon(\phi(x))| \, dx$$

(4)

where $M_1^\epsilon(\phi) = H^\epsilon(\phi)$ and $M_2^\epsilon(\phi) = 1 - H^\epsilon(\phi)$. $H^\epsilon(\phi)$ is a smoothly approximating the Heaviside function. The energy functional (4) is a three-variable optimal function. We make use of standard gradient descent to minimize the equation.

By calculus of variations for a fixed level set function ϕ, the optimal functions $f_1(x)$ and $f_2(x)$ that minimize $F(\phi, f_1, f_2)$ are gotten by:

$$f_i(x) = \frac{K_\sigma(x) * [M_i^\epsilon(\phi(x)) I(x)]}{K_\sigma(x) * [M_i^\epsilon(\phi(x))]}, i = 1, 2 \qquad (5)$$

Keeping $f_1(x)$ and $f_2(x)$ fixed, the level set function ϕ that minimizes $F(\phi, f_1, f_2)$ can be obtained by solving the following gradient flow equation according to standard gradient descent method:

$$\frac{\partial \phi(x)}{\partial t} = -\delta_\epsilon(\phi)(\lambda_1 e_1 - \lambda_2 e_2) + \nu \delta_\epsilon div\left(\frac{\nabla \phi}{|\nabla \phi|}\right) \qquad (6)$$

where e_1 and e_2 are the functions:

$$e_i(x) = \int K_\sigma(y-x) \, | I(x) - f_i(y) |^2 \, dy, i = 1, 2 \qquad (7)$$

III. The improvement of Region-Scalable fitting model

A. Global Convex Optimization of Region-Scalable Fitting Model

Due to level set formulation of Region-Scalable fitting model, the energy functional is non-convex, so the evolution of level set function ϕ can be easily trapped to a local minimum. In Ref. [11], Chan,Esedo□lu,Nikolova(CEN) transformed non-convex function into globally minimization convex formulation based on Strang's observations. Hence the formulation overcomed naturally the drawback of local minima.

Inspired by CEN's idea， we incorporate global convex optimization into Region-Scalable Fitting model. Because $H_\varepsilon(z)$ is noncompactly supported, smooth approximation for $H(z)$,The steady state solution of the gradient flow is the same as:

$$\frac{\partial \phi(x)}{\partial t} = \nu div\left(\frac{\nabla \phi}{|\nabla \phi|}\right) - (\lambda_1 e_1 - \lambda_2 e_2) \qquad (8)$$

This equation, in turn, is gradient descent for minimizing the following energy:

$$F(\phi, c_1, c_2, \lambda) = \int_\Omega |\nabla \phi| \, dx + \int_\Omega (\lambda_1 e_1 - \lambda_2 e_2) \phi dx \qquad (9)$$

Energy (9) is homogeneous of degree 1 in ϕ ,so the energy does not have a unique global minimization. According to [19],non-convex set is relaxed to continuous space, if the u is restricted as functions of bounder variations taking the values in [0, 1], then for almost every $\mu \in [0,1]$, we have the characteristic function $1_{\Omega_c(\mu)=\{x:\phi(x)>\mu\}}(x)$ that separates the image into the two regions, the global minimum can be guaranteed, the constrained minimization problem to carry out the segmentation task is as follows:

$$\min_{0 \le \phi \le 1} \left\{ F_{GCS}(\phi, e_1, e_2) = \int_\Omega |\nabla \phi| \, dx + \int_\Omega (\lambda_1 e_1 - \lambda_2 e_2) \phi dx \right\} \qquad (10)$$

Energy F_{GCS} is convex but not strictly convex,but its minima is a global minimizer. if $\phi(x)$ is any minimizer of F_{GCS}, for almost any positive constant $\mu \in [0,1]$, the set of points $\Omega_1 = \{x : \phi(x) \ge \mu\}$ in the function $\phi(x)$ such as arbitrary constant,e.g. $\mu = 0.5$ is a global minimize of energy F_{RSF} .

$$\Omega_1 = \{x : \phi(x) \ge \mu\} \qquad (11)$$

B. Hybrid Model based on Edge and Region Information

From the energy functional F_{GCS}, we know that the driving force of the curve evolution mainly comes from local region information, but it does not contain edge information of the object region. In order to take advantage of local region and edge information, we introduce image edge information by adding the gradient information into the length term of (10).The edge information can be expressed as potential function and the potential function as follows:

$$g(x) = \frac{1}{1 + \left| \nabla G_\sigma * f \right|^2} \qquad (12)$$

So we get the new edge and region-based hybrid Region-Scalable Fitting energy model:

$$\min_{0 \le \phi \le 1} \left\{ F_{GCS}(\phi, e_1, e_2) = \int_\Omega g(x) |\nabla \phi| \, dx + \int_\Omega (\lambda_1 e_1 - \lambda_2 e_2) \phi dx \right\} \qquad (13)$$

IV. Numerical Implementation Scheme of Improved Region-Scalable Fitting Model

We can compute a global minimize of F_{GCS} with the standard Euler-Lagrange equations technique and the algorithm based on explicit gradient descent. However, this numerical minimization method is very slow because of the regularization of the TV-norm. Indeed, on account of convex in u, we can solve the problem through powerful tools from convex theory. We use two efficient optimization strategies that can take advantage of the convexity of F_{GCS}.

A. Fast duality Algorithm

We use the fast duality projection algorithm of Chambolle[11].The variational problem is regularized as the following equation by introducing a new function $\nu : \Omega \to R$.

$$\min_{\phi, \nu} \left\{ F^r(\phi, \nu) = \int_\Omega g(x) |\nabla \phi| \, dx + \frac{1}{2\theta} \int_\Omega (\phi - \nu)^2 dx + \int_\Omega (\lambda_1 e_1 - \lambda_2 e_2) \nu + \alpha \nu(\upsilon) dx \right\}$$

(14)

Where $\lambda_1, \lambda_2, \theta > 0$ are three constants,the parameter θ must be small enough. Since Functional F^r is convex, its minimum can be computed by minimizing F^r with respect to ϕ and ν separately, and to iterate until convergence.

（1） ϕ being fixed, we search for ν as a solution of

$$\min_{\nu}\left\{\frac{1}{2\theta}\int_{\Omega}(\phi-\nu)^2dx+\int_{\Omega}(\lambda_1e_1-\lambda_2e_2)\nu+\alpha\nu(\upsilon)dx\right\} \quad (15)$$

The solution of (15) is given by:

$$\nu = min\left\{max\left\{\phi(x)-\theta\left(\lambda_1e_1-\lambda_2e_2\right),0\right\},1\right\} \quad (1)$$

（2） ν being fixed, we search for ϕ as a solution of

$$\min_{\phi}\left\{\int_{\Omega}g(x)\,|\nabla\phi|\,dx+\frac{1}{2\theta}\int_{\Omega}(\phi-\nu)^2dx\right\} \quad (2)$$

The solution of (17) is given by:

$$p^{n+1}=\frac{g(x)\left[p^n+\delta t\nabla\left(div(p^n)-\nu/\theta\right)\right]}{g(x)+\delta t\,|\nabla\left(div(p^n)-\nu/\theta\right)|} \quad (3)$$

Where

$$p^{n+1}=\frac{g(x)\left[p^n+\delta t\nabla\left(div(p^n)-\nu/\theta\right)\right]}{g(x)+\delta t\,|\nabla\left(div(p^n)-\nu/\theta\right)|} \quad (19)$$

The initial condition: $p^0=0$.

B. Split Bregman Algorithm

In order to enhance compute efficiency, we adopt another new fast minimization algorithm, i.e. Split Bregman algorithm.

First, we introduce a new vectorial function $d \leftarrow \nabla\phi$,

$$\min_{\phi\in[0,1],d}\left\{\int_{\Omega}g(x)\,|d|\,dx+\int_{\Omega}(\lambda_1e_1-\lambda_2e_2)\phi dx\right\} \quad (20)$$

Then we add a quadratic penalty function, and get the following unconstrained problem:

$$(\phi^{(n+1)},d^{(n+1)})=arg\min_{\phi\in[0,1],d}\int_{\Omega}g(x)\,|d|\,dx+\int_{\Omega}(\lambda_1e_1-\lambda_2e_2)\phi+\frac{\mu}{2}|d-\nabla\phi|^2\,dx \quad (21)$$

The constraint $d=\nabla u$ is enforced by the efficient Bregman iteration approach, and the resulting optimization problem is:

$$(\phi^{(n+1)},d^{(n+1)})=arg\min_{\phi\in[0,1],d}\int_{\Omega}g(x)\,|d|\,dx+\int_{\Omega}(\lambda_1e_1-\lambda_2e_2)\phi+\frac{\mu}{2}|d-\nabla\phi-b^n|^2\,dx$$

$$b^{(n+1)}=b^n+\nabla\phi^{n+1}-d^{n+1} \quad (22)$$

d being fixed, The minimizing solution ϕ^{k+1} is characterized by the optimality condition:

$$\mu\Delta\phi=(\lambda_1e_1-\lambda_2e_2)+\mu(b^n-d^n),\phi\in[0,1] \quad (23)$$

We get the fast approximated solution by a Gauss-Seidel iterative scheme:

$$\begin{aligned}\gamma_{i,j} &= d^{x,k}_{i-1,j}-d^{x,k}_{i,j}-b^{x,k}_{i-1,j}+b^{x,k}_{i,j}+d^{y,k}_{i,j-1}-d^{y,k}_{i,j}-b^{y,k}_{i,j-1}+b^{y,k}_{i,j}\\ \mu_{i,j} &= \frac{1}{4}\left(\phi^{k,n}_{i-1,j}+\phi^{k,n}_{i+1,j}+\phi^{k,n}_{i,j+1}+\phi^{k,n}_{i,j+1}-\frac{\lambda}{\mu}h_{ri,j}+\gamma_{i,j}\right)\\ \phi^{k+1,n+1}_{i,j} &= \max\left\{\min\left\{\mu_{i,j},1\right\},0\right\}\end{aligned} \quad (24)$$

Finally, the minimization solution d^{k+1} is given by soft-thresholding by fixing ϕ.

$$d^{k+1}_{i,j}=\frac{\nabla\phi^{k+1}+b^k}{|\nabla\phi^{k+1}+b^k|}\max\left(|\nabla u^{k+1}+b^k|-\mu^{-1}g_b,0\right) \quad (25)$$

V. Experimental Result and Analysis

The present experiment compares the algorithm provided in the paper with the original RSF model. The source images in the paper are synthetic images and the real medical images. The program of algorithms has run in the MATLAB 7.10 on Windows XP. The CPU times were obtained by running the program on a lenovo Desktops with Intel(R) Core(TM) E7200，2GB memory.

A. The Segmentation of T-shaped Object Image

Fig.1 shows the result of a T-shaped object which size is 96*127. Fig.1 a) is the original image with intensity inhomogeneity due to nonuniform illumination. The white solid line shows the segmentation result of the original RSF model in Fig.1 b), the white dot square is used as the initial contour which is specially chosen. Fig.1 c) is the result of the dual projection algorithm on our proposed method. Fig.1 d) is the result of the Split Bregman on our proposed method. From the experimental results, we can see that our proposed method has faster computation time than the original RSF model by using two fast numerical implementation scheme. The experimental data can be seen in table 1.

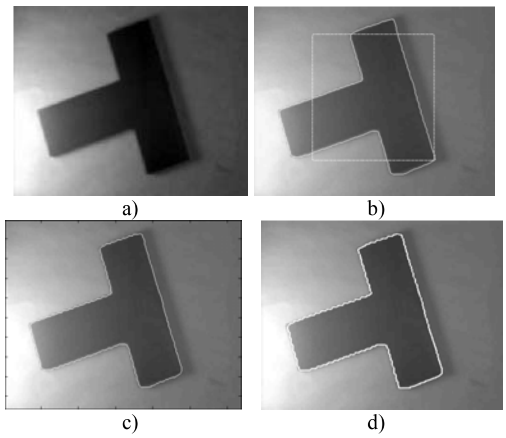

Figure 1. Result of T-shaped object image. a):original image. b):the result of RSF model. c):the result of the proposed model with dual algorithm. d): the result of the proposed model with Split Bregman algorithm

B. The Segmentation of Blood Vessel Image

We apply our proposed model to the segmentation of a vessel in Fig.2, Fig.2 a) is the original image with intensity inhomogeneity, its size is 110*111. The lower part of the vessel boundaries are quite weak. The white solid line shows the segmentation result of the original C-V model in Fig.2 b), the white dot square is used as the initial contour which is chosen specially. Fig.2 c) is the result of the dual projection algorithm on our proposed method. Fig.2 d) is the result of the Split Bregman on our proposed method. From the experimental results, we can see that our proposed method is independent of the initial condition. And we can see that our proposed method has faster computation time than the original RSF model by using two fast numerical implementation scheme.

The results of the proposed method are similar to the result with the original RSF model. But the proposed method is more efficient than the RSF model because we apply the fast implementation method. It can be demonstrated by comparing the iteration number and computation time in table 1.

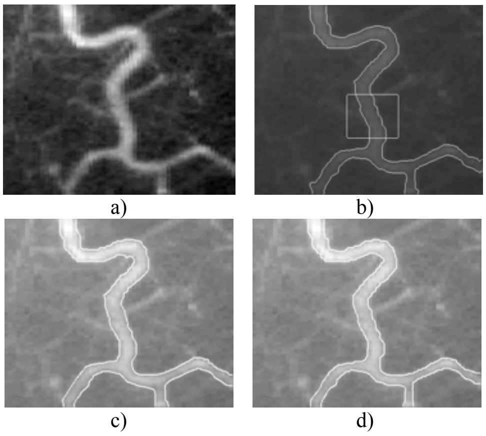

Figure 2. Result of vessel image.a): original image. b):the result of RSF model. c):the result of the proposed model with dual algorithm. d): the result of the proposed model with Split Bregman algorithm

C. The Segmentation of Another Blood Vessel Image

Fig. 3 is the X-ray image of vessel. In the image, the lower part of the vessel boundaries is quite weak, which renders it a nontrivial task to segment the vessels in the image. Fig.3 a) is original image which size is 131*103. Fig.3 b) shows that original RSF model succeed in partitioning the image when initial condition is white square. Fig.3 c) shows the corresponding segmentation results of the proposed method with dual projection method. Fig.3 d) is the result of the proposed method with Split Bregman method.

The above segmentation results are independent of initial condition because of convex property of the proposed model. Furthermore, the algorithm has higher segmentation efficiency. The evolution of energy minimization based on dual formulation converges in 25 iterations and takes 0.9 second, while the original RSF model makes 220 iterations nearly and takes for 1.68 second. The Split Bregman method only takes 0.82 second.

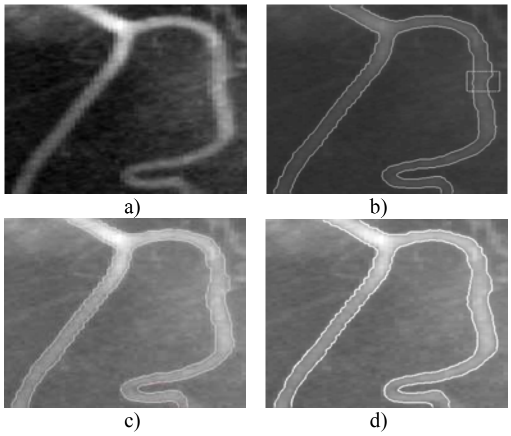

Figure 3. Result of another vessel image. a): original image. b):the result of RSF model. c):the result of the proposed model with dual algorithm. d): the result of the proposed model with Split Bregman algorithm

D. The Segmentation of MR Image

Intensity inhomogeneity often occurs in MR images, it arises from nonuniform magnetic fields as well as from variations in object susceptibility. In fig. 4 we apply our proposed model to the segmentation of MR image of a brain. We can see that some intensities of the white matter in the upper part are even lower than those of the gray matter in the lower part. Fig.4 a) is the original image whose size is 78*119. The white solid line shows the segmentation result of the original RSF model in Fig.4 b), the white dot square is used as the initial contour which is chosen specially. If we choose a non-ideal initial contour, the RSF model fails to segment the object no matter what parameter we choose, which can be seen from Fig.4 c). Fig.4 d) is the result of the dual projection algorithm on our proposed method. Fig.4 e) is the result of the Split Bregman on our proposed method. From the experimental results, we can see that the proposed method is independent of the initial condition. The experimental data can be seen in table 1.

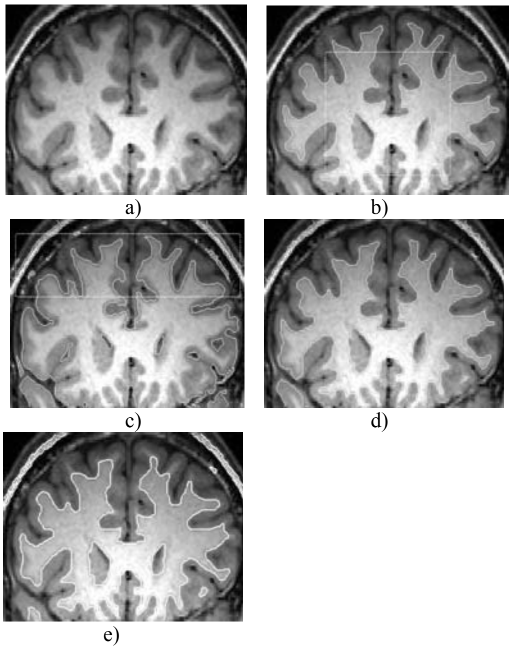

Figure 4. Result of MR image of a brain. a): original image. b):the result of RSF model. c):RSF model fails to segment the image because of non-ideal initial condition. d):the result of the proposed model with dual algorithm.e): the result of the proposed model with Split Bregman algorithm.

E. The Segmentation of the Synthetic Image

Fig. 5 demonstrates the global minimization property of the proposed method. Fig.5 a) is a synthetic image of 127×472 pixels, the brightness of the image is not uniform. Fig.5 b) shows the segmentation result of the original RSF model. Fig.5 c) is the result of the dual projection algorithm on our proposed method. Fig.5 d) is the result of the Split Bregman on our proposed method. From table 1, the fast numerical scheme on the proposed model demonstrates excellent performance.

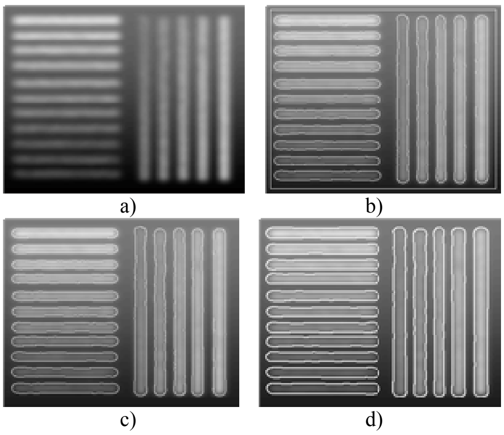

Figure 5. Result of the synthetic image. a): original image. b):the result of RSF model. c):the result of the proposed model with dual algorithm. d): the result of the proposed model with Split Bregman algorithm

VI. CONCLUSION

The paper has proposed a new hybrid active contour model by introducing the global convex optimization idea and edge detector operator into RSF model. The new model not only inherits the advantages of RSF model that can segment images with intensity inhomogeneity or weak object boundaries, but also has no sensitivity to initial contour, which arises from the convex property of the proposed model. We use fast numerical schemes to globally minimize the proposed model, and the experiment results show that the algorithm is more efficient and stable than the existing curve evolution Level Set Method. The analysis of theory and the experiment results prove the efficiency of the algorithm provided in the present paper.

TABLE I.
THE COMPARISON OF ITERATION NUMBER AND CPU TIME （IN SECOND）FOR RSF MODEL AND THE PROPOSED MODEL ON DUAL PROJECTION ALGORITHM AND SPLIT BREGMAN METHOD. THE IMAGES IN TABLE 1 FROM IMAGE1 TO IMAGE5 DENOTE T-SHAPED OBJECT IMAGE, BLOOD VESSEL IMAGE, ANOTHER BLOOD VESSEL IMAGE, BRAIN MRI, THE SYNTHETIC IMAGE, RESPECTIVELY.

		Image1	image2	image3	image4	image5
RSF model		300 (2.7)	180 (1.43)	220 (1.68)	220 (1.83)	40 (19.8)
The proposed model	Dual method	115 (1.5)	85 (1.0)	25 (0.9)	50 (1.3)	16 (5.97)
	Split Bregman	12 (1.15)	15 (0.92)	9 (0.82)	8 (0.72)	14 (4.99)

ACKNOWLEDGMENT

This work was supported by National Natural Science Foundation of China (61201431, 61201430), Shandong Province Natural Science Foundation(ZR2011FM024)

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