An Optimized Grey Cluster Model for Evaluating Quality of Labor Force

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***Abstract*— In this study, the concept of relative membership degree is proposed to investigate grey optimal cluster evaluation. Besides, the correlation coefficient between sample observation value and standard eigenvalue is used to reflect the similarity degree of evaluation objective and each cluster centre. Based on those mentioned above and concept of relative membership degree, an optimized grey cluster evaluation model is established. Meanwhile, Lagrange function is constructed to obtain the relative membership degree. According to the size of relative membership degree, evaluation objectives are classified. Thereby, the fuzzy membership degree information in classification is effectively integrated into grey cluster evaluation. Finally, the effectiveness and practicability of this model is verified through the labor force quality evaluation of three provinces in East China.**

***Index Terms*— grey cluster, triangle whiten weight function, grey correlation analysis, relative membership degree, dynamic quality evaluation**

I. INTRODUCTION

Grey cluster is also known as grey evaluation. It is a method that can cluster some observation indexes or observation objectives into several definable categories according to grey incidence matrix or the whitenization weight function of grey numbers [1][2]. In recent years, grey cluster analysis has been widely applied in the fields of natural science, social science, and economic management etc. and has solved a lot of uncertain issues in real life [3-7]. Using classification of cluster method, grey cluster can be divided into grey correlation cluster and grey whitenization weight function cluster. Among which, grey correlation cluster is mainly employed to incorporate the factors of same class to simplify complex systems. While grey whitenization weight function cluster is majorly applied to inspect whether or not observation objective belongs to preset different classes. If not, the observation objective can be treated differently.

Deng proposed grey correlation cluster method [1]. In this method, the correlation degree between index sequences should be firstly calculated and critical value should be given. When the correlation degree between two index sequences was bigger than the critical value, it was considered that these two indexes belonged to the same class. Meanwhile, Deng also put forward a grey whitenization weight function cluster method [1], in which, the weight of each index was calculated by the critical value of whitenization function. However, the weight obtained was more influenced by the magnitude of sample value. If the dimension between indexes was different, weight calculation shows no practical significance. Zhang conducted grey cluster on grey hazy set [8]. He investigated the greyness of cluster result by establishing grey cluster on grey hazy set and combining cluster result with cluster weight sequence. Xiao et al. put forward grey optimal cluster [9]. By their method, the whitenization weight function constructed was related to the standard value of each class. Thereby, arbitrary measured value of sample index was not zero to each class. Then the correlation degree and difference degree between classes were calculated, and cluster was performed by generalized a weighted distance method. Liu et al. carried out a grey fixed weight cluster decision analysis method [2]. The weight of each index was firstly determined by qualitative analysis or quantitative analysis (Delphi method or Analytic Hierarchy Process). Then grey cluster was performed by whitenization function; Liu & Xie put forward a grey cluster evaluation basing on triangle whitenization weight function [10], this method divided the value range of each index into s clusters according to evaluation requirements. Then calculation was conducted by grey fixed weight cluster. Qiu gave a grey correlation cluster analysis method [11]. In this method, grey similar matrix was calculated basing on the computation of grey correlation degree. Then cluster was performed according to maximal tree method or coding method. Dang et al. investigated the comprehensive cluster evaluation when cluster coefficient showed no significant differences. Moreover, they proved the condition of that the cluster obtained when grey cluster was aggregated by maximum principal was consistent with the results constructed by comprehensive cluster evaluation. Meanwhile, the grey cluster method of the two stages was studied [12].

To reflect the effect of fuzzy information in grey cluster evaluation more effectively, a grey optimal cluster

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model was built in this study basing on relative membership degree concept, triangle whitenization weight function, and grey correlation degree analysis. Whilst Lagrange function was constructed to find relative membership degree, according to which, the evaluation objective was classified. Finally, the effectiveness and practicability of this model was verified by the labor force quality evaluation of three provinces in East China.

II. METHODOLOGY

A. The Traditional Grey Cluster Model

Definition 1 Assume that there exists n objects, m criterions, p different grey classes, based on the observational value $x_{ij}\,(i=1,2,\cdots n,\ j=1,2,\cdots m)$ of the i th object $(i=1,\ 2,\ldots,n)$, at the j th criterion $(j=1,\ 2,\ldots,m)$, the i th object is classified into the k th $(k\in\{1,2,\ldots,p\})$ grey class, this is called grey assessment.

Definition 2 All the grey classes formed by the n objects, defined by their observational values at criterion j, are called the j-criterion subclasses. The whitening weight function of the k th subclass of the j-criterion is denoted $f_j^k(\bullet)$.

Proposition 1 1° The typical whitening weight function is given by

$$f_j^k(x)=\begin{cases}0 & x\notin[x_j^k(1),x_j^k(4)]\\ \dfrac{x-x_j^k(1)}{x_j^k(2)-x_j^k(1)} & x\in[x_j^k(1),x_j^k(2)]\\ 1 & x\in[x_j^k(2),x_j^k(3)]\\ \dfrac{x_j^k(4)-x}{x_j^k(4)-x_j^k(3)} & x\in[x_j^k(3),x_j^k(4)]\end{cases}.$$

2° The whitening weight function of lower measure is given by

$$f_j^k(x)=\begin{cases}0 & x\notin[0,x_j^k(4)]\\ 1 & x\in[0,x_j^k(3)]\\ \dfrac{x_j^k(4)-x}{x_j^k(4)-x_j^k(3)} & x\in[x_j^k(3),x_j^k(4)]\end{cases}.$$

3° The whitening weight function of moderate measure is given by

$$f_j^k(x)=\begin{cases}0 & x\notin[x_j^k(1),x_j^k(4)]\\ \dfrac{x-x_j^k(1)}{x_j^k(2)-x_j^k(1)} & x\in[x_j^k(1),x_j^k(2)]\\ \dfrac{x_j^k(4)-x}{x_j^k(4)-x_j^k(2)} & x\in[x_j^k(2),x_j^k(4)]\end{cases}.$$

4° The whitening weight function of upper measure is given by

$$f_j^k(x)=\begin{cases}0\ , & \mathrm{x}<x_j^k(1)\\ \dfrac{x-x_j^k(1)}{x_j^k(2)-x_j^k(1)}\ , & x\in[x_j^k(1),x_j^k(2)]\\ 1\ , & x\geq x_j^k(2)\end{cases}.$$

The whitening function $f_j^k(x)$ is studied out subjectively by experts' experience. $f_j^k(x_{ij})$ reflects the degree of i th object classified into k grey class at the j th criterion, while $0\leq f_j^k(x_{ij})\leq 1$.

B. Determining Triangle Whitenization Weight Function

We set n objectives, m evaluation indexes, s different grey classes, and sample observation value of objective i on index j be x_{ij}, $i=1,2,\cdots,n;\ \ j=1,2,\cdots,m$. According to the value of x_{ij}, corresponding objective i is evaluated and measured. The specific procedure of the grey evaluation method based on triangle whitenization weight function in indicated in the following.

In accordance with the grey class number s classified by evaluation requirements, the value range of each index is correspondingly divided into s grey classes, i.e. the value range $[a_1,a_{s+1}]$ of index j is divided into:

$$[a_1,a_2],\cdots,[a_{k-1},a_k],\cdots,[a_{s-1},a_s],[a_s,a_{s+1}].$$

Where, the value of a_k $(k=1,2,\cdots,s,s+1)$ can be determined by practical condition requirements or qualitative research results.

Let the whitenization weight function value of $\lambda_k=(a_k+a_{k+1})/2$ belonging to the k grey class be 1. Connecting $(\lambda_k,1)$ with the starting point a_{k-1} of the $k-1$ grey class and the ending point a_{k+2} of the $k+1$ grey class receptively, the triangle whitenization weight function $f_j^k(\bullet)$, $j=1,2,\cdots;m;k=1,2,\cdots,s$ of index j on k grey class is obtained. As for $f_j^1(\bullet)$ and $f_j^s(\bullet)$, the number field of index j can be extended right toward and left toward to a_0 and a_{s+2} respectively (as shown in Figure 1).

The triangle whitenization weight function of index j belonging to the grey class k can be acquired according to formula:

$$f_j^k(x)=\begin{cases}0 & x\notin[a_{k-1},a_{k+2}]\\ \dfrac{x-a_{k-1}}{\lambda_k-a_{k-1}} & x\in[a_{k-1},\lambda_k]\\ \dfrac{a_{k+2}-x}{a_{k+2}-\lambda_k} & x\in[\lambda_k,a_{k+2}]\end{cases} \quad (1)$$

By observation value x, the triangle whitenization weight function $f_j^k(x)$ of observation value x belonging to grey class $k(k=1,2,\cdots,s)$ can be calculated.

C. Calculating the Correlation Coefficient between Sample Observation Value and Standard Eigenvalue

Seen from formula (1), when $x=\lambda_k=(a_k+a_{k+1})/2$, the triangle whitenization weight function value of index j belonging to the k grey class is 1, that is $f_j^k(\lambda_k)=1$. Therefore, $\lambda_k^j(j=1,2,\cdots,m)$ is noted as the standard eigenvalue of index j belonging to the k grey class.

Let $y_k(j)=\lambda_k^j$, and

$$y_k=(y_k(1),y_k(2),\cdots,y_k(m))$$

The sample observation value of objective i on m indexes is

$$x_{ij}=(x_{i1},x_{i2},\cdots,x_{im}),i=1,2,\cdots,n.$$

Let $x_i(j)=x_{ij}$, and

$$x_i=(x_i(1),x_i(2),\cdots,x_i(m)),\quad i=1,2,\cdots,n.$$

Taking y_k as system characteristic sequence, and x_i as system factor sequence, the grey correlation coefficient of y_k and x_i on the j index is

$$\gamma(y_k(j),x_i(j))=\frac{\min_i\min_j|y_k(j)-x_i(j)|+\xi\max_i\max_j|y_k(j)-x_i(j)|}{|y_k(j)-x_i(j)|+\xi\max_i\max_j|y_k(j)-x_i(j)|} \quad (2)$$

Where, ξ is denoted as resolution coefficient, $\xi\in(0,1)$. Generally, we take $\xi=0.5$.

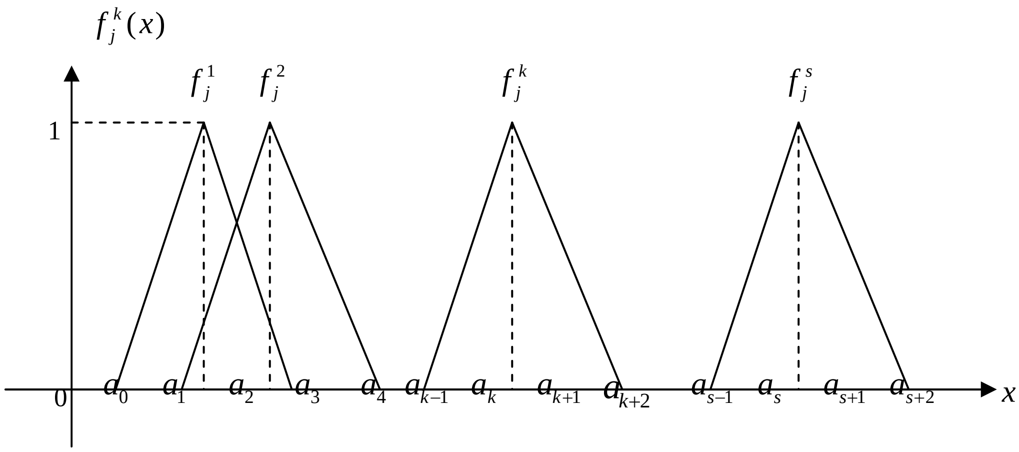

Figure 1. The triangle whitenization weight function of index j belonging to grey class k

D. Establishing Optimization Model

Set η_j^k as the weight of j index in k subclass. And $\sum_{j=1}^m\eta_j^k=1,0\le\eta_j^k\le1$ is named as the grey fixed weight of the objective i of η_j^k belonging to k grey class; if the weight η_j^k $(j=1,2,\cdots,m;\ k=1,2,\cdots,s)$ of j index of k subclass is independent from k, namely, for arbitrary $k_1,k_2\in\{1,2,\cdots,s\}$, there is always $\eta_j^{k_1}=\eta_j^{k_2}$, the superscript k of η_j^k can be omitted. Thus η_j^k $(j=1,2,\cdots,m;\ k=1,2,\cdots,s)$ is noted as η_j $(j=1,2,\cdots,m)$, and η_j is called as the grey fixed weight of objective i belonging to k grey class.

The difference between objective i and the k grey class can be presented as:

$$d_{ik}=\left\{\sum_{j=1}^m\left[\eta_j^k\left|\gamma(y_k(j),x_i(j))-f_j^k(\lambda_k)\right|\right]^p\right\}^{\frac{1}{p}}. \quad (3)$$

Where, when $p=2$, d_{ik} is Euclidean distance.

Assuming that u_{ik} is the relative membership degree of cluster objective i to the k grey class, weighted generalized distance is

$$D_{ik}=u_{ik}d_{ik}, \quad (4)$$

where, $\sum_{i=1}^m u_{ik}=1,0\le u_{ik}\le1$.

To find the optimal value of u_{ik}, the optimized model is established in the following:

$$\begin{cases}\min F(u_{ik})=\sum_{k=1}^s u_{ik}D_{ik}\\ s.t.\sum_{i=1}^s u_{ik}=1,0\le u_{ik}\le1\end{cases}. \quad (5)$$

The Lagrange function is constructed as:

$$L(u_{ik},\lambda_i)=\sum_{k=1}^{s}u_{ik}D_{ik}-\lambda_i\left(\sum_{k=1}^{s}u_{ik}-1\right). \quad (6)$$

Solving the function above, we obtain:

$$u_{ik}=\frac{d_{ik}^{-1}}{\sum_{k=1}^{s}d_{ik}^{-1}} = \frac{\left\{\sum_{j=1}^{m}\left[\eta_j^k\left|\gamma(y_k(j),x_i(j))-f_j^k(\lambda_k)\right|\right]^p\right\}^{\frac{1}{p}}}{\sum_{k=1}^{s}\left\{\sum_{j=1}^{m}\left[\eta_j^k\left|\gamma(y_k(j),x_i(j))-f_j^k(\lambda_k)\right|\right]^p\right\}^{\frac{1}{p}}}. \quad (7)$$

Since $f_j^k(\lambda_k)=1$, the following formula is acquired

$$u_{ik}=\frac{\left\{\sum_{j=1}^{m}\left[\eta_j^k\left|\gamma(y_k(j),x_i(j))-1\right|\right]^p\right\}^{\frac{1}{p}}}{\sum_{k=1}^{s}\left\{\sum_{j=1}^{m}\left[\eta_j^k\left|\gamma(y_k(j),x_i(j))-1\right|\right]^p\right\}^{\frac{1}{p}}} \quad (8)$$

Basing on $\max_{1\le k\le s}\{u_{ik}\}=u_{ik^*}$, objective i is judged as belonging to grey class k^*; Moreover, when there are multiple objectives belongs to k^* grey class, the merits and precedence of the objectives belonging to k^* grey class can be further determined by the size of relative membership degree.

To sum up, the procedure of relative membership degree-based grey optimal cluster can be indicated as follows:

a) According to the grey class number s divided by evaluation requirements, the value range of each index is correspondingly classified into s grey class.

b) Determining triangle whitenization weight function.

c) Calculating the correlation coefficient between sample observation value and standard eigenvalue.

d) Calculating the relative membership degree u_{ik} of objective $i(i=1,2,\cdots,n)$ belonging to grey class $k(k=1,2,\cdots,s)$.

e) According to $\max_{1\le k\le s}\{u_{ik}\}=u_{ik^*}$, judging that objective i belongs to grey class k.

Ⅲ. An actual example of evaluating quality of labor force

Labor force quality is an important factor influencing the economic and social development of one country or a region. Since labor force quality is a concept with uncertain connotation, and labor force quality evaluation system is a grey system, so grey evaluation method can be applied reasonably. The comprehensive evaluation of labor force quality can be performed in three aspects, which are cultural and educational quality, labor skill quality, and physiological physical quality. And each sub term of three aspects can be constituted by several evaluation factors and evaluation indexes. To enhance the reliability and comparability of comprehensive evaluation result, 3 sets, 12 statistical indexes in total are selected by analyzing the discriminability and membership degree of evaluation index and considering the feasibility of data collection. These statistical indexes constitute the labor force quality evaluation system in this study. Then, with the labor force evaluation of three provinces in East China proposed in [9][13] as an example, the relative membership degree-based grey cluster evaluation method put forward in this study is applied to comprehensively evaluate the labor force quality of three provinces in East China (noted as A, B, C respectively). Evaluation index system and evaluation index data are shown in Table 1. Classification standard and index weight are listed in Table 2 and Table 3.

TABLE 1.
THE EVALUATION INDEX SYSTEM AND PRACTICAL DATA OF LABOR FORCE QUALITY OF THREE PROVINCES IN EAST CHINA

Evaluation index	Practical data of three provinces		
	A	B	C
Education fund input(‰)	19	17	23
Educated people (one hundred)	72	78	70
Cultural fund input(‰)	6	8	5
Publishing input (owned by one hundred people)	32	33	30
Research fund input (‰)	2.4	6.3	7.2
People engaged in research (10 thousand)	48	78	75
Material production input (‰)	776	715	254
Overall labor force productivity (one thousand Yuan / one person)	41.6	46.3	46
Young and middle-aged people (one hundred)	58	52	68
Non-disabled people (one hundred)	95	94	92
Consumption level (one hundred Yuan /one person)	28.8	28	21.5
Average life(year)	68	66	63

TABLE 2.
THE CLASSIFICATION STANDARD OF LABOR FORCE QUALITY EVALUATION INDEXES

Index	Classification standard			
	Low level	Middle level	Relatively high level	High level
Education fund input(‰)	8—18	18—22	22—25	25—30
Educated people (one hundred)	5—35	35—60	60—75	75—85
Cultural fund input(‰)	2—7	7—10	10—13	13—15
Publishing input (owned by one hundred people)	20—32	32—38	38—42	42—50
Research fund input (‰)	1—5	5—10	10—12	12—14
People engaged in research (10 thousand)	10—40	40—60	60—70	70—80
Material production input (‰)	200—500	500—650	650—700	700—800
Overall labor force productivity (thousand Yuan / one person)	10—30	30—40	40—50	50—60
Young and middle-aged people (one hundred)	10—40	40—65	65—70	70—80
Non-disabled people (one hundred)	70—86	86—92	92—96	96—98
Consumption level(one hundred Yuan /one person)	10—22	22—28	28—35	35—50
Average life (year)	30—60	60—67	67—70	70—85

TABLE 3.
THE WEIGHT DISTRIBUTION OF LABOR FORCE QUALITY EVALUATION INDEX

Index	Weight
Education fund input(‰)	0.175
Educated people	0.075
Cultural fund input(‰)	0.046
Publishing input	0.0375
Research fund input(‰)	0.119
People engaged in research	0.064
Material production input(‰)	0.06
Overall labor force productivity	0.09
Young and middle-aged people	0.187
Non-disabled people	0.047
Consumption level	0.065
Average life	0.035

Firstly, according to the grey class number 4 classified via evaluation requirements, the value range of each index are correspondingly divided into 4 grey classes. That is, the first class is low level, the second class is middle level, the third class is relatively high level, and the fourth class is high level.

In accordance with the definition of triangle whitenization weight function, and triangle vertex is corresponded to the key point of each interval, the triangle whitenization weight function of each interval is determined. Based on the functions, the cluster centre of the 4 cluster class is further calculated, as shown in the following:

$$y_1 = (13,20,4.5,26,3,25,350,20,25,78,16,45)$$

$$y_2 = (20,47.5,8.5,35,7.5,50,575,35,52.5,89,25,63.5)$$

$$y_3 = (23.5,67.5,11.5,40,11,65,675,45,67.5,94,31.5,68.5)$$

$$y_4 = (27.5,80,14,46,13,75,750,55,75,97,42.5,77.5)$$

The relative membership degree matrix can be obtained by calculating the relative membership degree u_{ik} of objective $i(i=1,2,\cdots,n)$ belonging to grey class $i(i=1,2,\cdots,n)$. It is:

$$(u_{ik}) = \begin{bmatrix} 0.122 & 0.155 & 0.260 & 0.463 \\ 0.101 & 0.140 & 0.368 & 0.392 \\ 0.328 & 0.219 & 0.228 & 0.225 \end{bmatrix}$$

Through classification standard $\max_{1\le k\le s}\{u_{ik}\} = u_{ik^*}$, it is judged that objective i belongs to grey class k^*: objective A belongs to grey class 4, objective B belongs to grey class 3, objective C belongs to grey class 1.

IV. CONCLUSION

This paper introduces the concept of relative membership degree into the grey cluster model. Besides, the optimization model of cluster evaluation is established based on grey whitenization weight function and grey correlation degree. By constructing Lagrange function, relative membership degree is obtained. According to the size of relative membership degree, evaluation objectives are classified. Thereby, the fuzzy membership degree information in classification is effectively integrated into grey cluster evaluation. Therefore, the method proposed in this study can better reflect the fuzziness for people to comprehending the outside world.

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