

# A Modification of Relational Two-Stage DEA with Variable Returns to Scale

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**Abstract**—Traditional studies in two-stage data envelopment analysis (DEA) assumed that the outputs of the first stage completely input into the second stage. But we think that assumption is not realistic because of efficiency loss. This paper modifies the traditional relational two-stage DEA with Variable Returns to Scale (VRS) by abandon this assumption. By introduction of efficiency loss to measure the difference between Intermediate outputs and Intermediate inputs, the traditional model can be modified, and the new model is proved to be more powerful and useful in calculating efficiency. In order to test the model, a case of 14 Chinese listed banks is used as an example to illustrate the whole idea.

**Index Terms**—DEA, Two-stage DEA, Variable Returns to Scale, Efficiency loss

## I. INTRODUCTION

Data envelopment analysis (DEA) is an effective technique for measuring the relative efficiency of a set of DMUs which utilize the same inputs to produce the same outputs and it was initially developed by Charnes, Cooper and Rhodes (CCR model) [1], and further extended by Banker, Charnes and Cooper (BCC model) [2]. Since DEA was first introduced in 1978, researchers in a number of fields have quickly recognized that it is an excellent and easily used methodology for modeling operational processes for performance evaluations[3-7]benchmarking[8-12],and decision making[13-17].In traditional DEA models, it treat its reference technologies as black boxes, Fare and Grosskopf[18][19]and Fare et al. proposed several Network DEA to look into these boxes and to evaluate organizational performance and its component performance by decomposing the stages of the production process [20]. Lewis and Sexton developed the Network DEA to efficiency analysis of organizations with complex internal structure [21]. Nowadays, different from the traditional DEA model , the Network DEA doesn't has a standard form, Fare and Whittaker (1995), Lewis and Sexton (2004), developed static Network DEA to model intermediate products, where some outputs of certain processes are consumed by other processes as inputs. Opposed to the static model, there is a dynamic

model, in which some outputs of the process at a period are inputs of the process of the next period.

Kao and Hwang [22]investigate efficiency decomposition in a two-stage production process where the outputs of the first stage are the inputs of the second stage and they took takes the series relationship of the two sub-processes into account in measuring the efficiencies. Wei Huang [23] found that the relational network two-stage DEA proposed by Kao[22][24]is based on Constant Returns to Scale(CRS),so the efficiency model by this model would be influenced returns to scale, then Wei Huang [23]built the relational two-stage DEA with Variable Returns to Scale to solve that problem. We found that in previous studies, the outputs of the first stage completely input into the second stage. However, in the actual operation process there must be efficiency loss when put the Intermediate outputs of the first stage into Intermediate inputs of the second stage.

After joining the WTO, The market competition environment is becoming increasingly fierce, the enterprises in China are facing unprecedented opportunities and challenges, to maintain and enhance competitiveness, improve the efficiency is an effective way. Efficiency evaluation is an important content of efficiency and it is a basic guarantee for efficiency enhancing. Objectively and scientifically on efficiency evaluation of the listed company to found itself in the advantages and disadvantages, and through the improvement of management strategies to improve efficiency evaluation and market competitiveness, so it is an important premise for improving the competitiveness of listed companies. However the traditional efficiency evaluation method has great limitations, on one hand, it take the listed company as "black box", ignore the internal mechanism and just take the initial input and output to investigate the efficiency, it can't has comprehensive result, thus it is necessary to explore internal operation process from the Angle of the listed company efficiency evaluation, On the other hand, the traditional method for most static evaluation, and located in practical production process is a dynamic process, it will be as itself and the changes in the outside world and

the corresponding adjustment, static evaluation results do not reflect the real operation efficiency, which affects the correctness of the decision-making, even the company's strategic errors affect the correct operating company, therefore, it is necessary to study the dynamic process of production efficiency.

Based on the existing theory and empirical research, on the basis of the first two stages DEA model was studied, it has some problems, including the system elements, the system structure, and the mixed problem with system elements is deduced for the improvement of two phase correlation DEA model, including constant scale under two stages of remuneration DEA model, related variable scale under two stages of remuneration of the DEA model associated. According to the changes of the influence of efficiency, increase efficiency loss value on the concept of value, efficiency, efficiency loss (enhancement) in a certain extent, reflected elements loss (enhancement). Secondly, combined with the practical process of listed companies operating in China, with 14 listed Banks, as an example, the relative evaluation results improved two-phase association DEA model, applicability, and stronger ability to identify more close to reality the results. Finally, in the empirical research on the basis of Chinese listed the actual situation of Banks and puts forward the corresponding ways to improve efficiency and management level of Chinese listed Banks and market competitiveness has important reference value.

The objective of this paper is to take efficiency loss in to the relational two-stage DEA with Variable Returns to Scale. The efficiency calculated from the new model are more meaningful than those calculated from the traditional model. A case of 14 Chinese listed banks is used as an example to illustrate the whole idea.

The paper is organized as follows. In the next section, we modified the relational two-stage DEA with Variable Returns to Scale in Section 3, we used the 14 Chinese listed banks as an example to text the new model. Finally, we listed the result, then the conclusions.

## II. METHOD AND MATERIAL

### A .DEA Theory

One technique widely applied to measure the relative efficiency of a set of production systems, or decision making units (DMUs), which utilize the same inputs to produce the same outputs, is DEA (data envelopment analysis).

Hypothesis that there are  $n$  DMUs and they are comparable. Let  $X_{ij}$  and  $Y_{rj}$  be defined as the inputs and outputs of the system, respectively,  $i = 1, \dots, m, j = 1, \dots, n, r = 1, \dots, s$ . Denote  $u_r$  and  $v_i$  as the multipliers associated with the  $r$  th output and  $i$  th input. To evaluate the efficiency of  $DMU_0$ , we get programming model for it---model (1) base on CCR model (Charnes et al., 1978):

$$TE_0 = \max \sum_{r=1}^s u_r y_{r0} / \sum_{i=1}^m v_i x_{i0}$$

$$s.t. \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0 \quad j = 1, \dots, n$$

$$u_r, v_i \geq \varepsilon, r = 1, \dots, s; i = 1, \dots, m$$
(1)

$\varepsilon$  means a small non-Archimedean number (Charnes et al., 1979; Charnes and Cooper, 1984) which prohibits any input/output factor to be ignored.

Do Charnes-Cooper linear transformation to model (1), let  $t = (\sum_{i=1}^m v_i x_{i0})^{-1}, \mu_r = t u_r, \omega_i = t v_i$ , then we get an equivalent linear programming model (2):

$$TE_0 = \max \sum_{r=1}^s \mu_r y_{r0}$$

$$s.t. \sum_{r=1}^s \mu_r y_{rj} - \sum_{i=1}^m \omega_i x_{ij} \leq 0 \quad j = 1, \dots, n$$

$$\sum_{i=1}^m \omega_i x_{i0} = 1$$

$$\mu_r, \omega_i \geq \varepsilon, r = 1, \dots, s; i = 1, \dots, m$$
(2)

Model (2) has a dual of the following form and get model (3):

$$\min \theta_0^{CCR} - \varepsilon (\sum_{r=1}^s s_r^+ + \sum_{i=1}^m s_i^-)$$

$$s.t. \sum_{j=1}^n \lambda_j x_{ij} + s_i^- = \theta_0^{CCR} x_{i0} \quad i = 1, \dots, m$$

$$\sum_{j=1}^n \lambda_j y_{rj} - s_r^+ = y_{r0} \quad r = 1, \dots, s$$

$$\lambda_j, s_i^-, s_r^+ \geq 0, i = 1, \dots, m; r = 1, \dots, s; j = 1, \dots, n$$
(3)

For model (1), if  $TE_0 = 1, DMU_0$  is technical effective, or if  $TE_0 < 1$ , then  $DMU_0$  is technical inefficient,  $TE_0$  is the technical efficiency.

For model (3), if  $\theta_0^{CCR} = 1$  and  $s_i^- = s_r^+ = 0$ , then we call the unit evaluated id technical effective; if  $\theta_0^{CCR} = 1$ , but  $s_i^- > 0$  or  $s_r^+ > 0$ , then we call weakly effective; if  $\theta_0^{CCR} < 1$ , then it is technical inefficiency, and  $\theta_0^{CCR}$  is the technical efficiency.

Base on model (3), we can convert any unit  $DMU_0$  which is technical inefficiency into technical effective by transforming like below:

$$x_{i0}^* = \theta_0^{CCR} x_{i0} - s_i^- \quad i = 1, \dots, m$$

$$y_{r0}^* = y_{r0} + s_r^+ \quad r = 1, \dots, s$$
(4)

That is to say, if we want to convert any unit  $DMU_0$  which is technical inefficiency into technical effective, its  $i$  th ( $r = 1, \dots, s$ ) input should be reduced in volume  $(1 - \theta_0^{CCR})x_{i0} + s_i^-$ , and its  $r$  th ( $r = 1, \dots, s$ ) output should be increased in volume  $s_r^+$ .

**B. VRS Relational Two-stage DEA Model**

Wei Huang [23] proposed the two-stage relational DEA with Variable Returns to Scale (VRS): Denote  $x_{ij}$ ,  $i=1, \dots, m$  and  $y_{rj}$ ,  $r=1, \dots, s$  as the  $i$ th input and  $r$ th output, respectively, of DMU $_j$ ,  $j=1, \dots, n$ . Different from the conventional one-stage production process, here the production process is composed of two sub-processes with  $D$  intermediate products  $Z_{dj}$ ,  $d=1, \dots, D$ . Moreover, the intermediate products  $Z_{dj}$  are the outputs of stage 1 as well as the inputs of stage 2. Now, suppose a production process is composed of a series of two sub-processes as depicted in Figure 1.

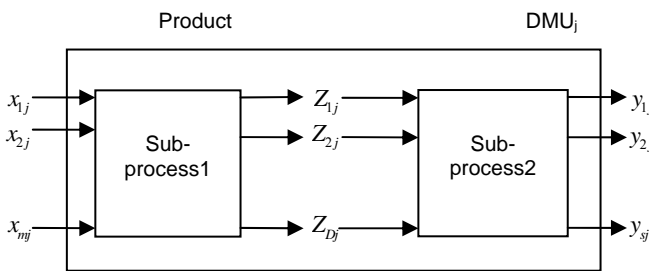


Figure 1. A tandem system with inputs X, outputs Y, and intermediate products Z

For the DMU0 (Decision Making Units) to be evaluated:

$$PTE_0 = \max(\sum_{r=1}^s u_r y_{r0} + \mu_0^{(1)} + \mu_0^{(2)}) / \sum_{i=1}^m v_i x_{i0}$$

$$s.t. \sum_{d=1}^D \delta_d z_{dj} + \mu_0^{(1)} - \sum_{i=1}^m v_i x_{ij} \leq 0 \quad j=1, 2, \dots, n$$

$$\sum_{r=1}^s u_r y_{rj} + \mu_0^{(2)} - \sum_{d=1}^D \delta_d z_{dj} \leq 0 \quad j=1, 2, \dots, n$$

$$u_r \geq \epsilon, v_i \geq \epsilon, \delta_d \geq \epsilon, \mu_0^{(1)} + \mu_0^{(2)} \in R^1$$

PTE is pure technical efficiency. where  $u_r$  and  $v_i$  are the most favorable multipliers to be applied to the  $r$ th output and  $i$ th input for DMU0 in calculating its efficiency PTE and  $\epsilon$  is a small non-Archimedean quantity [1][2] which prohibits any input/output factor to be ignored. Denote  $\delta_d$  as the multiplier, or the importance, associated with the  $d$ th intermediate product. Let  $\mu_0^{(1)}, \mu_0^{(2)}$  denote the slack associated respectively with stage 1 and stage 2,  $\mu_0^{(1)}, \mu_0^{(2)}$  can also reflect returns to scale.

**C. Modification of VRS Relational Two-stage DEA Model**

In previous studies, the outputs of the first stage completely input into the second stage. However, in the actual operation process, following problems should be considered.

(1) The system elements

Is the operation of the system fully effective? How to deal with the change of system elements?

(2) The system structure

How to add an affixation process to response to external space change, during the operation of the system?  
 (3) Mixed problem

How to deal with both system elements change and the affixation process?

This paper will discuss above problems and derive mathematical model of the first problem.

Now consider the two-stage DEA model that intermediate elements change (Figure 2)

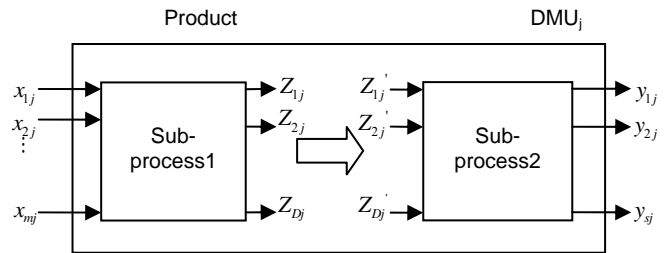


Figure 2. A tandem system with inputs X, outputs Y, and intermediate products changed from Z to Z'.

So the model should be changed to:

$$PTE_0 = \max(\sum_{r=1}^s u_r y_{r0} + \mu_0^{(1)} + \mu_0^{(2)}) / \sum_{i=1}^m x_{i0}$$

$$s.t. \sum_{r=1}^s u_r y_{rj} + \mu_0^{(1)} + \mu_0^{(2)} - \sum_{d=1}^D v_d z_{dj} \leq 0 \quad j=1, 2, \dots, n$$

$$\sum_{d=1}^D \delta_d z_{dj} + \mu_0^{(1)} - \sum_{i=1}^m v_i x_{ij} \leq 0 \quad j=1, 2, \dots, n$$

$$\sum_{r=1}^s u_r y_{rj} + \mu_0^{(2)} - \sum_{d=1}^D \delta_d z'_{dj} \leq 0 \quad j=1, 2, \dots, n$$

$$u_r \geq \epsilon, v_i \geq \epsilon, \delta_d \geq \epsilon, \mu_0^{(1)} + \mu_0^{(2)} \in R^1$$

Uses Charnes-Cooper transform, let

$$t = 1 / \sum_{i=1}^m v_i x_{i0}, \mu_r = t u_r, \varphi_d = t \delta_d, \omega_i = t v_i,$$

$$\eta_0^{(1)} = t \mu_0^{(1)}, \eta_0^{(2)} = t \mu_0^{(2)}, \tilde{\epsilon} = t \epsilon$$

$$PTE_0 = \max(\sum_{r=1}^s \mu_r y_{r0} + \eta_0^{(1)} + \eta_0^{(2)})$$

$$s.t. \sum_{r=1}^s \mu_r y_{rj} + \eta_0^{(1)} + \eta_0^{(2)} - \sum_{d=1}^D \omega_d z_{dj} \leq 0 \quad j=1, 2, \dots, n$$

$$\sum_{d=1}^D \varphi_d z_{dj} + \eta_0^{(1)} - \sum_{i=1}^m \omega_i x_{ij} \leq 0 \quad j=1, 2, \dots, n$$

$$\sum_{r=1}^s \mu_r y_{rj} + \eta_0^{(2)} - \sum_{d=1}^D \varphi_d z'_{dj} \leq 0 \quad j=1, 2, \dots, n$$

$$\sum_{i=1}^m \omega_i x_{i0} = 1$$

$$\mu_r \geq \tilde{\epsilon}, \varphi_d \geq \tilde{\epsilon}, \omega_i \geq \tilde{\epsilon}, \eta_0^{(1)}, \eta_0^{(2)} \in R^1$$

The dual program is:

$$PTE_0 = \min \theta^{BCC} - \epsilon (\sum_{i=1}^m s_i^- + \sum_{d=1}^D s_d^z + \sum_{r=1}^s s_r^+)$$

$$s.t. \theta^{CCR} x_{i0} - \sum_{j=1}^n \lambda_j x_{ij} - \sum_{j=1}^n \alpha_j x_{ij} - s_i^- = 0 \quad i=1, 2, \dots, m$$

$$\sum_{j=1}^n \beta_j y_{rj} + \sum_{j=1}^n \alpha_j y_{rj} - s_r^+ = y_{r0} \quad r = 1, 2, \dots, s$$

$$\sum_{i=1}^n \lambda_j z_{dj} - \sum_{i=1}^n \beta_j z'_{dj} - s_d^z = 0 \quad d = 1, 2, \dots, D \quad (7)$$

$$\sum_{j=1}^n \lambda_j + \sum_{j=1}^n \alpha_j = 1$$

$$\sum_{j=1}^n \beta_j + \sum_{j=1}^n \alpha_j = 1$$

$$\lambda_j, \beta_j, \alpha_j, s_i^-, s_r^+, s_d^z \geq 0 \quad j = 1, 2, \dots, n$$

Similar to the single-stage case, we can change any DMU0 from pure technical inefficient to pure technical efficient by adjusting its input and output as followed

$$x_{i0}^* = \theta x_{i0} - s_i^- \quad i = 1, \dots, m$$

$$y_{r0}^* = y_{r0} + s_r^+ \quad r = 1, \dots, s \quad (8)$$

$\theta$ ,  $s_i^-$  and  $s_r^+$  are the solution of (3). It should be noticed that  $s_d^z$  in model (3) does not have real meaning.

If  $\mu^*, \omega^*, \phi^*, \eta_0^{(1)*}, \eta_0^{(2)*}$  is the optimal solution of model (2-2),  $u_r^*, \delta_d^*, v_i^*, \mu_0^{(1)*}, \mu_0^{(2)*}$  is optimal solution of model (1) and for DMU0, pure technical efficiency of the whole system and two sub-processes are:

$$PTE_0 = (\sum_{r=1}^s u_r * y_{r0} + \mu_0^{(1)*} * \mu_0^{(2)*}) / \sum_{i=1}^m v_i * x_{i0}$$

$$PTE_0^{(1)} = (\sum_{d=1}^D \delta_d * z_{d0} + \mu_0^{(1)*}) / \sum_{i=1}^m v_i * x_{i0} \quad (9)$$

$$PTE_0^{(2)} = (\sum_{r=1}^s u_r * y_{r0} + \mu_0^{(2)*}) / \sum_{d=1}^D \delta_d * z'_{d0}$$

III. RESEARCH DESIGN AND MODEL USESD

A. Indicators

This paper uses banks to test the new model. In China, banking has an operation with a Series structure. Basically, it has two processes in its operation, the banking service itself and capital investment for profit. For the former, as payment intermediation, bank transacts payment and settlement business of currency, There are several studies which uses the DEA technique to measure the managerial performance of this industry. for the latter, it's a way to accept money deposits, bank can centralize idle funds and small amount of funds, then provide loans to monetary demanders. Simplified, fig. 2 shows operation process of banks in China: In order to uses the new model, the research abstract and simplified the business process of banks as followed:

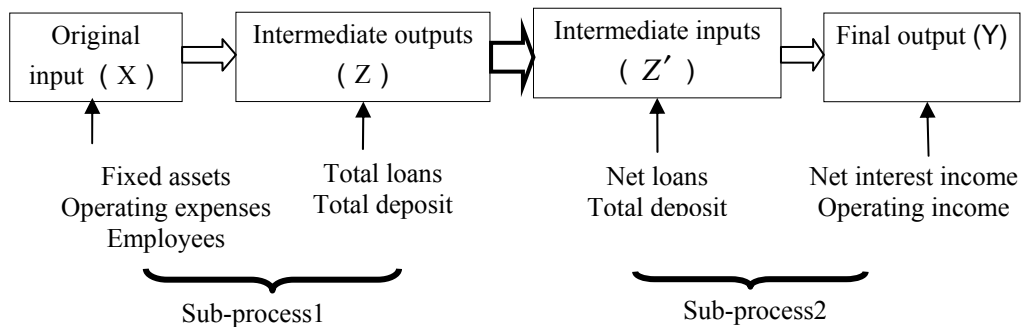


Figure 3. The business process of banks

Sub-process1.: We start from service of bank, Indicators of original input:

- (1) Fixed Asset: fixed asset reflect bank's scale of operation to some extent, and it's also motive power for bank to achieve profit;
- (2) Operating Expenses, during operation process of banking, there are many operating expenses, including management expenses, running expenses and so on;
- (3) Employees, human resources are basic investment for banks to operate, measuring with average number of employees during the period.

The reason why other information from income statement isn't selected is that they occur in the process in which banks are developing their business, their expenses can be offset by relational income, and don't occur in full.

Indexes of intermediate output:

- (1) Total Loans: loan is main source for bank gain income, so it reflects capability that bank gain income to some extent;

- (2) Total Deposit: deposit comes from clients, non-financial enterprises and also other banks. Deposit is equal to "raw material" for bank, there will be not credit and other investment activity without deposit, and if deposit cannot be used as investment funds, bank will not keep deposit and pay interest for its clients, so it's logical to classify like upside.

Sub-process2

Intermediate inputs Indicators are:

- (1) Net loans: because of some loans cannot be repaid, so not all loans can generate profits. This paper uses loan loss provisions to measure the loans that cannot be repaid, and net loans=totals loans—loans loss provisions.

- (2) Total deposit: There is no loss of total deposit when change it from Intermediate outputs to Intermediate inputs.

Final output indicators are:

(1) Net interest income: net interest income reflects the profitability of bank in traditional areas of financial services, and net interest=net interest income from loans—interest expense on deposits.

(2) Operating income: operating income can comprehensive reflect profits in operating.

*B. Data*

This paper selects 14 Chinese listed banks to be evaluated, because listed banks have many advantages, such as asset quality advanced management concept and as the leader in banking, non-listed banks to learn from.

The 14 Chinese listed banks are The industrial and commercial bank of China(ICBC),China construction bank(CCB), the bank of China(BOC), bank of communications(BCM), China citic bank(CNCB), China merchants bank(CMB), Shanghai pudong development bank(SPDB), Industrial bank(CIB), China minsheng bank corporation(CMBC), Hua Xia bank(HB), Shenzhen development bank(SDB), Bank of Beijing(BOB), Bank of Nanjing(NJCB) and Bank of Ningbo(NBCB). Data mainly comes from 2005 to 2009, China Financial Yearbook and Annual Report issued by the banks.

TABLE I  
THE RELEVANT DATA OF 14 LISTED BANKS IN 2009

2009	Employees	Fixed assets	Operating expenses	Total deposit	Total loans	Net loans	Operating income	Net interest income
BOC	262566	1099.54	810.76	71734.53	42088.3	40958.8	2321.98	1588.81
CCB	301537	746.93	879	73658.02	48197.73	47341.45	2671.84	2118.85
ICBC	391048	846.26	1017.03	107022.9	57286.26	55831.74	3094.54	2458.21
CIB	20146	36.23779	114.7356	9026.47	7015.975	7010.383	316.7905	272.0174
BCM	79122	237.28	260.43	24577.84	18393.14	18015.38	809.37	665.64
SPDB	19435	70.74942	132.5348	12991.17	9288.548	9105.08	368.2393	335.3839
SDB	12387	17.14461	63.11091	4622.053	3595.174	3555.625	151.1444	129.8437
CMBC	26039	74.4	177.37	1122.229	8829.79	8677.38	420.6	322.4
CMB	40340	125.19	230.78	16524.67	11858.22	11618.17	514.46	403.64
CNCB	24180	103.21	162.99	12845.16	9743.36	9597.16	408.01	359.84
HB	12301	44.69128	76.87893	5877.997	4302.256	4194.522	171.2964	158.0719
BOB	5681	17.34821	31.35597	4485.56	2734.809	2674.501	118.9411	109.5392
NJCB	2521	11.12035	11.38412	1051.451	670.2805	656.0777	36.27611	31.66853
NBCB	3012	9.26723	17.27595	1386.518	576.3416	532.1765	41.75506	35.539

IV. RESULT

The data of column 2—4 in table 1 is calculated from modified VRS relational two-stage DEA model, the data

of column 5—7 is calculated from the traditional VRS relational two-stage DEA model which was proposed by Wei Huang (2009).

TABLE II  
PURE TECHNICAL EFFICIENCY MEASURE RESULTS OF 14 LISTED BANKS IN 2009

2009	modified VRS relational two-stage DEA model			traditional VRS relational two-stage DEA model		
	PTE	PTE <sup>(1)</sup>	PTE <sup>(2)</sup>	PTE	PTE <sup>(1)</sup>	PTE <sup>(2)</sup>
BOC	0.9534	1.0000	0.9828	0.9709	0.9996	0.9761
CCB	0.9842	1.0000	1.0000	1.0000	1.0000	1.0000
ICBC	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
CIB	0.8752	1.0000	0.8742	0.8909	1.0000	0.8895
BCM	0.8183	1.0000	0.8395	0.8317	1.0000	0.8383
SPDB	0.8029	1.0000	0.8193	0.8172	1.0000	0.8202
SDB	0.8788	1.0000	0.7922	0.8892	1.0000	0.8035
CMBC	0.6754	0.6879	1.0000	0.6885	0.6885	1.0000
CMB	0.5644	0.7467	0.7857	0.5752	0.7467	0.7857
CNCB	0.6946	0.8345	0.8455	0.7073	0.8462	0.8494
HB	0.5660	0.7143	0.8165	0.5788	0.7143	0.8172
BOB	0.8919	1.0000	0.8339	0.9044	1.0000	0.8404
NJCB	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
NBCB	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Maximum	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Minimum	0.5644	0.6879	0.7857	0.5752	0.6885	0.7857
Average value	0.8361	0.9274	0.8993	0.8467	0.9282	0.9015
standard deviation	0.1561	0.1230	0.0906	0.1538	0.1223	0.0883

Seeing from column2, it shows that the PTE of ICBC, NJCB and NBCB achieve 1.0000 which means there is no efficiency loss during operating. CMB has the lowest PTE that is 56.44%, so, in CMB, investment in a unit can produce up to 0.5644 units of output and the other 0.4356 are wasted. It also shows that the average PTE of these 14 banks are 83.61%, overall, efficiency of these banks is perfect.

Compared column 2 with column 5, we can see that PTE calculated from modified VRS relational two-stage DEA model is less than PTE calculated from the traditional VRS relational two-stage DEA model, because in traditional VRS relational two-stage DEA model Intermediate outputs = Intermediate inputs, but in the new model the outputs of the first stage incompletely input into the second stage, this research uses net loans, not total loans as the Intermediate inputs. We can also find that there are 3 banks achieve efficiency of pure technical efficiency in the new model less than that in the traditional model which has four. So the new model is more powerful and useful in screening non-efficiency listed banks than the traditional model

Compared column 3 with column 6, PTE(1) calculated from modified VRS relational two-stage DEA model is approximately equal to PTE(1) calculated from the traditional VRS relational two-stage DEA model, owing to the new model has a change in stage 2 and no change in stage 1 compared with the traditional model.

## V. CONCLUSIONS

In previous studies, the outputs of the first stage completely input into the second stage. The questions are, firstly, the operation of the system may not fully effective and the traditional model has no way to deal with the change of system elements. Secondly, if external space changes, during the operation of the system, then how to add affixations process to response it. This article has attempted to modify the relational two-stage DEA with Variable Returns to Scale by taking efficiency loss into it. Such processes are important in many real-world settings, so the new model is more powerful and useful in calculating efficiency than the traditional one.

By summarizing the previous literature on data envelopment analysis, it showed that the outputs of the

first stage could not completely input into the second stage, in the actual operation process there must be efficiency loss when put the Intermediate outputs of the first stage into Intermediate inputs of the second stage. using 14 Chinese listed banks as an example to illustrate the whole idea, the paper found that PTE calculated from modified VRS relational two-stage DEA model is less than PTE calculated from the traditional VRS relational two-stage DEA model, because of efficiency loss, and the new model is more powerful and useful in screening non-efficiency listed banks than the traditional model.

However, there were several limitations in this study. The research just proposed the concept of efficiency loss, but not study it in quantitative. So the explanation of efficiency loss may not have strength probative force and we can also don't know impact of efficiency loss in two-stage DEA. Otherwise, there should more empirical research to text the ideal. The further research can be done to solve the limitations above.

#### ACKNOWLEDGMENTS

We should say thanks to the funding of Key Project of National Natural Science Foundation in China (No.91024028), Humanities and Social Sciences Foundation of Chinese Ministry of Education (No.10YJC860040), Social Science Foundation of Heilongjiang Province in China (No.11D080) and National Soft Science Foundation in China (No.2008GXS5D113).

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