Constraint Cellular Ant Algorithm for the Multi-Objective Vehicle Routing Problem

Yuanzhi Wang
Geography Department, Dezhou University, Dezhou, Shandong 253023, China
Email: wangyuanzhi1@sina.com

***Abstract*—Constraint Cellular ant algorithm is a new optimization method for solving real problems by using both constraints method, the evolutionary rule of cellular, graph theory and the characteristics of ant colony optimization. Multi-objective vehicle routing problem is very important and practical in logistic research fields, but it is difficult to model and solve because objectives have complicated relationship and restriction. Constraint Cellular ant algorithm has more obvious advantages to solve such kind of combinatorial optimization problems than many other algorithms. The test results show that the constraint cellular ant algorithm is feasible and effective for the MOVRP. The clarity and simplicity of the constraint cellular ant algorithm is greatly enhanced to ant colony optimization.**

***Index Terms*—Constraint cellular ant algorithm, Graph theory, Multi-objective, Vehicle routing problem**

I. INTRODUCTION OF THE MULTI-OBJECTIVE VEHICLE ROUTING PROBLEM

The vehicle routing problem (VRP) has been first proposed by Dantzig and Ramser [1][2]. It aims at the determination of the optimal set of routes to serve a given set of customers using a fixed fleet of vehicles. Following this study, many algorithms have been presented. Both optimal and approximated approaches have been considered [3] [4]. This problem is known to be NP-hard [5].

The multi-objective vehicle routing problem (MOVRP) is the important version of the VRP. MOVRP is very important and practical in logistic research fields, but it is difficult to model and solve because objectives have complicated relationship and restriction.

MOVRP is a kind of problem of multi-objective optimization. Multi-objective optimization (or multi-objective programming or pareto optimization), also known as multi-criteria or multi-attribute optimization, is the process of simultaneously optimizing two or more conflicting objectives subject to certain constraints [6].

II. CONSTRAINTS METHOD FOR MULTI-OBJECTIVE OPTIMIZATION

Project supported by Shandong province office of education (No.S07WZ35)
Corresponding author: wangyuanzhi1@sina.com (Yuanzhi Wang).

doi:10.4304/jsw.8.6.1339-1345

In mathematical terms, the multi objective problem can be written as:

$$\max_x [Z_1(x), Z_2(x), ..., Z_p(x)]^T$$

s.t.

$$g_i(x) \le \forall, \quad i = 1,2,...,c$$

$$x \in X \ .$$

The constraint method, typically employed with traditional mathematical programming methods, generates the non-inferior set for multiple objectives through iterative solution of the following single objective problem

$$\text{Maximize } Z_h(x)$$
$$\text{Subject to } g_i(x) \le \forall, \quad i = 1,2,...,c$$
$$Z_l(x) \ge Z_l^t, \quad l = 1,2,..., p, \quad l \ne h$$
$$x \in X$$

The problem is assumed to be a maximization problem without loss of generality. Z_h is one of k objectives, Z_l^t is the constraint value for objective l , $\{ \ x = x_j : j = 1,2,...,k \ \}$ represents the decision vector, X represents the decision space, c is the total number of constraints, and $g_i(x)$ is the ith constraint. The value of Z_l^t is varied incrementally, making the search migrate from one non-inferior solution to another.

III. THE MATHEMATICAL MODEL OF MOVRP

In order to simplify the problem, MOVRP in the research refers to the single depot vehicle routing problem (SDVRP).

In general, MOVRP meets the following requirements:
(1) Solving MOVRP is the process of simultaneously optimizing two or more conflicting objectives subject to certain constraints.
(2) Each customer can get the timely delivery service, and each customer is only visited by one vehicle only one time.

(3) Every vehicle can only service one route, and the distribution vehicle is initial and terminated in only one depot.
(4) On each distribution lines, the total demand of each customer is no more than the capacity of one vehicle.

MOVRP assume that there is one depot and n customers. The depot is numbered 0 and customer is numbered 1, 2…, n. The transportation cost between i customer and j customer is c_{ij}. The demand of i customer is q_i and the maximum load of the vehicles is T. The number of vehicle is m.

First, the following variables are defined:

$$y_{ik}=\begin{cases}1 & \text{, If the delivery of customer i is finished by vehicle k} \\ 0 & \text{, otherwise}\end{cases} \quad (1)$$

$$x_{ijk}=\begin{cases}1 & \text{, If the vehicle k drives from customer i to customer j} \\ 0 & \text{, otherwise}\end{cases} \quad (2)$$

The objective function:

$$\min Z = \min[z_1(x), z_2(x), \ldots z_n(x), \sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=1}^{m} c_{ij}x_{ijk}] \quad (3)$$

$$\text{s.t.} \sum_{k=1}^{m} y_{ik} = 1, i = 1,2,\ldots,n \quad (4)$$

$$\sum_{i=0}^{n}\sum_{k=1}^{m} x_{ijk} = 1, j = 0,1,\ldots,n \quad (5)$$

$$\sum_{j=1}^{n}\sum_{k=1}^{m} x_{ijk} = 1, i = 0,1,\ldots,n \quad (6)$$

$$\sum_{i} q_i y_{ik} \le T \quad (7)$$

$$\sum_{i=1}^{n}\sum_{k=1}^{m} x_{0ik} = \sum_{j=1}^{n}\sum_{k=1}^{m} x_{j0k} \quad (8)$$

Equation (4), (5) and (6) guarantee that each customer is only visited by one vehicle only one time; Equation (7) ensures that the total demand of each customer is no more than the capacity of one vehicle on each distribution lines; Equation (8) guarantees the distribution vehicle is initial and terminated in only one depot.

In order to make it easier to discuss the problem, the objective function (9) is adopted in the following discussion.

$$\min Z = \min[K, \sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=1}^{m} c_{ij}x_{ijk}] \quad (9)$$

Where, K represents the total vehicle number used.

IV. The Interpretation of Cellular Ant Algorithm

A. The Summary of Cellular Ant Algorithm Research

The Cellular Ant Algorithm combines ant colony optimization with cellular automaton to refrain from falling in to partial optimization while the good optimization ability of ant colony optimization is reserved.

ZhuGang first completely proposes Cellular Ant Algorithm for function and discrete systems optimization based on ant algorithm and cellular automata in his Ph.D. thesis. His research provides a new kind algorithm for NP-hard problems and gives convergence proof of the algorithm. His paper solves the classical TSP by Cellular Ant Algorithm through series of typical instances. The computational results show the effectiveness of the algorithm in numerical simulation [7].

Wang Zhoumian et al. solve the PCB routing problem by using the idea of cellular ant colony optimization. Assisted route distributing and the rule of obstacle avoidance are used for via and route minimization. The algorithm is coded in Delphi. A real world instance is solved and the results are within satisfaction compared with that of Protel [8].

LiuJixin et al. propose that the path planning problem of the mobile robot based on discrete mathematics can be solved by combining CA and ACO. The tests show that both the state result and the numerical result were satisfied with the application requirement. Using the model of CA and ACO to solve the path planning problem is feasible [9].

ZhangJin et al. design an improved cellular automata ant algorithm based on the discovery of the similar structure between the cellular automata and the search area of the rectilinear Steiner minimum tree. The computational results showed that this algorithm can improve the total length about 15% than that of the minimum spanning tree, so the effectiveness of the algorithm has been validated [10].

WangYu et al. give a Cellular Ant Algorithm and its mathematical description that can be used for solving the optimization problem of slope stability. The algorithm is coded in MATLAB and is tested through one of typical engineering date that gives promising results [11].

YeWen et al. propose to use Cellular Ant Algorithm in path planning of Unmanned Aerial Vehicle(UAV). The improved ant colony algorithm was used together with evolutionary rule of cellular in cellular space. The simulation results showed that the Cellular Ant Algorithm could help the solutions to escape from their local optimum and could find a better path at higher convergence speed and with a higher precision [12].

The above algorithms have different features and advantages, but they are not designed for MOVRP. In order to presents the Cellular Ant Algorithm for MOVRP, introduction and improvement of ant colony optimization and cellular automaton is necessary.

B. The Combination of Cellular Automaton and Graph

An cellular automaton is represented formally by a 4-tuple $A = (L, S, N, f)$, where

L is cellular spaces;
S is a finite set of states;

$N = (s_1, s_2, ..., s_n)$, n is the neighbor number of center cell;

s_i represents a state of cell;

f is the transition function, that is, f: $s_n \rightarrow S$.

In order to use cellular automata in solving MOVRP, it is needed to combine CA and graph theory.

Firstly, all the vertices in graph will be defined as cells which are the basic components of CA. So cellular spaces in graph is

$L = \{ v_1, v_2, ..., v_n \}$, $\{ v_1, v_2, ..., v_n \}$ is the set of vertices of G = (V, E).

In CA theory, the von Neumann neighborhood and the Moore neighborhood are the two most commonly used neighborhood types.

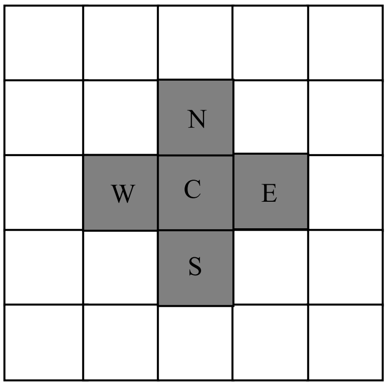

Figure 1. The von Neumann neighborhood

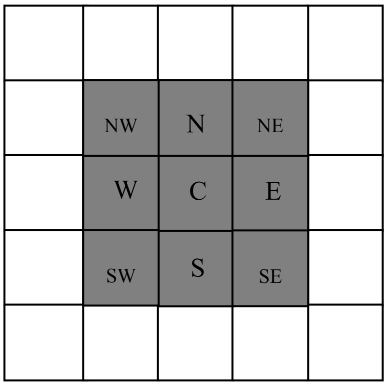

Figure 2. The Moore neighborhood

The von Neumann neighborhood comprises the four cells orthogonally surrounding a central cell on a two-dimensional square lattice. The neighborhood is named after John von Neumann, which is illustrated in Figure 1.

The Moore neighborhood comprises the eight cells surrounding a central cell C on a two-dimensional square lattice. The neighborhood is named after Edward F. Moore, which is shown in Figure 2.

Of course, there are other neighborhood types, but the basic principle of these types is that cells can influence each other, whose distance between each other is short. The principle can be called close principle. At certain times, the close principle is right to biological cells in the body, but in some cases, two cells which are farther together, the interaction between them are relatively large.

The close principle is not fit for the cellular automaton on graph. In graph, obviously to two vertices which are close together, if there is no edge directly connected, the interaction between them is weak. Also, to two vertices which are farther together, if there is edge directly connected, the interaction between them is relatively large. For example, in Figure 3, vertex 1 is closer to vertex 5 than vertex 2, but there is no edge directly connecting vertex 1 and vertex 5, so the interaction between vertex 1 and vertex 5 is weaker than the interaction between vertex 1 and vertex 2. The same relation lies in vertex 1, vertex 2 and vertex 5; vertex 1, vertex 4 and vertex 5; vertex 3, vertex 2 and vertex 6; vertex 3, vertex 4 and vertex 6.

Finally, in graph, the definition of central cell's neighborhood is

$$N(v_x) = \{ v | (v_x, v) \in E \vee (v, v_x) \in E \vee v = v_x \}.$$

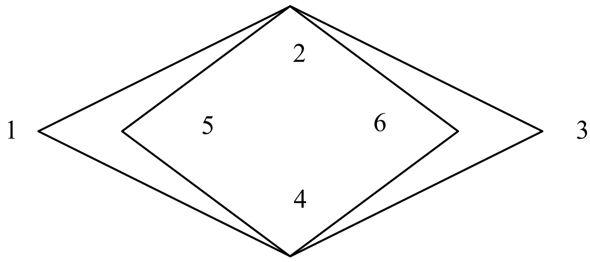

Figure 3. The cellular neighborhood on graph

C. The Model of ACO on Graph

If ant colony is put on graph G = (V, E), For ant k, the probability P_{ij}^k of moving from vertex i to vertex j depends on the combination of two values, viz., the attractiveness η_{ij} of the move, as computed by some heuristic indicating the a priori desirability of that move and the trail level τ_{ij} of the move, indicating how proficient it has been in the past to make that particular move.

In general, the k'th ant moves from vertex i to vertex j with probability:

$$P_{ij}^k = \frac{(\tau_{ij}^{\alpha})(\eta_{ij}^{\beta})}{\sum (\tau_{ij}^{\alpha})(\eta_{ij}^{\beta})}$$

Where,

τ_{ij} is the amount of pheromone deposited for transition from cell i to j,

$0 \le \alpha$ is a parameter to control the influence of τ_{ij},

η_{ij} is the desirability of vertex transition ij (a priori knowledge, typically 1 / c_{ij}, where c_{ij} is the transportation cost between vertex i and vertex j),

$\beta \ge 1$ is a parameter to control the influence of η_{ij}.

Pheromone update is as follows:

When all the ants have completed a solution, the trails are updated by

$$\tau_{ij}^{k} = (1-\rho)\tau_{ij}^{k} + \Delta\tau_{ij}^{k}$$

Where,

τ_{ij}^{k} is the amount of pheromone deposited for a transition i to j,

$\tau_{ij}(0)$ is the initial pheromone level assumed to be a small positive constant distributed equally on all the paths of the network since the start of the survey,

n is the number of vertexes needing service,

ρ is the pheromone evaporation coefficient,

$\Delta\tau_{ij}^{k}$ is the amount of pheromone deposited,

$$\Delta\tau_{ij}^{k} = \begin{cases} Q/L_k & \text{if ant k uses curve ij in its tour} \\ 0 & \text{otherwise} \end{cases}$$

where L_k is the cost of the k'th ant's tour (typically length) and Q is a constant.

D. The Cellular Ant Algorithm on Graph

Cellular automata $A = (L, S, N, f)$ is constructed on n vertex weighted graph G (V, E), where, cellular space is

$$L = \{v_0, v_1, ..., v_n\};$$

v_0 represents the depot. $v_1, v_2 ... v_n$ represent the customers. State set is

$$S = (S\_N, S\_M)$$

where, S_N means that the vertex is in initial state, and is not searched by ant; S_M indicates that the vertex is in mature state, that is the road has been searched by ant.

V. Constraint Cellular Ant Algorithm

As a global searching approach, cellular ant algorithm has some characteristic, such as positive feedback, distributing, paralleling, self- organizing, etc, so it is well suited to solve multi-objective optimization by extending the algorithm. Since all the different objectives interrelate to and interact with each other in multi-objective optimization, constraints method was introduced into cellular ant algorithm.

For the p objectives $Z_1(x), Z_2(x), ..., Z_p(x)$ of multi-objective optimization, p ant colonies are built. Each ant colony consists of a group of ants with own pheromone which is independent to the other ant colony. Although each ant colony walks on the same routes in the same region, the different ant colony which optimizes different objective cannot interference each other. The pheromone released from one ant colony has no effect on ants of other ant colony. In this way, it can ensure that each optimization objective is independent, and initial solution optimized respectively is obtained. For a static moment of running system, ant colony Ant_h communicates with $Ant_l(l = 1,2,..., p, \ l \neq h)$, and gains the best solution of the other objective. The goal of Ant_h is optimization of the objective $Z_h(x)$, so the $Ant_l(l = 1,2,..., p, \ l \neq h)$ is limited on a value greater than Z_l^t , and the objective $Z_h(x)$ is optimized. The optimization process of other ant colony coincides with Ant_h .Therefore, the each objective is optimized individually, but through information exchange of different ant colony, the individual optimization is based on the excellent solution of other ant colony optimization. The solution set of the multi objective problem is obtained by cellular ant algorithm, which is the each objective approach gradually to the best solution.

VI. The Implementation Procedure of Constraint Cellular Ant Algorithm for MOVRP

The constraint Cellular Ant Algorithm for MOVRP includes the four procedures: CCAA_MOVRP_MAIN, CCAA_MOVRP_VM, CCAA_MOVRP_LEN, ANT_CONSTRUCT.

A. The CCAA_MOVRP_MAIN Procedure

Sep1. initialization, initial solution is built by cellular ant algorithm, and saved in ψ_g .

Step2. The main loop

Circulation is stopped until stop conditions is meet (designated iteration times or the difference between the former optimization and the following optimization is little).

{

step2.1 CMCS_MOVRP_VM procedure is called. The objective Z_2 optimized is obtained through exchanging information with the Ant_2 .On that basis, the objective of vehicle number is optimized, and the better solution is obtained. If the optimization can not continue, the objective Z_1 maintain invariable.

step2.2 CMCS_MOVRP_LEN procedure is called. The objective Z_1 optimized is obtained through exchanging information with the Ant_1 .On that basis, the objective of path length is optimized, and the better solution is obtained. If the optimization can not continue, the objective Z_2 maintain invariable.

step2.3 If the solution obtained from Ant_1 optimization or Ant_2 optimization is better than ψ_g , ψ_g is updated.

}

B. The CCAA_MOVRP_VM Procedure

For the ant colony aiming at the objective of vehicle number, concrete process CCAA_MOVRP_VM is the following:

Step1. ant colony, pheromone array Pheromone_vei, ant individuals are initialized. The total city number which has been visited to the current best solution is calculated and saved in Cur_City_Counts.

Step2. The main loop

Circulation is stopped until stop conditions is meet (designated iteration times or the difference between the former optimization and the following optimization is little).

{

Step2.1 The objective Z_2 optimized is obtained through exchanging information with other ant colony.

Step2.2 for (i=0; i<m; i++)

{

On the base which the objective Z_2 is limited, $\psi_k = ConstructTours$ that could be infeasible solution.

}

Step2.3 if the solution satisfies: the city number that has been visited is the biggest one; the total path length meets requirements of the objective Z_2 ; the other constraints, the solution is saved in ψ_{best} which represents local optimal solution.

Step2.4 pheromone array Pheromone_vei is updated by the local optimal solution.

}

Step3 ψ_{best} is returned.

The procedure CCAA_MOVRP_LEN that optimizes the path length is similar as the CCAA_MOVRP_VM.

C. The ANT_CONSTRUCT Procedure

The procedure ANT_CONSTRUCT that ant colony optimizes the single objective is following.

Step1: initialization:

t=0, $\tau_{ij}(0) = \text{const}, \Delta\tau_{ij} = 0$, The m ants is located in the depot, NC=0, $w_k = 0$, $CA_{(\min)} = 0$;

Step2: let s=1

for (k=1; k<=m; k++)

After k ant start from the depot, the state of the first visiting cellular (customer) is to S-M and the cause of S-M is remembered as k ant;

Step3: while (the state of all cellulars is not S-M)

{

s=s+1;

for (k=1; k<=m; k++)

{

According to probability P_{ij}^k , the next destination (cellular or customer) is chosen. The k ant is moved to cellular j. Next the state of the cellular j is to S-M and the cause of S-M is remembered as k ant. If cellular j is the depot, the finding route activity of the ant is stopped.

}

}

Step4: for(k=1; k<=m; k++)

if (the ant k returns to the depot)

{

The total path cost W_k of k ant is calculated; the minimum cost routes found are updated; the cost $CA_{\min}$ which corresponds to the minimum cost route is recorded.

}

For (k=1; k<=m; k++)

The pheromone is updated;

Step5: calculates all the $\tau_{ij}(t+1)$

t=t+1;

NC=NC+1;

$\Delta\tau_{ij} = 0$;

Step6: if (NC<NC$_{(MAX)}$)

{

all cellulars convert to the state of S _N;

back to step1;

}

else back to step5

Step7: The minimum cost routes found are updated; the cost $CG_{\min}$ which corresponds to the minimum cost route is recorded.

If (| $CG_{(\min)}$ - $CA_{(\min)}$ | < Given any small positive Numbers)

The minimum cost routes are found and the whole program is terminated;

Else

Back to step2

VII. Test Results and Analysis

The experimental data in the paper is from the classic instances of VRP. In TABLE I and TABLE II, the name of example is listed in column “instances”, and “nodes” represents node number. “Best Known Solution” represents the best solution found by the researcher up till now. CMCA Solution represents the solution obtained by cellular ant algorithm based on constraints method. "Number of Iterations" represents the iterations which the optimal solution comes firstly.

TABLE I.
BEST KNOWN SOLUTION TO CLASSIC INSTANCES

instances	nodes	Best Known Solution	
		length	vehicle
Eil30	30	542	3
Eil51	51	521	5
A-n39-k6	39	831	6
B-n45-k5	45	751	5
P-n16-k8	16	450	8
P-n20-k2	20	216	2
P-n70-k10	70	827	10

TABLE II.
CMCA SOLUTION TO CLASSIC INSTANCES

instances	nodes	CMCA Solution		Number of Iterations
		length	vehicle	
Eil30	30	542	3	57
Eil51	51	534	5	46
A-n39-k6	39	841	6	23
B-n45-k5	45	757	5	56
P-n16-k8	16	450	8	23
P-n20-k2	20	216	2	8
P-n70-k10	70	885	10	68

Parameter setting is as the following: $m = n/2$ (m is the number of ants, and n is city number); $Q = 1$; α, β, ρ is dynamically set; Iterations is set to 100.

The analysis of test process shows that there are two factors to impact the solution quality:

1) The initial pheromone value $\tau_{ij}(0)$ setting, if $\tau_{ij}(0) = \rho$ /(the city number * the path length of the initial solution), the mean value of the solutions is ten percent lower.

2) The parameter dynamically set. At the start of iteration, α, β, ρ is set to a relatively low value as $(\alpha, \beta, \rho) = (1,1,0.1)$, so the solution space is increasing. When iteration to a certain stage, the parameters are enlarged as $(\alpha, \beta, \rho) = (1,2,0.3)$, thus solution space was narrowed and ants are nudged closer to the better routs.

VIII. CONCLUSIONS

This paper has focused on the constraint cellular ant algorithm for the MOVRP. The algorithm combines constraint method, cellular automaton and ant colony optimization which cooperating together to optimize the multi-objective vehicle routing problem.

In the test, the constraint cellular ant algorithm can obtain the approximate solution or the best solution of the classic instances, and the results show that this algorithm is convergent and efficient. Through the Combination of the individual optimization of ant colony and the information exchange between different ant colony, the constraint cellular ant algorithm make each objective approach to the best solution gradually under constraints. To weighting method for optimization, the constraint cellular ant algorithm avoids the hard estimate of the weight.

REFERENCES

[1] Jianwei Zhang, Si Liu, Zhen He, Zengyu Cai, “A Physical Topology-Aware Chord Model based on ACO”, *Journal of Computers*, Vol.6, No.12, pp.2711-2718, 2011.

[2] Dantzig George B., Ramser John H, “The truck dispatching problem”, *Management Science* 6, pp.80-91, 1959.

[3] uanping Zhao, Xiuhui Fu, “Improved Ant Colony Optimization Algorithm and Its Application on Path Planning of Mobile Robot”, *Journal of Computers*, Vol.7, No.8, pp.2055-2062, 2012.

[4] Jianping Wang, Yuesheng Gu, Xiaomin Li, “Multi-robot Task Allocation Based on Ant Colony Algorithm”, *Journal of Computers*, Vol.7, No.9, pp.2160-2167, 2012.

[5] Lenstra Jan K, Rinnooy Kan, Alexander H.G, “Complexity of vehicle routing and scheduling problems”, *Networks* 11, 221-228, 1981.

[6] Wenbin Hu, Zhengbing Hu, Yuheng Cheng,Hai Zhang,Wei Song, "Modeling and Simulation on Dynamic Allocation and Scheduling of Multi-resource problem", *Journal of Computers*, Vol.6, No.7, pp.1369-1377, 2011.

[7] ZhuGang, Cellular ant algorithm and its application, *PhD thesis*, University of Shanghai for Science and Technology, 2007.

[8] Wang Zhoumian et al, “Application of cellular ant colony optimization to pcb routing”, *Journal of Beijing Normal University*(Natural Science), 02,2007.

[9] LiuJixin et al, “Application of ant colony algorithm and cellular automata model”, *Journal of Nanjing University of Technology* (Natural Science Edition), 06, 2008.

[10] ZhangJin et al, “Solving rectilinear Steiner minimum tree problem by improved cellular ant algorithm”, *Computer Engineering and Applications*, 20, 2008.

[11] WangYu et al, “Evaluation of Slope Stability Factor Based on Cellular Ant Algorithm”, *Journal of Wuhan University of Technology* (Transportation Science & Engineering), 06, 2009.

[12] YeWen et al, “Path Planning Algorithm for UAV”, *Electronics Optics & Control*, 02, 2011.

Yuanzhi Wang, Geography Department, Dezhou University, Dezhou, Shandong Province, China, P.C. 253023
Email: wangyuanzhi1@sina.com
Personal Details & Education:
Born Date: October, 1977
Nationality: Hebei Province, China
Major: Computer Science and Application in Geography
Master of Engineering, 2001.9- 2004.7, Henan Polytechnic University, Henan, China
Research Interests & Experience:

- GIS Applications
- Optimization theory and algorithms
- Data Mining
- VRP