

Research on the Optimal Design of Repeaters for WSNs Based on CTCSS

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Abstract—Recently, the speed of development and application domain technology of the wireless communication have already surpassed the fixed service technology. Repeater is the critical role in the message transmission. Thus, how to minimize the number of repeaters and meanwhile serves a certain number of users with limited resource is an important issue. This paper is constructed to solve the related issue in this paper. To discuss the repeaters coverage, the cellular network is to discover the best partition solution with the least overlap coverage and seamless. To consider the number of users and to design the full network coverage solution, a Hamilton circuit with relevant requirements which transmits every hexagon's repeaters has been searched and frequency band of every hexagon has been found by improved backtracking algorithm.

Index Terms—CTCSS protocol, optimal design, Okumura-Hata model

I. INTRODUCTION

In the modern life, telecommunication makes it possible for us to communicate with each other in foreign areas. VHF is the radio frequency which ranges from 30 MHz to 300 MHz. A repeater is an electronic device that receives a signal and retransmits it at a higher level and higher power, or onto the other side of an obstruction, so that the signal can cover longer distances [1].

However, repeaters can interfere with one another unless they are far enough apart or transmit on sufficiently separated frequencies. Therefore, the sub audio frequency technology has been developed, which is an audio signal with low frequency. It mainly opens squelch, not only can improve the sensitivity, but also can avoid disturbances and noises. The most common sub audio frequency is “continuous tone-coded squelch system” (CTCSS) [2], which has been applied to mitigate interference problems. In telecommunications, CTCSS is

a circuit that is used to reduce the annoyance of listening to other users on a shared two-way-radio communications channel.

In early systems, it was common to avoid the use of adjacent tones. On channels where every available tone is not in use, this is good engineering practice. For example, an ideal would be to avoid using 97.4 Hz and 100.0 Hz on the same channel [3]. The tones are so close that some decoders may periodically false trigger. The user occasionally hears a syllable or two of co-channel users on a different CTCSS tone talking. As electronic components age, or through production variances, some radios in a system may be better than others at rejecting nearby tone frequencies.

CTCSS is an analog system. A later digital system was developed by Motorola and is called Digital Private Line, or DPL. General Electric responded with the same system under the name of Digital Channel Guard. The use of digital squelch on a channel that has existing tone squelch users precludes the use of the 131.8Hz and 136.5 Hz tones as the digital bit rate is 134.4 bits per second and the decoders set to those two tones will sense an intermittent signal (referred to in the two-way field as “falling” the decoder) [4].

Gary David Gray has studied the simulating technique to total-area radio coverage [5]. The simulating technique described provides total-area radio coverage on an automatic basis where the use of multiple transmitter and receiver sites is required to achieve acceptable communications coverage over rugged terrain or wide areas. In this technique, the same audio information is simultaneously broadcast over several transmitters operating on a single nominal-carrier frequency. Gary David Gray described the application of the simulating technique to a coordinated FM-UHF law enforcement communications system in Orange County, CA, and because of the use of the simulating technique, it reduces effects of fading in a mobile environment through transmitter space diversity. In our paper, we also should take consideration into the total-area radio coverage and set the fewest repeaters.

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II. CIRCULAR COVERAGE AREA MODEL

Before determining the minimum number of repeaters, this paper should investigate the transmission process of VHF. Because of the using of CTCSS technology to reduce the inferences among the repeaters, this paper assumes that the region of the signal can transmit is a circle and the center of the circle has one or more repeaters [6]. However, repeaters in the same circle interfere with one another unless transmit on sufficiently separated frequencies, so repeaters with different tones in a circle have the same frequency band. Different classes of users in a circle can transmit and receipt message via repeater.

Since, VHF/UHF and higher frequencies generally don't bounce off the upper layers of the atmosphere, so two repeaters' signals propagate along the straight line [7]. Calculating the maximum distance of antenna round coverage on the earth as shown in Fig.1. A is the repeater transmission limit point on the earth, B is the point of repeater, H is the height of repeater, R is earth radius and O is the center of the earth. AB tangent to OA is the limiting case when two repeaters can line-of-sight transmits and receipt. On this case, angle α and projected radius r get the largest.

$$\begin{cases} \cos \alpha = \frac{R}{R+H} \\ r = \alpha R \end{cases} \quad (1)$$

Hence,

$$r = R \arccos \frac{R}{R+H} \quad (1)$$

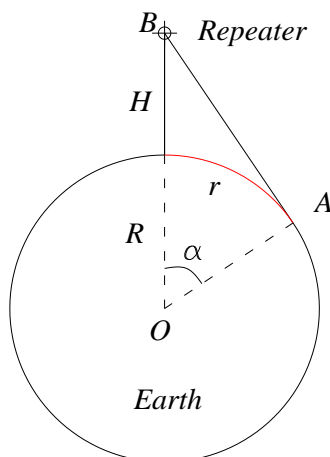


Fig.1 Repeater's Transmit Area

From Fig.1, the largest length radius of repeater's circular coverage area can be calculated as $R \arccos \frac{R}{R+H}$. What's more, the limit case when the

two repeaters can just connect with each other, length projecting on the earth is $l = 2r$.

III. COMPLETELY SEAMLESS COVERAGE AND HEXAGONAL GRID PARTITION

Since the locations of these repeaters are arbitrary, the problem of repeater coordination in a circular flat area with minimum number of repeaters is quite difficult [8]. To cover a circular flat area with several circles of radius r , no matter how small r is, small circles can't join together to make up a complete large circle without holes. Therefore, in order that users can use simultaneously, repeaters' coverage areas should completely seamless cover the circular area and this paper should find the most efficiency way of completely seamless coverage.

The radius of every repeater is same, to accomplish coverage in a circular area with the minimum repeaters, this model need to make repeater cover the maximum area by using mathematic formula to indicate it

$$\forall p \in F, \exists i, \text{ let } p \in D_i \text{ and } \max_{j \in N} \otimes D_i \quad (2)$$

Where p is any point; F is the area to cover; D is area covered by repeater; N is the number of repeaters.

Using regular polygons to cover seamlessly equals to put several interior angles on the same apex. An interior angle of a regular polygon is $\frac{(n-2)180^\circ}{n}$, n is the side number of the regular polygon. So equation to satisfy an apex's requirement with the number of x interior angle:

$$\begin{cases} x \frac{(n-2)180^\circ}{n} = 360^\circ \\ x = 2 + \frac{4}{n-2} \end{cases} \quad (3)$$

To make x to be a positive integer, n can only be 3, 4 or 6, with x is 6, 4 or 3. In other words, the polygons can be equilateral triangle, square and regular hexagon, which are all inscribed polygons (see Fig. 2).

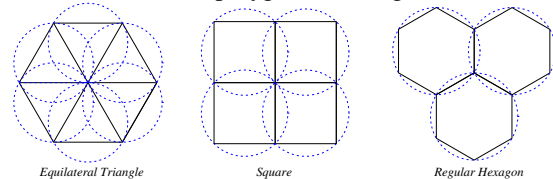


Fig.2 Regular Polygons Cover Seamlessly

According to Fig.3, the overlap area as formula (5) (6) shows as following:

$$S_1 = S_2 - S_3 = \pi r^2 - \frac{\pi r^2}{2} \sin \frac{2\pi}{n} \quad (4)$$

$$S_{\min} = (\pi - \frac{3\sqrt{3}}{2})r^2 \quad (n=3,4,6). \quad (6)$$

Therefore, hexagon coverage is the most efficient completely seamless coverage. Circle is simplified by regular hexagon and there is no overlap between regular hexagons. This method called hexagonal grid partition method [9]. The coverage rate is deduced as (7).

$$\eta = \frac{3\sqrt{3}r^2/2}{\pi r^2} \times 100\% = 82.7\% \quad (7)$$

According to the conclusion of the largest length of projected radius $r = R \arccos \frac{R}{R+H}$, the least hexagon

number n in the circular flat area is almost $n \approx \frac{S}{S_3}$.

Actually, the accurate hexagon number n depends on the actual relative between of the cellular networks and the circular flat area. Since, it may be incomplete hexagons near the circular flat area boundary, so we have to adjustment the circular flat area position to surround the least hexagon number.

IV. COMPLETELY SEAMLESS COVERAGE AND HEXAGONAL GRID PARTITION

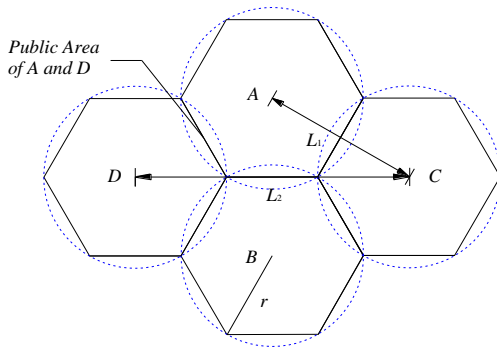


Fig.3 Relative Position of Hexagon

In Fig.3, A, B, C, D are four repeaters, L_1 is length of two adjacent hexagon's center, and $L_1 = \sqrt{3}r < 2r$; L_2 is the length of recent non-adjacent hexagon's center, and $L_2 = 3r > 2r$. Since the limit length when the two repeaters can just connect with each other and transmit signals is $l = 2r$, then $L_1 < l < L_2$. So that two repeaters of adjacent hexagon's center can transmit and receipt signal with each other while non-adjacent ones can't interfere.

There are two cases of the message transmission process [10]: one is only repeaters of adjacent hexagon's center can connect with each other. Any two non-adjacent repeaters can't contact directly, if they have to, they can communicate with each other by a range of adjacent repeaters. Such as Fig.3 shows, if repeater C want to transmit message to another non-adjacent repeater D, firstly C transmit message to adjacent A or B, and then A or B transmit the message to D. However, B may transmit

message to A directly; the second one is if two adjacent repeater can pass message, their frequency band are continuous.

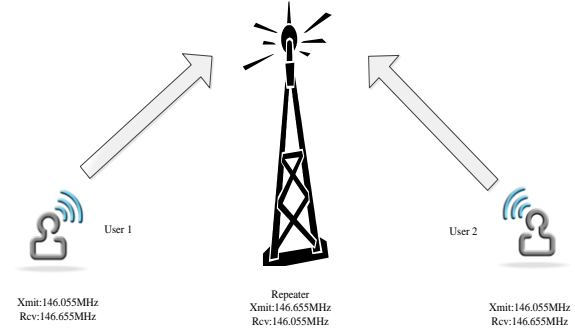


Fig.4 Message Transmission Via One Repeater

According to Fig.4, user 2 is going to receive the repeater's transmitted frequency (146.655MHz), and user 1 is going to transmit to the repeater's receipt frequency (146.055MHz). There is no interfere between user 1 and user 2, for the PL line is used to connect them through repeater. It's also suitable to repeaters in adjacent hexagon transmitting and receiving. Such as in the Fig.5, adjacent repeaters which can transmit and receipt message has continuous frequency band.

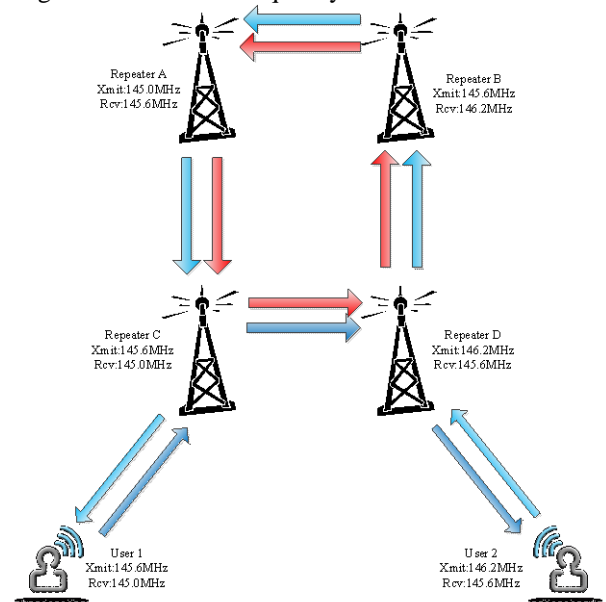


Fig.5 Message Transmission Via Several Repeater

If a class of users in a hexagon, they share the same frequency band and transmit and receipt message via repeaters in the hexagon. As shown in Fig.4. If a class of users in different hexagons, they have different frequency band, so they have to transmit message along directed circuit via adjacent hexagons with continuous frequency band. As is showed in Fig.5, in the dark blue path, user 1 transmit message to repeater C at the frequency of 145Mhz, while repeater receive this message and retransmit to repeater D, finally D transmit to user 2. On the other hand, in the light blue path, user 2 transmit message to repeater B, and B retransmit it to A, and then C, at last the message transmit to user 1. Through the analysis on the above routes, all repeaters should be

joined into a line through adjacent hexagons with continuous frequency band (The red line path in Fig.5).

In a conclusion, covering all the users equals that according to cellular hexagons picture, find a line to run across every hexagon's repeaters with one time. Therefore, finding a one-way and whole nodes interconnected line can be abstracted to find a Hamilton circuit question with certain requirements.

V. The Requirements for Hamilton Circuit

The Hamilton circuit factors consist of shift, direction and repeaters. The shift is defined as the difference in frequency between the sender and the repeater. The transmitter frequency in a repeater is either 600 kHz above or 600 kHz below the receiver frequency, so the shift is 600KHz. If transmitted at a lower frequency than received (-) or higher frequency than received (+), the spectrum available is 145 to 148 MHz", so it's a three meter band with 0.6 shift. Therefore, the band divided into at most 5 bands and plus 2 (with the direction + and -), it get 10 types (see Table 1).

Table 1. 10 Types of Repeaters

Band	1	2	3	4	5
Direction	+	+	+	+	+
Rev(MHz)	145	145.6	146.2	146.8	147.4
Xmit(Mhz)	145.6	146.2	146.8	147.4	148
Band	6	7	8	9	10
Direction	-	-	-	-	-
Rev(MHz)	148	147.4	146.8	146.2	145.6
Xmit(Mhz)	147.4	146.8	146.2	145.6	145

Table 1 shows ten types of repeaters. For example, band 2 and 3 are positive and band 8 is negative. So frequency 146.2MHz is not only band 3's receiving frequency but also the band 8's receiving frequency. So when band 2 transmits message, band 3 and band 8 try to receive at the same time. Furthermore, when any repeater of band 1 to 4 and 6 to 9 transmit, 2 repeaters will try to receive. According to Fig.3, a hexagon B is surrounded by 6 hexagons, so B's type is different from 6 surrounded ones and any three adjacent hexagons aren't the same type. What's more, B has to transmit to other hexagon, so the six adjacent hexagons have only one band which can receive. This must be paid more attention when B is the type of 1 to 4 and 6 to 9.

In a conclusion, requirements for Hamilton circuit are: the next node's receiving frequency is the latter node's transmitting frequency; Any of the two adjacent nodes are 2 of 10 types, and when one transmit, only one will receive.

VI. DETERMINE THE NUMBER OF SIMULTANEOUS USERS

Different classes of users in a hexagon get a single narrow frequency band sharing the hexagon's total frequency band. A class of user's frequency band is 6.25Khz. For Hamilton circuit is a single line without branches and each paragraph bear how much of the information is random, so any pieces of the Hamilton circuit bear the same amount of frequency band. When all

classes of users are in the same hexagon and they want to transmit message at the same time, Hamilton line can still bear it. In that case, Hamilton circuit gets the most width frequency. If only one repeater is in the center of a hexagon, the largest number of users as following:

$$n' = \frac{600}{6.25} = 96 \quad (8)$$

However, more repeaters can use the same frequency with different tones in a hexagon. Based on the above conclusion "any pieces of the Hamilton circuit bear the same amount of frequency band.", equal numbers of repeaters are in the different hexagons. In other words, the amount of tones in every hexagon is the same. So the tone number of every hexagon is $m = \frac{k}{n'}$.

VII. SIMULATION

Average of radius of the earth is $R = 6.4 \times 10^6 m$, commonly the height of repeaters $H = 30m$, radius of the circular flat area to be petitioned $d = 40miles = 64373.76m$. From formula (2), the radius of hexagon's circum circle as following:

$$r = R \arccos\left(\frac{R}{R+H}\right) = 19596m \quad (9)$$

And the least hexagon number is almost as following:

$$n \approx \frac{S}{S_3} = \frac{\pi d^2}{\frac{3\sqrt{3}}{2} r^2} = 13.0491 \quad (10)$$

Therefore there are almost 13 hexagons in the circular flat area.

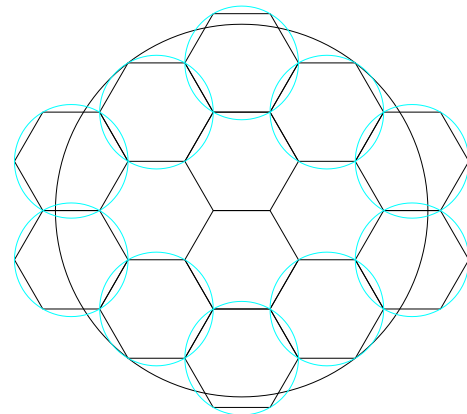


Fig.6 Circular Flat Partition

Adjustment the circular flat area's position to surround the least hexagon number, we find the Fig.6. Using 14 hexagons but the boundary of the circular still exists some dispersive holes. But the holes are not only scattered but also with small area, so it's no use to set some other repeaters to cover them.

Based on the requirements to find the Hamilton circuit by backtracking algorithm, we can get several circuits. A possible Hamilton circuits is found as Fig.7 shows

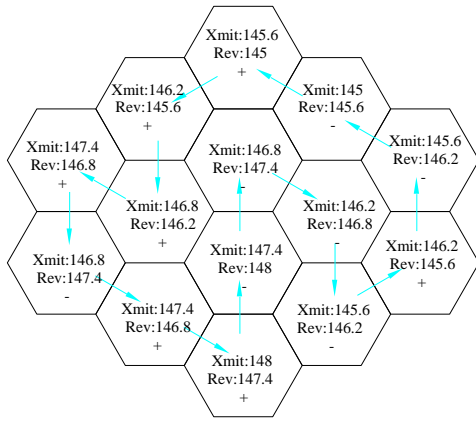


Fig.7 A Possible Hamilton Circuit With Requirements

As Fig.7 shows, the circuit along the purple arrows satisfies all the requirements: The circuit has one-way run across all the hexagons; the next node's receiving frequency is the latter node's transmitting frequency; Any of the two adjacent nodes are 2 of 10 types, and when one transmit, only one will receive; any three of adjacent nodes are different from each other.

This paper can get from the line that the positive shift nodes number equals to the negative ones' number. Therefore, number of the nodes has to be double number. If the area is partitioned into odd number, we have to ignore some area to cover or put more repeaters to make the number double.

When the number of users is 1000, tone number m is $m = \frac{k}{n'} = \frac{1000}{96} \approx 11$. It means every hexagon has 11 tones and repeaters.

In a conclusion, to accommodate 1000 simultaneous users in the circular flat area, we provide a directed circuit as the purple arrows show in Fig.7. The flat area is divided into 14 hexagons. Frequency band of 11 tones or 11 repeaters in a hexagon is showed on the figure. The total number of repeaters is 154 (14*11).

In this simulation, assuming that height of repeater is 30m and a user's channel bandwidth is 6.25kHz. We analyze the effect of parameter changes on the model.

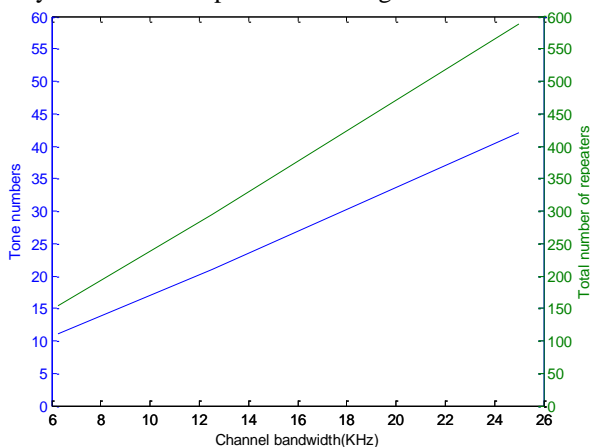
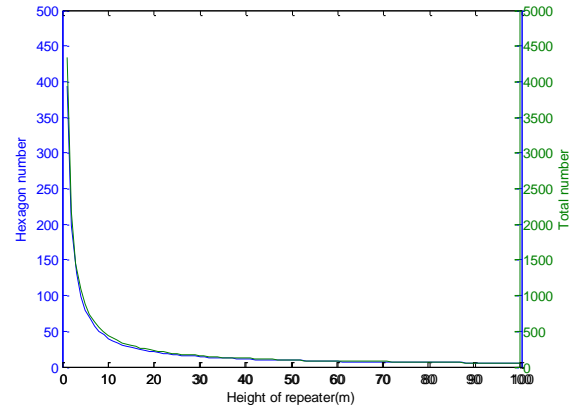


Fig.8 The Sensitivity Analysis of Tone Number and Total Number of Repeaters

From formula (8), we have the least hexagon number is almost

$$n \approx \frac{S}{S_3} = \frac{\pi d^2}{\frac{3\sqrt{3}}{2} r^2} = \frac{\pi d^2}{\frac{3\sqrt{3}}{2} R^2 \arccos^2\left(\frac{R}{R+H}\right)} \quad (11)$$

So this paper can get the relation among n , repeaters' number and H .

Fig.9 The Relation Among n , Repeater's Number and H

As is shown in Fig.11, with the increasing of H , we can get that: When $0 \leq H \leq 10$, n is dropping fast; $20 \leq H$, n change slowly; $30 \leq H$, n is basically unmoved.

In another word, the thing that the experimental results close to reality also proves the feasibility of the model.

VIII. CONCLUSION.

In this paper, the influence of landform to transmit and receipt signal has been fully considered. According to the perceptions of the repeaters, we had proved and used nested hexagons to cover the radius of 40miles area with non-holes and the least overlap area.

To make the users connected anytime, the mechanism of repeaters communication is analyzed with the structure of the nested hexagons' coverage area, so that the issue is deduced into finding a Hamilton circuit with requirements. Through the programming with backtracking algorithm, the model of repeaters distribution that can guarantee the entire users connective anywhere and anytime is constructed.

Considering the limitations of repeaters bandwidth, the number of simultaneous users is below a certain number. Thus, we can increase the users' number by putting the same number of tones and repeaters in the same hexagon. As a result, the case of 1000 simultaneous users has been successfully solved. However, it can't apply to 10000 users at the same time with all the 54 PL tones, so the above model should be modified. With the given data, we had improved the backtracking algorithm and got two independent Hamilton circuits with the same points, so the new wireless network system can solve the problem of 10000 simultaneous users.

Finally, results of the model were analyzed. With the analysis, we found out how does height of repeater and channel bandwidth influence to our model. Comparing the calculation and reality's results, we found experimental results are accurate and efficient. So it has proved feasibility and validity of the proposed model.

Commonly, there are three kinds of channel bandwidth available, namely 25 kHz, 12.5 kHz, 6.25 kHz, the corresponding tone number are 11, 21, 42, and total number of repeaters are 154, 294, 588; we discuss the effect of changed channel bandwidth on the total number of repeaters given that the height of antenna is unchanged. As Fig.10 shows, total number of repeaters and tone number are all in direct proportion with channel bandwidth, and the total numbers of repeaters change faster than tone number when the channel bandwidth varies the same value

IX.ACKNOWLEDMENT

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