Joint Polarization and Angle Estimation for Robust Multi-Target Localization in Bistatic MIMO Radar

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Abstract—**In the paper, we propose a novel algorithm using joint polarization and angle information for robust and high-resolution multi-target localization in bistatic multiple-input multiple-output (MIMO) radar system. The proposed algorithm exploits the singular value decomposition (SVD) of cross-correlation matrix of the received data from two transmitter subarrays to obtain robust performance in noise. Polarization sensitive array-based ESPRIT technologies are employed to estimate the direction of departure (DOD), the direction of arrival (DOA) and the polarization parameters. The Cramer-Rao bounds (CRBs) are given. In the method, the closely spaced targets can be well distinguished by polarization diversity. Also, the DODs, DOAs, and polarizations of multiple targets can be well paired. The simulation results demonstrate that the proposed algorithm can work well and achieve high-resolution identification and robust localization of multiple targets.**

Index Terms— **MIMO radar, joint parameter estimation, polarization, singular value decomposition, Cramer-Rao bound**

I. INTRODUCTION

Multiple-input multiple-output (MIMO) radar [1] and its applications in localization and direction-finding [2]-[4] has recently become a hot topic research. Specifically, the methods of target localization for bistatic MIMO radar are studied to estimate both the direction of departure (DOD) and the direction of arrival (DOA) [5]-[9]. However, a situation must be paid attention that the resolution of these algorithms is greatly degraded when multiple targets are closely spaced and cannot be well distinguished from the spatial domain. Polarimetric radar reflects the tremendous advantages in target estimation, detection and tracking technology [10] [11]. The echoes with different states of polarizations of electromagnetic wave, can be independent of each other due to targets at different locations. By making full use of polarization diversity in MIMO radar, the accuracy of multi-target identification and localization can be improved.

In this paper, we propose a novel algorithm jointing polarization and angle information for robust multi-target localization in bistatic MIMO radar system. We use singular value decomposition (SVD) of cross-correlation matrix of the data received from two transmitter subarrays to obtain robust performance in a noise environment. Polarization sensitive array processing [12] and ESPRIT technologies are used to estimate targets for bistatic MIMO radar. By partitioning the transmitter array into two subarrays and matching the received data with the transmitted signals of two subarrays, we obtain two groups of received data from the transmitter subarrays. Then, the DOAs, DODs and polarizations of multi-targets can be effectively estimated and paired automatically.

This paper is organized as follows. A signal model for polarimetric MIMO radar is presented in Section 2. A novel algorithm for robust multi-target localization by jointing DOA, DOD and polarization estimation is proposed in Section 3. The Cramer-Rao bound is derived in Section 4. Some simulations are conducted to verify the performance of the proposed method in Section 5. Finally, a conclusion is drawn in Section 6.

II. SIGNAL MODEL

For a bistatic MIMO radar system, as shown in Fig.1, we assume that the transmitter is composed of two uniform linear subarrays having M_1 and M_2 sensors, and the receiver is a polarization sensitive array with N pairs of crossed dipoles. The inter-element spacings at the transmitter array and the receiver array are d_t and d_r, respectively, both having no more than half wavelength. Assume that the target's range is much larger than the apertures of the transmitter and receiver arrays. At the transmitter site, $M = M_1 + M_2$ elements of the transmitter, which are orthogonal each other, simultaneously transmit waveforms from the subarray 1 and subarray 2. They have identical center frequency and bandwidth but are temporally coded. P targets appear in the far-field of transmitter and receiver. For the p-th target, here $p = 1, \cdots, P$, its DOD and DOA are θ_p and ϕ_p, respectively, and two polarization phase factors γ_p and η_p denote its polarization information, respectively.

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For the l-th snapshot, $l = 1,\cdots,L$, the observed data matrix at the receiver array is denoted as

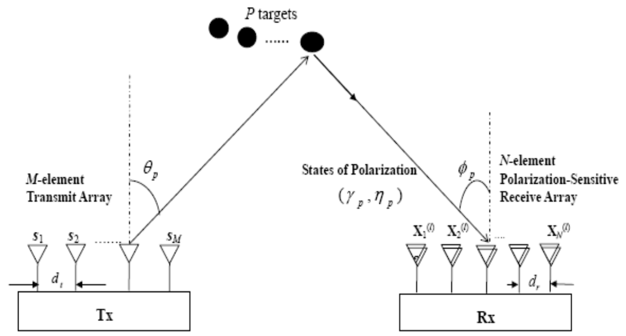


Figure 1. Bistatic MIMO radar with polarization sensitive receiver array

$$\mathbf{X}^{(l)} = \mathbf{A}_r(\phi,\gamma,\eta)\mathbf{B}^{(l)}\begin{bmatrix}\mathbf{A}_{t1}(\theta)\\ \mathbf{A}_{t2}(\theta)\end{bmatrix}^T \mathbf{S} + \mathbf{Z}^{(l)} \tag{1}$$

where $\mathbf{S} \in \mathbb{C}^{M\times K}$ denotes the transmitted baseband coded signal matrix. $\mathbf{B}^{(l)} \in \mathbb{C}^{P\times P}$ denotes the reflected target signal matrix. $\mathbf{A}_{t1}(\theta_1) \in \mathbb{C}^{M_1\times P}$ and $\mathbf{A}_{t2}(\theta_2) \in \mathbb{C}^{M_2\times P}$ denote the transmitter steering matrices of the subarray 1 and 2, respectively. $\mathbf{A}_r(\phi,\gamma,\eta) \in \mathbb{C}^{2N\times P}$ denotes the manifold matrix of receiver array. $\mathbf{Z}^{(l)} \in \mathbb{C}^{2N\times K}$ denotes the noise matrix. $\mathbf{S} = \left[S_1{}^T, S_2{}^T, \cdots, S_M{}^T\right]^T$, where $s_m \in \mathbb{C}^{K\times 1}$ is signal vector of the m-th transmit element, with length K in a repetition interval T_s. Let $\mathbf{SS}^H = K\mathbf{I}$ for M orthogonal transmitted waveforms. $\mathbf{B}^{(l)} = diag\left\{\beta_1^{(l)} e^{j2\pi f_{d1} t_l}, \cdots, \beta_p^{(l)} e^{j2\pi f_{dp} t_l}\right\}$, where f_{dp} is the Doppler frequency of the p-th target. $\beta_p^{(l)}$ is complex amplitudes proportional to the RCSs of the p-th target, which is time-varying in each snapshot. $t_l = lT_S$ is the slow time. $\mathbf{A}_{ti}(\theta) = \left[\mathrm{a}_{ti}(\theta_1), \cdots, \mathrm{a}_{ti}(\theta_p)\right]$, where $\mathbf{a}_{ti}(\theta_p) \in \mathbb{C}^{M_i\times 1}$ is the p-th steering vector of the i-th transmiter subarray, i=1,2.

$$\mathbf{a}_{t1}(\theta_p) = \left[1, \mathrm{e}^{-\mathrm{j}2\pi(d_t/\lambda)\sin\theta_p}, \ldots, \mathrm{e}^{-\mathrm{j}2\pi(M_1-1)(d_t/\lambda)\sin\theta_p}\right]^{\mathrm{T}} \tag{2a}$$

$$\mathbf{a}_{t2}(\theta_p) = \left[1, \mathrm{e}^{-\mathrm{j}2\pi M_1(d_t/\lambda)\sin\theta_p}, \ldots, \mathrm{e}^{-\mathrm{j}2\pi(M-1)(d_t/\lambda)\sin\theta_p}\right]^{\mathrm{T}} \tag{2b}$$

The manifold matrix of receiver array denotes $\mathbf{A}_r(\phi,\gamma,\eta) = \left[\mathbf{a}_r(\phi_1,\gamma_1,\eta_1), \cdots, \mathbf{a}_r(\phi_p,\gamma_p,\eta_p)\right]$, where $\mathbf{a}_r(\phi_p,\gamma_p,\eta_p) \in \mathbb{C}^{2N\times 1}$ is the p-th manifold vector of receiver array.

$$\mathbf{a}_r(\phi_p,\gamma_p,\eta_p) = \mathbf{q}_p \otimes \mathbf{v}_p \tag{3}$$

where $\otimes$ denotes Kronecker product. $\mathbf{q}_p \in \mathbb{C}^{N\times 1}$ is the p-th steering vector of arrival signal,

$$\mathbf{q}_p = \left[1, \mathrm{e}^{-\mathrm{j}2\pi(d_r/\lambda)\sin\phi_p}, \ldots, \mathrm{e}^{-\mathrm{j}2\pi(N-1)(d_r/\lambda)\sin\phi_p}\right]^T \tag{4}$$

and $\mathbf{v}_p \in \mathbb{C}^{2\times 1}$ is the p-th polarization vector arrival signal,

$$\mathbf{v}_p = \begin{bmatrix}-\cos\gamma_p\\ \sin\gamma_p\cos\phi_p e^{j\eta_p}\end{bmatrix} \tag{5}$$

The received signals are matched with $M = M_1 + M_2$ transmitted waveforms. The matched filter matrix is $\left(1/\sqrt{K}\right)\mathbf{S}^H$. The output of the matched filter is $\mathbf{Y}^{(l)} \in \mathbb{C}^{2N\times M}$, which can be written by $\mathbf{Y}^{(l)} = \left(1/\sqrt{K}\right)X^{(l)}\mathbf{S}^H$, i.e.,

$$\mathbf{Y}^{(l)} = \sqrt{K}\mathbf{A}_r(\phi,\gamma,\eta)\mathbf{B}^{(l)}\begin{bmatrix}\mathbf{A}_{t1}(\theta)\\ \mathbf{A}_{t2}(\theta)\end{bmatrix}^T + (1/\sqrt{K})\mathbf{Z}^{(l)}\mathbf{S}^H = \left[\mathbf{Y}_1^{(l)} \,|\, \mathbf{Y}_2^{(l)}\right] \tag{6}$$

where $\left(1/\sqrt{K}\right)\mathbf{Z}^{(l)}\mathbf{S}^H$ denotes the noise matrix after matched filter. $\mathbf{Y}_1^{(l)} \in \mathbb{C}^{2N\times M_1}$ and $\mathbf{Y}_2^{(l)} \in \mathbb{C}^{2N\times M_2}$ denote the two sub-matrices of $\mathbf{Y}^{(l)}$.

Vectorize the sub-matrices $\mathbf{Y}_1^{(l)}$ and $\mathbf{Y}_2^{(l)}$, respectively, we have

$$\mathbf{y}_1^{(l)} = vec\left(\mathbf{Y}_1^{(l)}\right) \tag{7}$$

$$\mathbf{y}_2^{(l)} = vec\left(\mathbf{Y}_2^{(l)}\right) \tag{8}$$

Thus, the two column vectors $\mathbf{y}_1^{(l)} \in \mathbb{C}^{2M_1N\times 1}$ and $\mathbf{y}_2^{(l)} \in \mathbb{C}^{2M_2N\times 1}$ are composed of M_1 and M_2 column vectors of $2N\times 1$, respectively. $\mathbf{y}_1^{(l)}$ and $\mathbf{y}_2^{(l)}$ can be further written as

$$\mathbf{y}_1^{(l)} = \mathbf{A}_1(\theta,\phi,\gamma,\eta)\mathbf{b}^{(l)} + \mathbf{v}_1^{(l)} \tag{9}$$

$$\mathbf{y}_2^{(l)} = \mathbf{A}_2(\theta,\phi,\gamma,\eta)\mathbf{b}^{(l)} + \mathbf{v}_2^{(l)} \tag{10}$$

where $\mathbf{A}_1(\theta,\phi,\gamma,\eta) \in \mathbb{C}^{2M_1N\times P}$ and $\mathbf{A}_2(\theta,\phi,\gamma,\eta) \in \mathbb{C}^{2M_2N\times P}$ denote the two manifold matrix with respect to the two transmitter subarrays,

$$\begin{aligned}\mathbf{A}_1(\theta,\phi,\gamma,\eta) &= \mathbf{A}_{t1}(\theta)\Diamond\mathbf{A}_r(\phi,\gamma,\eta)\\ &= \left[\mathrm{a}_{t1}(\theta_1)\otimes\mathrm{a}_r(\phi_1,\gamma_1,\eta_1), \cdots, \mathrm{a}_{t1}(\theta_p)\otimes\mathrm{a}_r(\phi_p,\gamma_p,\eta_p)\right]\end{aligned} \tag{11}$$

$$\begin{aligned}\mathbf{A}_2(\theta,\phi,\gamma,\eta) &= \mathbf{A}_{t2}(\theta)\Diamond\mathbf{A}_r(\phi,\gamma,\eta)\\ &= \left[\mathrm{a}_{t2}(\theta_1)\otimes\mathrm{a}_r(\phi_1,\gamma_1,\eta_1), \cdots, \mathrm{a}_{t2}(\theta_p)\otimes\mathrm{a}_r(\phi_p,\gamma_p,\eta_p)\right]\end{aligned} \tag{12}$$

where $\Diamond$ denotes Khatri-Rao product. After matched filtering with M_1 and M_2 transmitted waveforms which are orthogonal with each other, the two noise vectors $\mathbf{v}_1^{(l)}$ and $\mathbf{v}_2^{(l)}$ are uncorrelated, zero-mean complex Gaussian distributed. Here $\mathbf{b}^{(l)} = \sqrt{K}\left[\beta_1^{(l)} e^{j2\pi f_{d1} t_l}, \cdots, \beta_P^{(l)} e^{j2\pi f_{dp} t_l}\right]^T$ is the reflected signal vector.

Based on the signal model in (9) and (10), the problem of interest is to jointly estimate multiple parameters $(\theta_P, \phi_P, \gamma_P, \eta_P)$ for the *p*-th target.

III. Joint Polarization and Angle Estimation Algorithm for Robust Multi-Target Localization

The cross-correlation matrix between $\boldsymbol{y}_1^{(l)}$ and $\boldsymbol{y}_2^{(l)}$ is given by

$$\boldsymbol{R}_c = E\{\boldsymbol{y}_1^{(l)} \boldsymbol{y}_2^{(l)^H}\} = \boldsymbol{A}_1(\theta,\phi,\gamma,\eta)\boldsymbol{R}_b\boldsymbol{A}_2(\theta,\phi,\gamma,\eta)^H \quad (13)$$

where $\boldsymbol{R}_b$ is the covariance matrix of $\boldsymbol{b}^{(l)}$. Here the cross-correlation matrix of noise $\boldsymbol{v}_1^{(l)}$ and $\boldsymbol{v}_2^{(l)}$ has been canceled since $\boldsymbol{v}_1^{(l)}$ and $\boldsymbol{v}_2^{(l)}$ are uncorrelated. Performing a singular value decomposition (SVD) on it, we obtain

$$\boldsymbol{R}_c = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^H \quad (14)$$

where $\boldsymbol{U} \in \mathbb{C}^{2M_1N \times 2M_1N}$ and $\boldsymbol{V} \in \mathbb{C}^{2M_2N \times 2M_2N}$ are two unitary matrices composed of left and right singular vectors corresponding to all the singular values, respectively.

$$\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma}_0 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \quad (15)$$

where $\boldsymbol{\Sigma}_0 = diag(\sigma_1, \sigma_2, \cdots \sigma_P)$ is a diagonal matrix , and $\sigma_1, \sigma_2, \cdots \sigma_P$ are the first P non-zero singular value of the matrix $\boldsymbol{R}_c$, which are real and positive, such that $\sigma_1 \geq \cdots \geq \sigma_P > 0$. Partition $\boldsymbol{U}$ into

$$\boldsymbol{U} \triangleq [\boldsymbol{U}_s | \boldsymbol{U}_n] \quad (16)$$

where $\boldsymbol{U}_s$ and $\boldsymbol{U}_n$ are the first P columns and the last $2M_1N-P$ columns of $\boldsymbol{U}$, respectively, then, we have the following relations:

$$span\{\boldsymbol{U}_s\} = span\{\boldsymbol{A}_1(\theta,\phi,\gamma,\eta)\} \quad (17)$$

That is to say, the columns in $\boldsymbol{U}_s \in \mathbb{C}^{2M_1N \times P}$ span the same signal subspace as the column vectors in $\boldsymbol{A}_1(\theta,\phi,\gamma,\eta)$. Thus, there is a unique non-singular matrix $\boldsymbol{T}$ such that

$$\boldsymbol{U}_s = \boldsymbol{A}_1(\theta,\phi,\gamma,\eta)\boldsymbol{T} \quad (18)$$

Thus $\boldsymbol{U}_s$ can be used to form multiple appropriate subsets because it preserves the invariance properties of $\boldsymbol{A}_1(\theta,\phi,\gamma,\eta)$.

For both transmitter array and receiver array possess shift-invariance properties of ESPRIT method, we can obtain the estimation of DOD, DOA and polarization parameters based on the matrix $\boldsymbol{U}_s$.

A. DOD Estimation

Since $\boldsymbol{A}_1(\theta,\phi,\gamma,\eta)$ is composed of M_1 blocks according to row, each is a $2N \times P$ matrix. If we let $\boldsymbol{A}_{f1}$ and $\boldsymbol{A}_{f2}$ be the two $2(M_1-1)N \times P$ sub-matrices of $\boldsymbol{A}_1(\theta,\phi,\gamma,\eta)$ formed with the first and last M_1-1 blocks of $\boldsymbol{A}_1(\theta,\phi,\gamma,\eta)$, respectively, then, $\boldsymbol{A}_{f2} = \boldsymbol{A}_{f1}\boldsymbol{\Lambda}_f$, where $\boldsymbol{\Lambda}_f$ is a $P \times P$ diagonal matrix,

$$\boldsymbol{\Lambda}_f = diag\left\{e^{-j2\pi d_t/\lambda \sin\theta_1}, e^{-j2\pi d_t/\lambda \sin\theta_2}, \cdots e^{-j2\pi d_t/\lambda \sin\theta_P}\right\} \quad (19)$$

Since the Kroneker product is used in the structure of the column in $\boldsymbol{U}_s$, we divide $\boldsymbol{U}_s$ into M_1 blocks according to row, each of which is a $2N \times P$ matrix. The subspace $\boldsymbol{U}_s$ spanned by the column in $\boldsymbol{A}_{f1}$ and $\boldsymbol{A}_{f2}$ are the same except for the phase rotation caused by the diagonal matrix $\boldsymbol{\Lambda}_f$. Let $\boldsymbol{U}_{f1} \in \mathbb{C}^{2(M_1-1)N \times P}$ be a sub-matrix formed with the first M_1-1 blocks of $\boldsymbol{U}_s$, and let $\boldsymbol{U}_{f2} \in \mathbb{C}^{2(M_1-1)N \times P}$ be a subset formed with the last M_1-1 blocks of $\boldsymbol{U}_s$ in the same way the $\boldsymbol{A}_{f1}$ and $\boldsymbol{A}_{f2}$ are formed from $\boldsymbol{A}_1(\theta,\phi,\gamma,\eta)$. We have

$$\boldsymbol{U}_{f1} = \boldsymbol{A}_{f1}\boldsymbol{T} \quad (20)$$

$$\boldsymbol{U}_{f2} = \boldsymbol{A}_{f2}\boldsymbol{T} = \boldsymbol{A}_{f1}\boldsymbol{\Lambda}_f\boldsymbol{T} \quad (21)$$

Then $span\{\boldsymbol{U}_{f1}\} = span\{\boldsymbol{U}_{f2}\} = span\{\boldsymbol{A}_{f1}\}$, and

$$\boldsymbol{U}_{f2} = \boldsymbol{U}_{f1}\boldsymbol{T}^{-1}\boldsymbol{\Lambda}_f\boldsymbol{T} = \boldsymbol{U}_{f1}\boldsymbol{\Psi}_f \quad (22)$$

where

$$\boldsymbol{\Psi}_f = \boldsymbol{T}^{-1}\boldsymbol{\Lambda}_f\boldsymbol{T} = \boldsymbol{Q}\boldsymbol{\Lambda}_f\boldsymbol{Q}^T \quad (23)$$

where the diagonal elements of $\boldsymbol{\Lambda}_f$ are P eigenvalues of matrix $\boldsymbol{\Psi}_f$, and $\boldsymbol{Q} = \boldsymbol{T}^{-1}$ composed of the eigenvectors corresponding to the eigenvalues of matrix $\boldsymbol{\Psi}_f$. Therefore, DOD of the *p*-th target can be obtained from $a_{tp} = e^{-j2\pi d_t/\lambda \sin\theta_p}$, $p = 1, 2, \cdots, P$, i.e., from the diagonal elements of $\boldsymbol{\Lambda}_f$.

B. DOA Estimation

For the estimation of DOAs, let $\boldsymbol{A}_{q1}$ and $\boldsymbol{A}_{q2}$ be the two $2M_1(N-1) \times P$ sub-matrices of $\boldsymbol{A}_1(\theta,\phi,\gamma,\eta)$ formed with the first and the last $2(N-1)$ rows of each blocks of $\boldsymbol{A}_1(\theta,\phi,\gamma,\eta)$, respectively. Then $\boldsymbol{A}_{q2} = \boldsymbol{A}_{q1}\boldsymbol{\Lambda}_q$, where $\boldsymbol{\Lambda}_q$ is also a $P \times P$ diagonal matrix,

$$\boldsymbol{\Lambda}_q = diag\left\{e^{-j2\pi d_r/\lambda \sin\phi_1}, e^{-j2\pi d_r/\lambda \sin\phi_2}, \cdots e^{-j2\pi d_r/\lambda \sin\phi_P}\right\} \quad (24)$$

Similarly, we could form two sub-matrices based on $\boldsymbol{U}_s$ in the same way $\boldsymbol{A}_{q1}$ and $\boldsymbol{A}_{q2}$ are formed from $\boldsymbol{A}_1(\theta,\phi,\gamma,\eta)$. However, DOD and DOA for each target may not be well paired in such a way. In order to make DODs and DOAs be paired automatically, we first form a new matrix

$$\tilde{\boldsymbol{U}}_s = \boldsymbol{U}_s\boldsymbol{Q} \quad . \tag{25}$$

Substitute (18) into (25), and consider $\boldsymbol{Q} = \boldsymbol{T}^{-1}$ yielding

$$\tilde{\boldsymbol{U}}_s = \boldsymbol{A}_1(\theta,\phi,\gamma,\eta)\boldsymbol{T}\boldsymbol{Q} = \boldsymbol{A}_1(\theta,\phi,\gamma,\eta) . \tag{26}$$

Therefore, $\tilde{\boldsymbol{U}}_s$ has the same structure with $\boldsymbol{A}_1(\theta,\phi,\gamma,\eta)$. Thus, we divide $\tilde{\boldsymbol{U}}_s$ into M_1 blocks according to row, each of which is a $2N\times P$ matrix. Let $\boldsymbol{U}_{q1}\in\mathbb{C}^{2M_1(N-1)\times P}$ be a sub-matrix formed with the first $2(N-1)$ rows of each blocks of $\tilde{\boldsymbol{U}}_s$, and $\boldsymbol{U}_{q2}\in\mathbb{C}^{2M_1(N-1)\times P}$ be a subset formed with the last $2(N-1)$ rows of each blocks of $\tilde{\boldsymbol{U}}_s$ in the same way $\boldsymbol{A}_{q1}$ and $\boldsymbol{A}_{q2}$ are formed from $\boldsymbol{A}_1(\theta,\phi,\gamma,\eta)$, we have

$$\boldsymbol{U}_{q1} = \boldsymbol{A}_{q1}\boldsymbol{T}\boldsymbol{Q} = \boldsymbol{A}_{q1} \tag{27}$$

$$\boldsymbol{U}_{q2} = \boldsymbol{A}_{q2}\boldsymbol{T} = \boldsymbol{A}_{q1}\boldsymbol{\Lambda}_q\boldsymbol{T}\boldsymbol{Q} = \boldsymbol{A}_{q1}\boldsymbol{\Lambda}_q \tag{28}$$

$span\{\boldsymbol{U}_{q1}\} = span\{\boldsymbol{U}_{q2}\} = span\{\boldsymbol{A}_{q1}\}$, and

$$\boldsymbol{U}_{q2} = \boldsymbol{U}_{q1}\boldsymbol{\Lambda}_q \tag{29}$$

Thus, the DOA of the *p*-th target can be obtained from the *p*-th diagonal element of $\boldsymbol{\Lambda}_q$, i.e., $q_p = e^{-j2\pi \frac{d_r}{\lambda}\sin\phi_p}$, and the DOA and DOD of the *p*-th target can be well paired with each other.

C. Polarization Estimation

For the estimation of two polarization parameters, let $\boldsymbol{A}_{r1}$ and $\boldsymbol{A}_{r2}$ be the two $M_1N\times P$ sub-matrices of $\boldsymbol{A}_1(\theta,\phi,\gamma,\eta)$ formed with the even and the odd rows of $\boldsymbol{A}_1(\theta,\phi,\gamma,\eta)$, respectively. Then $\boldsymbol{A}_{r2} = \boldsymbol{A}_{r1}\boldsymbol{\Lambda}_r$, where $\boldsymbol{\Lambda}_r$ is the diagonal matrix,

$$\boldsymbol{\Lambda}_r = diag\{r_1, r_2, \cdots r_P\} \tag{30}$$

where r_p is the ratio of the first element and the second element of the vector in (5),

$$r_p = \frac{-\cos\gamma_p}{\sin\gamma_p\cos\phi_p e^{j\eta_p}} \tag{31}$$

Similarly, according to (26), let $\boldsymbol{U}_{r1}\in\mathbb{C}^{M_1N\times P}$ be a sub-matrix formed with the even and the odd rows of $\tilde{\boldsymbol{U}}_s$, and let $\boldsymbol{U}_{r2}\in\mathbb{C}^{M_1N\times P}$ be a subset formed with the even and the odd rows of $\tilde{\boldsymbol{U}}_s$ in the same way the $\boldsymbol{A}_{r1}$ and $\boldsymbol{A}_{r2}$ are formed from $\boldsymbol{A}_1(\theta,\phi,\gamma,\eta)$, we have

$$\boldsymbol{U}_{r1} = \boldsymbol{A}_{r1}\boldsymbol{T}\boldsymbol{Q} = \boldsymbol{A}_{r1} \tag{32}$$

$$\boldsymbol{U}_{r2} = \boldsymbol{A}_{r2}\boldsymbol{T}\boldsymbol{Q} = \boldsymbol{A}_{r1}\boldsymbol{\Lambda}_r\boldsymbol{T}\boldsymbol{Q} = \boldsymbol{A}_{r1}\boldsymbol{\Lambda}_r \tag{33}$$

Then $span\{\boldsymbol{U}_{r1}\} = span\{\boldsymbol{U}_{r2}\} = span\{\boldsymbol{A}_{r1}\}$, and

$$\boldsymbol{U}_{r2} = \boldsymbol{U}_{r1}\boldsymbol{\Lambda}_r \tag{34}$$

Therefore, polarization parameters of the *p*-th target can be attained from the the diagonal elements of $\boldsymbol{\Lambda}_r$, i.e., $r_p = \frac{-\cos\gamma_p}{\sin\gamma_p\cos\phi_p e^{j\eta_p}}$, and the polarization parameters of the *p*-th target can be paired with DOD of the *p*-th target.

The DOD and DOA of the *p*-th target are calculated by

$$\theta_p = \arcsin\left\{-\frac{\lambda}{2\pi d_t}\angle(a_{tp})\right\} \tag{35}$$

$$\phi_p = \arcsin\left\{-\frac{\lambda}{2\pi d_r}\angle(q_p)\right\} \tag{36}$$

The two polarization parameters of the *p*-th target are calculated by

$$\gamma_p = \arctan\left(\frac{1}{r_p\cos\phi_p}\right) \tag{37}$$

$$\eta_p = \angle\left(-\frac{1}{r_p\cos\phi_p}\right) \tag{38}$$

The steps for the proposed algorithm are given as follows:

Step1: Contrust a cross-correlation matrix $\boldsymbol{R}_c$ between $\boldsymbol{y}_1^{(l)}$ and $\boldsymbol{y}_2^{(l)}$, and perform SVD on it.

Step2: Form $\boldsymbol{U}_s$ from the left singular vector of $\boldsymbol{R}_c$. Divide $\boldsymbol{U}_s$ into blocks, form two sub-matrices $\boldsymbol{U}_{f1}$, $\boldsymbol{U}_{f2}$, and calculate $\boldsymbol{\Psi}_f = \boldsymbol{U}_{f2}{}^{+}\boldsymbol{U}_{f1}$.

Step3: Perform eigen-decomposition of $\boldsymbol{\Psi}_f$ to obtain its eigenvalue matrix $\boldsymbol{\Lambda}_f$ and eigen vector matrix $\boldsymbol{Q}$.

Step4: Form a matrix $\tilde{\boldsymbol{U}}_s = \boldsymbol{U}_s\boldsymbol{Q}$, and divide $\tilde{\boldsymbol{U}}_s$ into M_1 blocks according to row.

Step5: Form two sub-matrices $\boldsymbol{U}_{q1}$, $\boldsymbol{U}_{q2}$, and two sub-matrices $\boldsymbol{U}_{r1}$, $\boldsymbol{U}_{r2}$ respectively using different blocks of $\tilde{\boldsymbol{U}}_s$, then calculate $\boldsymbol{\Lambda}_q = \boldsymbol{U}_{q2}{}^{+}\boldsymbol{U}_{q1}$ and $\boldsymbol{\Lambda}_r = \boldsymbol{U}_{r2}{}^{+}\boldsymbol{U}_{r1}$ based on (29) and (34), respectively.

Step6: Estimate θ_p, ϕ_p, γ_p and η_p from the diagonal elements of Λ_f, Λ_q and Λ_r using (35)~(38).

For a bistatic MIMO radar system, the distance between the transmitter and the receiver is known. Thus we can localize a target in a 2-D plane. θ_p and ϕ_p can be used to determine the position of a target via trilateration, and γ_p and η_p can be used in target identification and improvement of resolution.

IV. CRAMER-RAO BOUND

The Cramer-Rao bound(CRB) provides a lower bound on the variance of any unbiased estimator. The CRBs of the parameters of DOA, DOD and polarizations in MIMO radar are considered here. Based on a composite data of $\mathbf{y}_1^{(l)}$ and $\mathbf{y}_2^{(l)}$, the signal model in (9) and (10) can be jointly written as

$$\boldsymbol{y}^{(l)} = \boldsymbol{A}(\theta,\phi,\gamma,\eta)\boldsymbol{b}^{(l)} + \boldsymbol{v}^{(l)} \tag{39}$$

where $\mathbf{A}(\theta,\phi,\gamma,\eta) = \begin{bmatrix} \mathbf{A}_1(\theta,\phi,\gamma,\eta) \\ \mathbf{A}_2(\theta,\phi,\gamma,\eta) \end{bmatrix}$ and $\mathbf{v}^{(l)} = \begin{bmatrix} \mathbf{v}_1^{(l)} \\ \mathbf{v}_2^{(l)} \end{bmatrix}$.

Here the CRB expression of JADE (Joint Angle and Delat Estimation) Approach is extended to the polarimetric MIMO Radar system, we give the CRB for the parameters of interest as

$$CRB(\theta,\phi,\gamma,\eta) = \frac{\sigma_v^2}{2}\{\sum_{l=1}^{L} real[\mathbf{B}_b^{(l)H}\mathbf{D}_A^H\mathbf{P}_A^{\perp}\mathbf{D}_A\mathbf{B}_b^{(l)}]\}^{-1} \tag{40}$$

where $\mathbf{P}_A^{\perp} = \mathbf{I} - \mathbf{A}(\theta,\varphi,\gamma,\eta)\mathbf{A}^H(\theta,\varphi,\gamma,\eta)$, $\mathbf{B}_b^{(l)} = \mathbf{I}_4 \otimes diag\{\mathbf{b}^{(l)}\}$, and $\mathbf{I}_4$ denotes an identity matrix of order 4. $\mathbf{D}_A$ denotes the differentiation matrix, where each column is differentiated with respect to the corresponding parameter. $\mathbf{D}_A$ can be expressed as

$$\begin{aligned}\mathbf{D}_A &= [\dot{\mathbf{A}}_t(\theta)\Diamond\mathbf{A}_r(\phi,\gamma,\eta), \mathbf{A}_t(\theta)\Diamond\dot{\mathbf{A}}_r(\phi,\gamma,\eta)] \\ &= [\dot{\mathbf{A}}_\theta, \dot{\mathbf{A}}_\phi, \dot{\mathbf{A}}_\gamma, \dot{\mathbf{A}}_\eta]\end{aligned} \tag{41}$$

where $\mathbf{A}_t(\theta) = \left[\mathbf{A}_{t1}^T(\theta), \mathbf{A}_{t1}^T(\theta)\right]^T$, and

$$\dot{\mathbf{A}}_\theta = \left[\frac{\partial \mathbf{a}_t(\theta_1)}{\partial \theta_1} \otimes \mathbf{a}_r(\phi_1,\gamma_1,\eta_1), \cdots, \frac{\partial \mathbf{a}_t(\theta_p)}{\partial \theta_p} \otimes \mathbf{a}_r(\phi_p,\gamma_p,\eta_p)\right]$$

$$\dot{\mathbf{A}}_\phi = \left[\mathbf{a}_t(\theta_1) \otimes \frac{\partial \mathbf{a}_r(\phi_1,\gamma_1,\eta_1)}{\partial \phi_1}, \cdots, \mathbf{a}_t(\theta_p) \otimes \frac{\partial \mathbf{a}_r(\phi_p,\gamma_p,\eta_p)}{\partial \phi_p}\right]$$

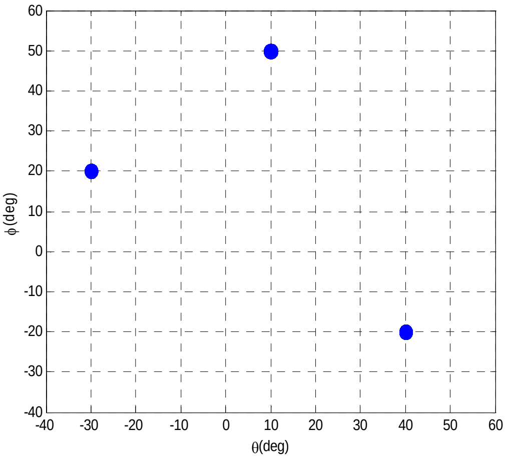
$$\dot{\mathbf{A}}_\gamma = \left[\mathbf{a}_t(\theta_1) \otimes \frac{\partial \mathbf{a}_r(\phi_1,\gamma_1,\eta_1)}{\partial \gamma_1}, \cdots, \mathbf{a}_t(\theta_p) \otimes \frac{\partial \mathbf{a}_r(\phi_p,\gamma_p,\eta_p)}{\partial \gamma_p}\right]$$

$$\dot{\mathbf{A}}_\eta = \left[\mathbf{a}_t(\theta_1) \otimes \frac{\partial \mathbf{a}_r(\phi_1,\gamma_1,\eta_1)}{\partial \eta_1}, \cdots, \mathbf{a}_t(\theta_p) \otimes \frac{\partial \mathbf{a}_r(\phi_p,\gamma_p,\eta_p)}{\partial \eta_p}\right]$$

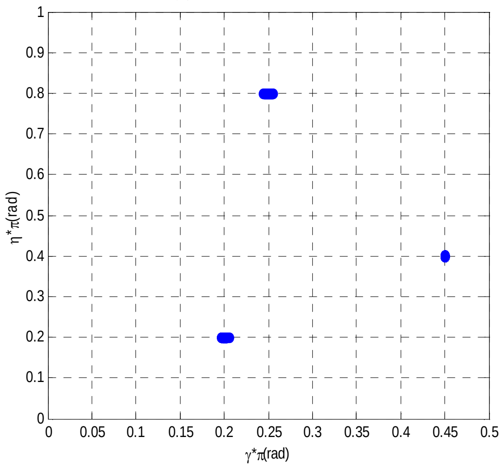
V. SIMULATION RESULTS

Simulations are conducted to verify the effectiveness of the proposed method in this section. The intervals between adjacent array elements of transmit and receive arrays are all half wavelength. The length of sequences is 1024. The RCSs are given by $\left|\boldsymbol{\beta}_p^{(l)}\right| = 1$. The number of snapshots is L=100, and the number of Monte Carlo trials is 100. The performance of the algorithm will be evaluated through root-mean-square error (RMSE).

Example1: Suppose that the transmit array and the receive array consist of $M_1 = 3$ and $M_2 = 2$ elements and $N = 4$ pairs of crossed dipoles in the polarimetric MIMO radar, respectively. There are antennas in the first subarray and $P = 3$ targets. Their parameters of the DOD, the DOA and the states of polarization are $\theta = (10^\circ, 40^\circ, -30^\circ)$, $\phi = (50^\circ, -20^\circ, 20^\circ)$, $\gamma = (9\pi/20, \pi/5, \pi/4)$ and $\eta = (2\pi/5, \pi/5, 4\pi/5)$. The Doppler frequencies of the three targets are 100Hz, 2000Hz and 5000Hz, respectively. The estimation results of DOD, DOA and polarization parameters when SNR is 10dB are shown in Fig.2 (a),(b), respectively. The performances of RMSE and Root CRB versus SNR for the DOD, DOA and polarizations estimation are shown in Fig. 3.



(a) DOD and DOA



(b) Two polarization parameters

Figure 2. The estimation results of DOD, DOA and polarizations when SNR is 10 dB

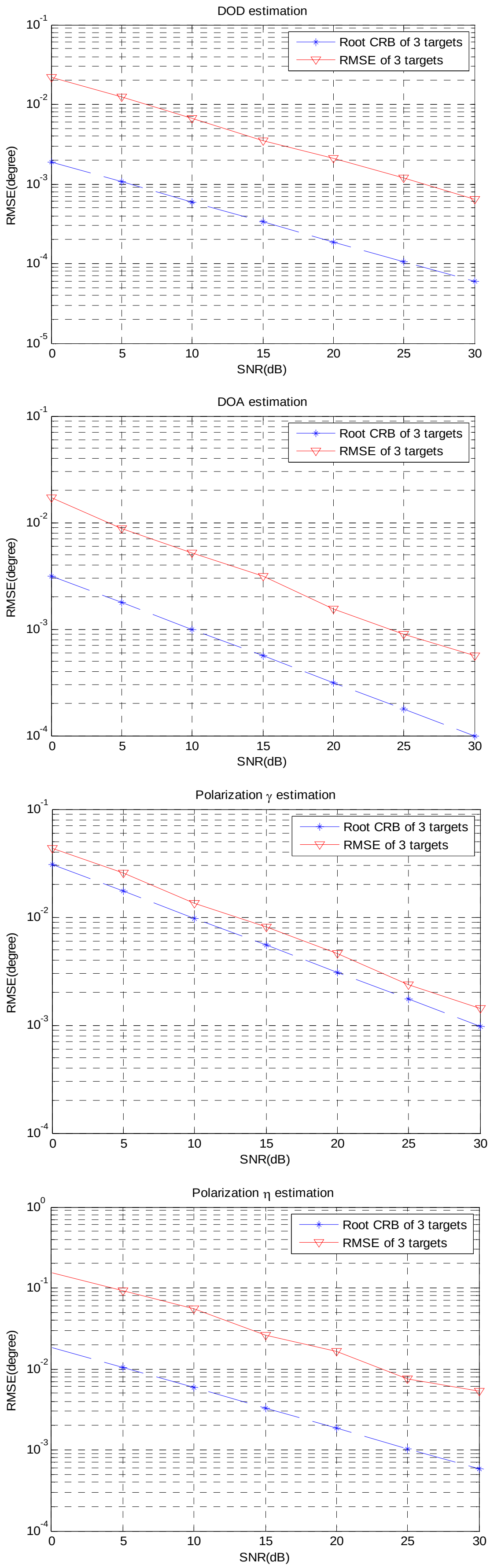


Figure 3. The performances of RMSE and Root CRB versus SNR for the DOD, DOA and polarizations estimation

From Fig. 2 and Fig. 3, it is shown that the proposed method could effectively estimate DOD, DOA and polarization parameters for bistatic MIMO radar. In addtion, the automatical pairing between DOD, DOA and polarization parameters can be obtained.

Example 2: The number of transmitter array, receiver array and snapshot is the same as Example 1. The number of targets is 2. SNR changes from 0dB to 30dB. We consider two targets which are closely spaced, $\theta=\left(10^{\circ},11^{\circ}\right)$, $\phi=\left(20^{\circ},20^{\circ}\right)$, $\gamma=\left(\pi/6,\pi/3\right)$ and $\eta=\left(0,\pi/4\right)$.

We compare the performance of the proposed DOD/DOA/polarization parameter estimation algorithm with Chen's method [5] of DOD/DOA estimation. The performance of RMSE with SNR for the two targets is shown in Fig. 4 .

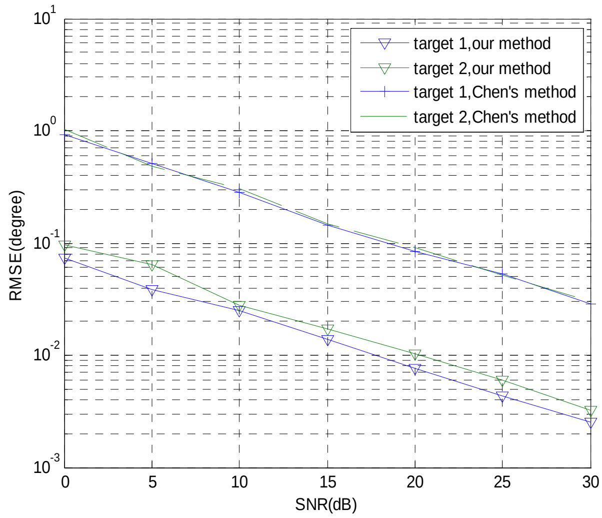


Figure 4. The performance comparison of DOD/DOA estimation.

The simulation shows that the proposed algorithm obviously performs better than the algorithm in previous research. Because polarization diversity is adopted in our algorithm, the multi-target resolution of DOD/DOA estimation is obviously improved.

VI. Conclusion

In the paper, we propose a robust algorithm to jointly estimate DOD, DOA and polarization parameters for multi-targets in bistatic MIMO radar system via ESPRIT algorithm, polarization sensitive array processing and SVD of cross-correlation matrix of the received data from two transmitter subarrays. The simulation results show that the proposed method can effectively estimate multiple parameters for each target, i.e. the angles of departure and arrival, and two polarization parameters, in a noise environment.

Also, the parameters can be paired automatically. Using polarization diversity technique, the estimation performance is improved, especially when two targets are closely spaced and cannot be well separated in spatial domain.

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