Multi-attribute Group Decision-making Method Based on Triangular Intuitionistic Fuzzy Number and 2-Tuple Linguistic Information

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***Abstract*—For multiple attribute group decision-making problems, in which the attribute values are triangular intuitionistic fuzzy numbers and the attribute weight information is the linguistic evaluation, a multiple attribute group decision-making method is proposed. In the method, first, to transform the 2-tuple linguistic attribute weights to the real numbers. Then, to aggregate the experts' preferences with triangular intuitionistic fuzzy number by the extended aggregation operators, the group overall evaluation values of the alternatives were obtained. The ranking could be present according to the weighted average area. Triangular intuitionistic fuzzy number and 2-tuple linguistic information are easier to deal with the fuzzy and the uncertain information of different decision makers. Finally, a numerical example was used to illustrate the proposed method. The result shows the approach is simple, effective, and easy to calculate.**

***Index Terms*—multiple attribute group decision-making, triangular intuitionistic fuzzy number, 2-tuple linguistic information, aggregation operators**

I. INTRODUCTION

Zadeh [1] introduced the concept of fuzzy set whose basic component is only a membership function with the non-membership function being one minus the membership function. However, in real-life situations, when a person is asked to express his/her preference degree to an object, it is possible that he/she is not so sure about it, that is, there usually exists a hesitation or uncertainty about the degree, and there is no means to incorporate the hesitation or uncertainty in a fuzzy set. Later, Atanassov [2] gave a generalized form of fuzzy set, called Intuitionistic Fuzzy Set (IFS), which is characterized by a membership function and a non-membership function.

The intuitionistic fuzzy set has received more and more attention since its appearance [3-9]. Mitchell [10] and Nayagam [11] introduced the concept of the intuitionistic fuzzy number (IFN) and studied the ranking method. Grzegoraewski [12] suggested some methods for measuring distances between intuitionistic fuzzy sets and/or interval-valued fuzzy sets, based on the Hausdorff metric. The proposed new distances are generalizations of the Hamming distance, the Euclidean distance and their normalized counterparts. Literature [13-19] applied to multi-attribute decision. Wang and Xu [14-15] expanded intuitionistic fuzzy set to interval-valued intuitionistic fuzzy set. Chen [17] and Hwang [18] expanded triangular intuitionistic fuzzy number's score function and gave ranking method. Sun [20] defined triangle intuitionistic fuzzy number (TIFN), gave us four kinds of algorithms, and applied it to a decision tree analysis. Li [21] gave a intuitionistic fuzzy set note for fault-tree analysis on printed circuit board assembly. Literature [22-23] defined the intuitionistic trapezoidal fuzzy number (ITFN) and gave a multi-criteria decision-making method, which is the TIFN extension. The TIFN and ITFN extended intuitionistic fuzzy set from another direction. It was that a discrete set extended to a continuous set.

However, most related literatures are all researches of multiple attribute decision making. It seems that the multiple attribute group decision making literature on the TIFN is less. In this paper, we further develop a new multiple attribute group decision making method based on the TIFN and 2-Tuple Linguistic Information. To do so, this paper is structured as follows. In Sect. 2, we introduce some basic concepts, operational laws and aggregation operators of the TIFN. In Sect. 3, we introduce concepts and operational laws of 2-tuple linguistic variable information. In Sect. 4, we develop a practical method based on the TIFN and the 2-tuple

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linguistic information for triangular intuitionistic fuzzy group decision making problem, which is straightforward and has no loss of information. And in Sect. 5, we give an illustrative example to verify the developed approach and to demonstrate its feasibility and practicality. Finally, in Sect. 6, we conclude the paper and give some remarks.

II. TRIANGULAR INTUITIONISTIC FUZZY NUMBER

In the following, we introduce some basic concepts related to triangular intuitionistic fuzzy number (TIFN).

A. Related Definitions and Operational Laws of TIFN

Definition 1 If $\tilde{a}$ be intuitionistic fuzzy number (IFN), $\tilde{a} \in R$, membership function be defined as

$$\mu_{\tilde{a}}(x) = \begin{cases} f_{\tilde{a}}^{L}(x) & a \le x < b \\ w_{\tilde{a}} & b \le x \le c \\ f_{\tilde{a}}^{R}(x) & c < x \le d \\ 0 & x < a \text{ or } x > d \end{cases}$$

non-membership function be defined as

$$v_{\tilde{a}}(x) = \begin{cases} g_{\tilde{a}}^{L}(x) & a_1 \le x < b \\ u_{\tilde{a}} & b \le x \le c \\ g_{\tilde{a}}^{R}(x) & c < x \le d_1 \\ 0 & x < a_1 \text{ or } x > d_1 \end{cases}$$

Since $f_{\tilde{a}}^{L}(x)$ and $g_{\tilde{a}}^{R}(x)$ are continuous and monotonically increasing function, on the contrary , $f_{\tilde{a}}^{R}(x)$ and $g_{\tilde{a}}^{L}(x)$ are continuous and monotonically decreasing functions, $f_{\tilde{a}}^{L}(x)$ and $f_{\tilde{a}}^{R}(x)$ are known as the membership degrees of left, right benchmark function; while $g_{\tilde{a}}^{L}(x)$ and $g_{\tilde{a}}^{R}(x)$ are known as the non-membership degrees of left, right benchmark function; $w_{\tilde{a}}$ denotes the maximum membership degree and $u_{\tilde{a}}$ denotes the minimal non-membership, with $0 \le w_{\tilde{a}} \le 1$, $0 \le u_{\tilde{a}} \le 1$, $0 \le w_{\tilde{a}} + u_{\tilde{a}} \le 1$. The intuition fuzzy number be denoted as

$$\tilde{a} = \left\langle \left[(a,b,c,d); w_{\tilde{a}} \right], \left[(a_1,b,c,d_1); u_{\tilde{a}} \right] \right\rangle$$

When $f_{\tilde{a}}^{L}(x) = \frac{x-a}{b-a} w_{\tilde{a}}$, $f_{\tilde{a}}^{R}(x) = \frac{d-x}{d-c} w_{\tilde{a}}$, $g_{\tilde{a}}^{L}(x) = \frac{b-x+u_{\tilde{a}}(x-a_1)}{b-a_1}$, $g_{\tilde{a}}^{R}(x) = \frac{x-c+u_{\tilde{a}}(d_1-x)}{d_1-c}$, The intuition fuzzy number is called the intuitionistic trapezoidal fuzzy number. Specially, when $a = a_1$, $d = d_1$, the ITFN is denoted as $\tilde{a} = \left\langle (a,b,c,d); w_{\tilde{a}}, u_{\tilde{a}} \right\rangle$. When $b = c$, the intuitionistic trapezoidal fuzzy number degenerates into the intuitionistic triangular fuzzy number (the triangular intuitionistic fuzzy number). So the triangular intuitionistic fuzzy number be defined as follows:

Definition 2 Let $\tilde{a} =< (\underline{a}, a, \overline{a}); w_{\tilde{a}}, u_{\tilde{a}} >$ be a TIFN, $\tilde{a} \in R$, its membership function be defined as

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{x-\underline{a}}{a-\underline{a}} w_{\tilde{a}} & \underline{a} \le x < a \\ w_{\tilde{a}} & x = a \\ \frac{\overline{a}-x}{\overline{a}-a} w_{\tilde{a}} & a < x \le \overline{a} \\ 0 & x < a \text{ or } x > \overline{a} \end{cases} \tag{1}$$

non-membership function be defined as

$$v_{\tilde{a}}(x) = \begin{cases} \frac{a-x+u_{\tilde{a}}(x-\underline{a})}{a-\underline{a}} & \underline{a} \le x < a \\ u_{\tilde{a}} & x = a \\ \frac{x-a+u_{\tilde{a}}(\overline{a}-x)}{\overline{a}-a} & a < x \le \overline{a} \\ 1 & x < \underline{a} \text{ or } x > \overline{a} \end{cases} \tag{2}$$

where $w_{\tilde{a}}$ be maximal membership degree and $u_{\tilde{a}}$ be minimal non-membership degree, with $0 \le w_{\tilde{a}} \le 1$, $0 \le u_{\tilde{a}} \le 1$, $0 \le w_{\tilde{a}} + u_{\tilde{a}} \le 1$.

Let $\pi_{\tilde{a}}(x) = 1 - \mu_{\tilde{a}}(x) - v_{\tilde{a}}(x)$, $\pi_{\tilde{a}}(x)$ be called the degree of indeterminacy of the the element x to $\tilde{a}$, it reflects hesitancy degree of the element x to $\tilde{a}$.

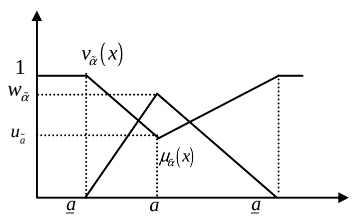

Fig.1 Triangular intuitionistic fuzzy number $\tilde{a}$

If $\underline{a} \ge 0$ and $\underline{a}, a, \overline{a}$ are not all zero, then $\tilde{a} =< (\underline{a}, a, \overline{a}); w_{\tilde{a}}, u_{\tilde{a}} >$ be called the positive TIFN, it denotes as $\tilde{a} >_{IFN} 0$. Similarly, if $\underline{a} \le 0$ and $\underline{a}, a, \overline{a}$ are not all zero, then $\tilde{a} =< (\underline{a}, a, \overline{a}); w_{\tilde{a}}, u_{\tilde{a}} >$ be called the negative TIFN. It denotes as $\tilde{a} <_{IFN} 0$, where “ $<_{IFN}$ ” means intuition fuzzy “less than”.

The TIFN $\tilde{a} =< (\underline{a}, a, \overline{a}); w_{\tilde{a}}, u_{\tilde{a}} >$ represents a approximate real number. Namely, the uncertainties $\tilde{a}$ with arbitrary real number representation is between $\underline{a}$ and $\overline{a}$, and every real number has different degrees of membership degree, non-membership degree and hesitancy degree. The uncertainty $\tilde{a}$ is the most probable value a , its membership degree and non-membership degree are $w_{\tilde{a}}$ and $u_{\tilde{a}}$ respectively. The most pessimistic and optimistic values of $\tilde{a}$ are $\underline{a}$ and $\overline{a}$ respectively, its membership degree and non-membership degree are between 0 and 1 respectively. If $x \in (\underline{a}, \overline{a})$, the membership degree and non-membership degree of the uncertainty $\tilde{a}$ are $\mu_{\tilde{a}}(x)$ and $v_{\tilde{a}}(x)$ respectively.

Relative to the definition of intuitionistic fuzzy set, the membership degree and non-membership degree of the TIFN are not only relative to a vague concept of "excellent" or "good", but relative to the TIFN, and that is decision makers' hesitancy degree, so as to it reflects the decision-makers information more accurately and objectively.

For example, the TIFN $\tilde{5} =< (3,4,9); 0.6, 0.2 >$, then if $x = 4$, the membership degree of TIFN $\tilde{5}$ is 0.6, the non-membership of TIFN $\tilde{5}$ is not 0.2, at the same time, hesitancy degree is 0.1.

Sun [20] defined four algorithms of the TIFN, but there were some errors. Li [21] modified them and defined four algorithms of triangular intuitionistic fuzzy number as follows:

Definition 3 Let $\tilde{a} =< (\underline{a}, a, \overline{a});\ w_{\tilde{a}}, u_{\tilde{a}} >$ and $\tilde{b} =< (\underline{b}, b, \overline{b});\ w_{\tilde{b}}, u_{\tilde{b}} >$ be two TIFNs, λ be a real number, then defined as [21]

$$\tilde{a}+\tilde{b} = \left\langle \left(\underline{a}+\underline{b}, a+b, \overline{a}+\overline{b}\right); \min\{w_{\tilde{a}}, w_{\tilde{b}}\}, \max\{u_{\tilde{a}}, u_{\tilde{b}}\} \right\rangle$$

$$\tilde{a}-\tilde{b} = \left\langle \left(\underline{a}-\overline{b}, a-b, \overline{a}-\underline{b}\right); \min\{w_{\tilde{a}}, w_{\tilde{b}}\}, \max\{u_{\tilde{a}}, u_{\tilde{b}}\} \right\rangle$$

$$\tilde{a}\tilde{b} = \begin{cases} \left\langle \left(\underline{ab}, ab, \overline{ab}\right); \min\{w_{\tilde{a}}, w_{\tilde{b}}\}, \max\{u_{\tilde{a}}, u_{\tilde{b}}\} \right\rangle, & \text{if } \tilde{a} > 0 \text{ and } \tilde{b} > 0 \\ \left\langle \left(\underline{a}\overline{b}, ab, \overline{a}\underline{b}\right); \min\{w_{\tilde{a}}, w_{\tilde{b}}\}, \max\{u_{\tilde{a}}, u_{\tilde{b}}\} \right\rangle, & \text{if } \tilde{a} < 0 \text{ and } \tilde{b} > 0 \\ \left\langle \left(\overline{ab}, ab, \underline{ab}\right); \min\{w_{\tilde{a}}, w_{\tilde{b}}\}, \max\{u_{\tilde{a}}, u_{\tilde{b}}\} \right\rangle, & \text{if } \tilde{a} < 0 \text{ and } \tilde{b} < 0 \end{cases}$$

$$\tilde{a}/\tilde{b} = \begin{cases} \left\langle \left(\underline{a}/\overline{b}, a/b, \overline{a}/\underline{b}\right); \min\{w_{\tilde{a}}, w_{\tilde{b}}\}, \max\{u_{\tilde{a}}, u_{\tilde{b}}\} \right\rangle, & \text{if } \tilde{a} > 0 \text{ and } \tilde{b} > 0 \\ \left\langle \left(\overline{a}/\overline{b}, a/b, \underline{a}/\underline{b}\right); \min\{w_{\tilde{a}}, w_{\tilde{b}}\}, \max\{u_{\tilde{a}}, u_{\tilde{b}}\} \right\rangle, & \text{if } \tilde{a} < 0 \text{ and } \tilde{b} > 0 \\ \left\langle \left(\overline{a}/\underline{b}, a/b, \underline{a}/\overline{b}\right); \min\{w_{\tilde{a}}, w_{\tilde{b}}\}, \max\{u_{\tilde{a}}, u_{\tilde{b}}\} \right\rangle, & \text{if } \tilde{a} < 0 \text{ and } \tilde{b} < 0 \end{cases}$$

$$\lambda\tilde{a} = \begin{cases} \left\langle \left(\lambda\underline{a}, \lambda a, \lambda\overline{a}\right); w_{\tilde{a}}, u_{\tilde{a}} \right\rangle, & \text{if } \lambda > 0 \\ \left\langle \left(\lambda\overline{a}, \lambda a, \lambda\underline{a}\right); w_{\tilde{a}}, u_{\tilde{a}} \right\rangle, & \text{if } \lambda < 0 \end{cases}$$

$$\tilde{a}^{-1} = \left\langle \left(1/\overline{a}, 1/a, 1/\underline{a},\right); w_{\tilde{a}}, u_{\tilde{a}} \right\rangle$$

It's worth noting that the addition and substruction calculation results are still the TIFNs, but multiplication or division results of the TIFNs are not the TIFNs themselves. For simplicity, result often approximates the TIFN.

B. The TIFN Ranking Method based on the λ Weighted Average Area

Let $\tilde{a} =< (\underline{a}, a, \overline{a}); w_{\tilde{a}}, u_{\tilde{a}} >$ be a TIFN, $\alpha \in [0, w_{\tilde{a}}]$, $m(\tilde{a}_\alpha)$ and $m(\tilde{a}_\beta)$ respectively are average values of α-cut set $\tilde{a}_\alpha$ and β-cut set $\tilde{a}_\beta$ of the TIFN $\tilde{a}$, we give triangular intuitionistic fuzzy number average area.

$$m(\tilde{a}_\alpha) = \left[2\alpha a + (w_{\tilde{a}} - \alpha)(\underline{a} + \overline{a})\right] / 2w_{\tilde{a}}$$

$$m(\tilde{a}_\beta) = \left[2(1-\beta)a + (\beta - u_{\tilde{a}})(\underline{a} + \overline{a})\right] / 2(1 - u_{\tilde{a}})$$

Then, the average areas of $\tilde{a}$ about the membership degree $\mu_{\tilde{a}}(x)$ and non-membership degree $v_{\tilde{a}}(x)$ are respectively defined as

$$S_\mu(\tilde{a}) = \int_0^{w_{\tilde{a}}} m(\tilde{a}_\alpha) d\alpha = \left(2a + \underline{a} + \overline{a}\right) w_{\tilde{a}} \Big/ 4 \quad (3)$$

$$S_v(\tilde{a}) = \int_{\mu_{\tilde{a}}}^{1} m(\tilde{a}_\beta) d\beta = \left(2a + \underline{a} + \overline{a}\right)\left(1 - u_{\tilde{a}}\right) \Big/ 4 \quad (4)$$

Let

$$S_\lambda(\tilde{a}) = \lambda S_\mu(\tilde{a}) + (1-\lambda) S_v(\tilde{a}) \quad (5)$$

where λ be the weighting vector, with $\lambda \in [0,1]$. $S_\lambda(\tilde{a})$ be defined as the λ weighted average area of $\tilde{a}$, and $S_\lambda(\tilde{a})$ reflects the membership degree and non-membership degree at different confidence levels. Since some decision makers pay attention to membership degree of the TIFN, but other decision makers more concern about non-membership degree of the TIFN, decision makers select the λ value according to their preference information. The larger the $S_\lambda(\tilde{a})$ value, the larger the TIFN. So we get the following TIFN ranking method.

Definition 4 Let $S_\lambda(\tilde{a})$ and $S_\lambda(\tilde{b})$ are respectively the λ weighted average areas of two TIFNs

$$\tilde{a} =< (\underline{a}, a, \overline{a}); w_{\tilde{a}}, u_{\tilde{a}} >$$

and

$$\tilde{b} =< (\underline{b}, b, \overline{b}); w_{\tilde{b}}, u_{\tilde{b}} >$$

then

1. If $S_\lambda\left(\tilde{a}\right) < S_\lambda\left(\tilde{b}\right)$, then $\tilde{a} <_{IFN} \tilde{b}$;
2. If $S_\lambda\left(\tilde{a}\right) > S_\lambda\left(\tilde{b}\right)$, then $\tilde{a} >_{IFN} \tilde{b}$;
3. If $S_\lambda\left(\tilde{a}\right) = S_\lambda\left(\tilde{b}\right)$, then $\tilde{a} =_{IFN} \tilde{b}$.

Example Comparing to two TIFNs

$$\tilde{a} = \left\langle (0.2, 0.4, 0.6); 0.6, 0.3 \right\rangle$$

and

$$\tilde{b} = \left\langle (0.1, 0.6, 0.9); 0.5, 0.2 \right\rangle$$

By formulas (3) and (4), we respectively calculate the average area values of the membership degree and the non-membership degree:

$$s_\mu\left(\tilde{a}\right) = 0.24, \quad s_v\left(\tilde{a}\right) = 0.28$$
$$s_\mu\left(\tilde{b}\right) = 0.275, \quad s_v\left(\tilde{b}\right) = 0.44$$

By formulas (5), we calculate the weighting average area values:

$$s_\lambda\left(\tilde{a}\right) = 0.28 - 0.04\lambda, \quad s_\lambda\left(\tilde{b}\right) = 0.44 - 0.165\lambda$$

Through analysis and discussion, we get

1. If $\lambda \in [0, 0.5), s_\lambda\left(\tilde{a}\right) < s_\lambda\left(\tilde{b}\right)$, then $\tilde{b} >_{IFN} \tilde{a}$;
2. If $\lambda = 0.5, s_\lambda\left(\tilde{a}\right) = s_\lambda\left(\tilde{b}\right) = 0.325$, then $\tilde{b} =_{IFN} \tilde{a}$;
3. If $\lambda \in (0.5, 1], s_\lambda\left(\tilde{a}\right) > s_\lambda\left(\tilde{b}\right)$, then $\tilde{a} >_{IFN} \tilde{b}$.

C. Aggregation Operators of The TIFN [17-22]

Definition 5 Let $\tilde{a}_j\ (j = 1, 2, \cdots, n)$ be TIFN, and $TI-WAA_\omega : \Omega^n \to \Omega$, if

$$TI-WAA_\omega\left(\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n\right) = \sum_{j=1}^{n} \omega_j \widetilde{a_j} \tag{6}$$

where $\omega = (\omega_1, \omega_2, \cdots \omega_n)^T$ is the weight vector of $\tilde{a}_j\ (j = 1, 2, \cdots, n)$, $\omega_j \in [0,1]$, $\sum_{j=1}^{n} \omega_j = 1$. Then the $TI-WAA$ is called the weighted arithmetic average operator of the TIFN. Especially, if $\omega = \left(\frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n}\right)^T$, the $TI-WAA$ is reduced to the arithmetic averaging operator $(IT-WA)$ of the TIFN.

From above definition 5 , we know that the $TI-WAA$ operator weights are the given arguments, and then aggregates these weighted arguments into a collective one.

By section Ⅱ. B operational laws, we easily get

Theorem 1 Let $\tilde{a}_j = \left\langle \left(\underline{a}_j, a_j, \overline{a}_j\right); w_{\tilde{a}_j}, u_{\tilde{a}_j} \right\rangle$ $(j = 1, 2, \cdots, n)$ be TIFN, then Definition 5 aggregation result still be the TIFN, and

$$TI-WAA_\omega\left(\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n\right)$$
$$= \left\langle \left(\sum_{j=1}^{n} \omega_j \underline{a}_j, \sum_{j=1}^{n} \omega_j a_j, \sum_{j=1}^{n} \omega_j \overline{a}_j \right); \right.$$
$$\left. \min\left\{ w_{\tilde{a}_1}, w_{\tilde{a}_2}, w_{\tilde{a}_3} \right\}, \max\left\{ u_{\tilde{a}_1}, u_{\tilde{a}_2}, u_{\tilde{a}_3} \right\} \right\rangle$$

Definition 6 Let $\tilde{a}_j\ (j = 1, 2, \cdots, n)$ be the TIFN, an ordered weighted average operator of dimension n is a mapping $TI-OWA : I^n \to I$, $TI-OWA$ be defined as

$$TI-OWA_\omega\left(\tilde{a}_1, \cdots, \tilde{a}_n\right) = \sum_{j=1}^{n} \omega_j \tilde{a}_{\sigma(j)}$$

where $\omega = \left(\omega_1, \cdots, \omega_n\right)$ be associated with a weight vector of the $TI-OWA$, with $0 \le \omega_j \le 1$, $\sum_{j=1}^{n} \omega_j = 1$, such that $\omega_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right)$ $i = 1, 2, \cdots, n$, Q be fuzzy operator, $a, b, r \in [0,1]$

$$Q(r) = \begin{cases} 0, & r < a; \\ \dfrac{r-a}{b-a}, & a \le r \le b \\ 1, & r > b \end{cases}$$

In "at least half", "fuzzy majority" and "as much as possible " principle, the corresponding parameters $[a, b]$ of the fuzzy quantization operator Q respectively $(0, 0.5)$, $(0.5, 0.8)$ and $(0.5, 1)$.

$\left(\sigma_{(1)}, \cdots, \sigma_{(n)}\right)$ is $(1, \cdots, n)$ any replacement, and satisfy $\tilde{a}_{\sigma(j-1)} \ge \tilde{a}_{\sigma(j)}$. Especially, if $\omega = (1/n, \cdots, 1/n)$, then the $TI-OWA$ is reduced to the $TI-WA$.

Theorem 2 If $\tilde{a}_j\ (j = 1, \cdots, n)$ be the TIFNs, $\left(\tilde{a}_{\sigma(1)}, \cdots, \tilde{a}_{\sigma(n)}\right)$ is $\left(\tilde{a}_1, \cdots, \tilde{a}_n\right)$ any replacement, $\tilde{a}_{\sigma(j-1)} \ge \tilde{a}_{\sigma(j)}$, then

$$TI-OWA_\omega\left(\tilde{a}_1, \cdots, \tilde{a}_n\right)$$
$$= \left\langle \left(\sum_{j=1}^{n} \omega_j \underline{a}_{\sigma(j)}, \sum_{j=1}^{n} \omega_j a_{\sigma(j)}, \sum_{j=1}^{n} \omega_j \overline{a}_{\sigma(j)} \right); \right.$$
$$\left. \min\left\{ w_{\tilde{a}_{\sigma(1)}}, w_{\tilde{a}_{\sigma(2)}}, w_{\tilde{a}_{\sigma(3)}} \right\}, \max\left\{ w_{\tilde{a}_{\sigma(1)}}, w_{\tilde{a}_{\sigma(2)}}, w_{\tilde{a}_{\sigma(3)}} \right\} \right\rangle$$

The proving procedures of theorem 2 are similar to the theorem 1.

III. 2-TUPLE LINGUISTIC INFORMATION

Many problems in the real world can’t be assessed precisely in a quantitative form, but it may be done in a qualitative one. In that case 2-tuple linguistic information was made by Professor Herrera of Spain in 2000. This method was defined as language phrases within the continuous variable, and it could express all information by one binary form of linguistic assessment information between pre-language phrases and a real value, which can not only effectively avoid the occurrence of loss and distortion but also make language information more accurate.

Let $S=\{s_i | i=0,1,\cdots,t\}$ be a linguistic term set with odd cardinality. Any label, s_i represents a possible value for a linguistic variable, and it should satisfy the following characteristics:

1. The set is linearly ordered: $s_i > s_j$, if $i > j$;

2. Max operator: $\max(s_i, s_j) = s_i$, if $s_i \ge s_j$;

3. Min operator: $\min(s_i, s_j) = s_i$, if $s_i \le s_j$;

4.Negation operator: $neg(s_\alpha) = s_{-\alpha}$, especially, $neg(s_0) = s_0$. Obviously, the mid linguistic label s_0 represents an assessment of “indifference”, and with the rest of the linguistic labels being placed symmetrically around it.

The linguistic labels in the above linguistic label sets are uniformly and symmetrically distributed. For example, S can be defined as $S=\{s_0=$ extremely poor (EP), $s_1=$ very poor (VP), $s_2=$ poor (P), $s_3=$ medium (M), $s_4=$ good (G), $s_5=$ very good (VG), $s_6=$ extremely good (EG) $\}$. However, in many realistic situations, the unbalanced linguistic information appears due to the nature of the linguistic variables used in the problem.

The 2-tuple fuzzy linguistic representation model represents the linguistic information by means of a 2-tuple (s,a), where s is a linguistic label and a is a numerical value that represents the value of the symbolic translation.

In this fuzzy linguistic context, if a symbolic method to aggregate linguistic information obtains a value $\beta \in [0, T]$ and $\beta \notin \{0,1,2,\cdots,T\}$, then an approximation function is used to express the result in S; called Δ :

$$\Delta : [0,T] \to S \times [-0.5, 0.5)$$

$$\Delta(\beta) = \begin{cases} s_i, & i = round(\beta) \\ \alpha_i = \beta - i, & \alpha_i \in [-0.5, 0.5) \end{cases}$$

In such a way, β is represented by means of a 2-tuple (s_i, α_i), where $s_i \in S$, s_i represents the linguistic label information, and α_i is a numerical value of the symbolic translation from the original result β to the most close index label i in the linguistic term set $s_i \in S$. On the other hand, there is always a function Δ^{-1}. Such that, a 2-tuple linguistic (s_i, α_i) returns its equivalent numerical value β ($\beta \in [0, T] \subset R$), $\Delta^{-1}(s_i, \alpha_i) = i + \alpha_i$. It is obvious that the conversion of a linguistic term s_i into a linguistic 2-tuple consists in adding a value 0 as symbolic translation $(s_i, 0)$.

The 2-tuple linguistic computational model is defined by presenting a negation operator, comparison of 2-tuple and aggregation operators:

1. A 2-tuple negation operator:

$$neg(s_i, \alpha) = \Delta\left(t - \left(\Delta^{-1}(s_i, \alpha)\right)\right)$$

where $t+1$ is the cardinality of S, $S=\{s_i | i=0,1,\cdots,t\}$.

2. A 2-tuple comparison operator

Let (s_k, α_1) and (s_l, α_2) be two 2-tuples. Then

- If $k < l$ then (s_k, α_1) is smaller than (s_l, α_2);
- If $k = l$ then

(1) If $\alpha_1 = \alpha_2$ then (s_k, α_1) ; (s_l, α_2) represents the same information;

(2) If $\alpha_1 < \alpha_2$ then (s_k, α_1) is smaller than (s_l, α_2);

(3) If $\alpha_1 > \alpha_2$ then (s_k, α_1) is larger than (s_l, α_2).

IV. An Approach to Group Decision Making under the TIFN and 2-TUPLE Linguistic Information

In this section, we present a computational model to multi-attribute group decision making (MAGDM) problem.

A. Group Decision Making Problem

The following assumptions or notations are used to represent group decision making problems, which take the forms of 2-tuple linguistic weight information and TIFN attribute values.

Let $A=\{A_1, A_2, \cdots, A_m\}$ be a finite set of m feasible alternatives. $a^*=\{a_1, a_2 \cdots, a_n\}$ is a finite set of attributes and $W=(w_1, \cdots, w_n)$ be the weight vector of attributes a_j $(j=1,2,\cdots n)$, where $w_j \in [0,1]$, $\sum_{j=1}^{n} w_j = 1$ and w_j is denoted by the 2-tuple linguistic term (s_j, α_j) $(j=1,2,\cdots n)$. $P=\{P_1, P_2, \cdots, P_k\}$ be a set of decision makers, Let $\tilde{A}^{(t)} = \left(\tilde{a}_{ij}^{(t)}\right)_{m\times n}$ be a triangular intuitionistic fuzzy group decision making matrix, where $\tilde{a}_{ij}^{(t)} = \left\langle \left(\underline{a}_{ij}^{(t)}, a_{ij}^{(t)}, \overline{a}_{ij}^{(t)}\right); w_{\tilde{a}_{ij}^{(t)}}, u_{\tilde{a}_{ij}^{(t)}} \right\rangle$ is an TIFN with $0 \le w_{\tilde{a}_{ij}^{(t)}} \le 1$, $0 \le u_{\tilde{a}_{ij}^{(t)}} \le 1$, $w_{\tilde{a}_{ij}^{(t)}} + u_{\tilde{a}_{ij}^{(t)}} \le 1$. $\omega = (\omega_1, \cdots, \omega_k)$ be weighting vector of decision makers with $\omega_j \in [0,1]$, $\sum_{j=1}^{k} \omega_j = 1$. $\tilde{a}_{ij}^{(t)}$ denotes that the t-th decision maker P_t $(t=1,2,\cdots k)$ gives the value of alternative A_i $(i=1,2,\cdots m)$ to attribute a_j $(j=1,2,\cdots n)$. So we get decision matrix $\tilde{A}^{(t)} = \left(\tilde{a}_{ij}^{(t)}\right)_{m\times n}$ by the decision maker $P_t \in P$.

B. Group Decision Making Algorithm

The steps of the decision making based on the triangular fuzzy inuitionistic fuzzy numbers are as follows:

Step1: The attribute weights transform the real numbers by the 2-tuple linguistic negation operator $\Delta^{-1}(s_i,\alpha_i) = i+\alpha_i = \beta$. Owing to $\beta \in [0, T]$, we must normalize $W' = \left(w_1',\cdots,w_n'\right)$ and satisfy $\sum_{i=1}^{n} w_i' = 1$;

Step2: According to the triangular intuitionistic fuzzy matrix $\tilde{A}^{(t)} = \left(\tilde{a}_{ij}^{(t)}\right)_{m\times n}$ by decision maker P_t, utilize the $TI-WAA$ operator and Theorem 1 to aggregate the i-th line $\tilde{a}_{ij}^{(t)}$ $(j=1,\cdots,n)$, we gain each scheme comprehensive TIFN information.

Step3: According to fuzzy quantization function method, to determine the weighting vector $\omega = \left(\omega_1,\cdots,\omega_k\right)$ with correlative $TI-OWA$. Then, to calculate the λ weighted average area of $\tilde{a}_j$ $\left(j=1,\cdots,n\right)$ and rank by definition 4. Finally, aggregate $\tilde{a}_i^{(t)}$ $\left(t=1,\cdots,k\right)$ by Theorem 2, and get all experts group integrative TIFN to A_i $\left(i=1,\cdots,m\right)$;

Step4: By Eqs (3-5), calculate the λ weighted average area value $S_\lambda\left(\tilde{a}_i\right)$ of every alternative group integrative TIFN;

Step5: Rank all the alternatives and select the best one in accordance with ranking law of Eq. (5).

V. ILLUSTRATIVE EXAMPLE

Suppose that there is an unconventional emergency problem, which needs to classification ranking. There is a panel with three decision makers P_1, P_2 and P_3, decision makers must take a classification decision according to the following attributes: (1) scale (a_1); (2) severity degree of loss (a_2); (3) effect to people living (a_3). The three possible alternatives emergency options A_i, $i=1,2,3$ are to be evaluated using the TIFN by the decision makers.

Let $S=\{s_0,s_1,s_2,s_3,s_4,s_5,s_6\}$, three decision makers give the weighting vectors as follows:

$$W_1 = \left(w_{11}, w_{12}, w_{13}\right) = \left((s_5,0.4),(s_4,-0.1),(s_2,-0.3)\right)$$
$$W_2 = \left(w_{21}, w_{22}, w_{23}\right) = \left((s_6,-0.2),(s_3,0.4),(s_2,-0.2)\right)$$
$$W_3 = \left(w_{31}, w_{32}, w_{33}\right) = \left((s_4,-0.2),(s_2,0.4),(s_1,0.2)\right)$$

Three decision makers respectively constructe the decision matrix datas as follows:

Table 1 Triangular intuitionistic fuzzy information by decision maker P_1

	w_1	w_2	w_3
A_1	$\langle(1,2,4);0.7,0.2\rangle$	$\langle(2,3,5);0.5,0.4\rangle$	$\langle(3,5,7);0.7,0.2\rangle$
A_2	$\langle(4,5,6);0.6,0.3\rangle$	$\langle(3,4,5);0.6,0.3\rangle$	$\langle(4,5,9);0.5,0.4\rangle$
A_3	$\langle(2,4,8);0.5,0.4\rangle$	$\langle(2,3,4);0.8,0.2\rangle$	$\langle(1,5,6);0.6,0.4\rangle$

Table 2 Triangular intuitionistic fuzzy information by decision maker P_2

	w_1	w_2	w_3
A_1	$\langle(3,5,8);0.5,0.4\rangle$	$\langle(2,3,5);0.8,0.2\rangle$	$\langle(2,4,7);0.7,0.1\rangle$
A_2	$\langle(1,2,3);0.8,0.0\rangle$	$\langle(3,4,8);0.5,0.4\rangle$	$\langle(3,4,6);0.7,0.2\rangle$
A_3	$\langle(2,3,6);0.7,0.2\rangle$	$\langle(1,4,8);0.6,0.2\rangle$	$\langle(1,3,6);0.7,0.2\rangle$

Table 3 Triangular intuitionistic fuzzy information by decision maker P_3

	w_1	w_2	w_3
A_1	$\langle(5,6,8);0.5,0.2\rangle$	$\langle(2,5,7);0.6,0.3\rangle$	$\langle(2,5,7);0.8,0.1\rangle$
A_2	$\langle(1,2,4);0.8,0.1\rangle$	$\langle(4,5,8);0.7,0.2\rangle$	$\langle(1,4,6);0.5,0.4\rangle$
A_3	$\langle(2,3,5);0.7,0.0\rangle$	$\langle(2,4,7);0.5,0.4\rangle$	$\langle(3,5,8);0.7,0.2\rangle$

In the following, we shall utilize the proposed approach in section Ⅳ. B gets the most desirable classification.

Step1: Transform weight vector by using the 2-tuple linguistic negation operator $\Delta^{-1}(s_i,\alpha_i) = i+\alpha_i = \beta$ and make them normalized. The results are as follows:

$$W_1' = \left(0.49, 0.36, 0.15\right)$$

$$W_2^{'} = (0.53, 0.31, 0.16)$$
$$W_3^{'} = (0.51, 0.32, 0.17)$$

Step2: By Table 1-3, we get triangular intuitionistic fuzzy group decision matrix $\tilde{A}^{(t)} = \left(\tilde{a}_{ij}^{(t)}\right)_{3\times3}$. Utilize the $TI-WAA$ operator and Theorem 1 to aggregate the i-th line of fuzzy information $\tilde{a}_{ij}^{(t)}$ $(j=1,2,3)$. We gain every comprehensive TIFN information to decision making expert:

$$P_1: \tilde{a}_1^{(1)} = \langle (1.66, 2.81, 4.81); 0.5, 0.4 \rangle$$
$$\tilde{a}_2^{(1)} = \langle (3.64, 4.64, 6.09); 0.5, 0.4 \rangle$$
$$\tilde{a}_3^{(1)} = \langle (1.85, 3.79, 6.26); 0.5, 0.4 \rangle$$
$$P_2: \tilde{a}_1^{(2)} = \langle (2.53, 4.22, 6.91); 0.5, 0.4 \rangle$$
$$\tilde{a}_2^{(2)} = \langle (1.94, 2.94, 5.03); 0.5, 0.4 \rangle$$
$$\tilde{a}_3^{(2)} = \langle (1.53, 3.31, 6.62); 0.6, 0.2 \rangle$$
$$P_3: \tilde{a}_1^{(3)} = \langle (3.53, 5.51, 7.51); 0.5, 0.3 \rangle$$
$$\tilde{a}_2^{(3)} = \langle (1.96, 3.3, 5.62); 0.5, 0.4 \rangle$$
$$\tilde{a}_3^{(3)} = \langle (2.17, 3.66, 6.15); 0.5, 0.4 \rangle$$

Step3: Select fuzzy quantization “majority” criterion, we calculate the weighting vector $\omega = (0, 0.57, 0.43)$ with correlative $TI-OWA$, to aggregate $\tilde{a}_i^{(t)}$ $(t=1,2,3)$ by Eq.（6） and get all experts group integrative TIFNs :

$$\tilde{a}_1 = \langle (2.16, 3.61, 6.01); 0.5, 0.4 \rangle$$
$$\tilde{a}_2 = \langle (1.95, 3.15, 5.37); 0.5, 0.4 \rangle$$
$$\tilde{a}_3 = \langle (2.03, 3.72, 6.20); 0.5, 0.4 \rangle$$

Step4: By Eqs. (3)-(5), we calculate the λ weighted average area value $S_\lambda(\tilde{a}_i)$ of every alternative group integrative TIFN:

$$S_\lambda(\tilde{a}_1) = 2.31 - 0.39\lambda$$
$$S_\lambda(\tilde{a}_2) = 2.04 - 0.34\lambda$$
$$S_\lambda(\tilde{a}_3) = 2.35 - 0.39\lambda$$

Step5: Rank all the alternatives $A_i(i=1,2,3)$ in accordance with the overall preference values $S_\lambda(\tilde{a}_3) > S_\lambda(\tilde{a}_1) > S_\lambda(\tilde{a}_2)$:

$$A_3 \succ A_1 \succ A_2$$

And thus, the most desirable is A_3. By the data of analyzing table 1-3, we know that group decision classification ranking result is reasonable.

VI. Conclusion

In this paper, we use the extended aggregation operators to ranking alternatives based on the TIFN attribute values and the 2-tuple linguistic attribute weights, which develop a new approach of group decision making approach. Triangular intuitionistic fuzzy number and 2-tuple linguistic are easier to deal with the fuzzy and the uncertain information of different decision makers. Therefore, the approach has a good prospect in application. Finally, We have also applied the proposed approach to a practical emergency problem. The numerical results show that the proposed method is simple, feasible and effective.

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