Convexity Conditions for Parameterized Surfaces

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***Abstract*—Based on a geometrical method, the internal relationships between locally parameterized curves and the local parameterized surfaces are analyzed. A necessary and sufficient condition is derived for the local convexity of parameterized surfaces and functional surfaces. A criterion for local convexity (concavity) of parameterized surfaces is found, also, the criterion condition of binary function convex surfaces is obtained. Finally, the relationships between a globally parameterized curves surfaces is discussed, a necessary condition is presented for the global convexity of parameterized surfaces , and it is proved that locally convex parameterized surfaces are also globally convex.**

***Index Terms*—local convexity, global convexity, gauss curvature, the second fundamental form**

I. INTRODUCTION

With the development of CAGD technology, the geometrical morphological analysis of parameterized surface has become an important subfield of CAGD, which is the research content of differential geometry in the analysis and study of free parameters surface. The geometrical design theorem is the foundation of the application, such as modeling and design of industrial product, Computer Aided Process and Analysis. In this paper, the local and global convexity analysis of the regular parameterized surface is studied.

To the convexity research of curves and surfaces, many scholars have obtained some results. Convexity problems of general and special parameterized curves, such as Bézier-Curves, B-splint-Curves, have been solved by C. Liu and C. R. Trass[1], Dingyuan Liu[2], B.de Boor[3]. However, the surfaces convexity study is always an interesting topic. H. Wai[4] derived the convexity condition of Bernstein-Bézier multinomial surface in rectangle region, G. D. Koras and P. D. Kaklis[5] presented several sufficient conditions for parameterized tensor product of B-splint convexity surface, especially, many excellent results as to the convexity research of Bezier-surface in triangle region were obtained by the research group leaded by G. Z. Chang. But those results can't be used to analyze the convexity of general parameterized surfaces, since the above results are obtained by particular methods. At present, there are still some scholars devote to the study of the convexity of binary function surface, Lia [7] and K. Fang generalized the determining method of convex function to binary functional surface, and they derived the necessary and sufficient condition of the binary functional surface. Dahmen[9] presented the convexity condition of multinomial Bernstein-Bézier and trunk splint.

Typical convexity surface is the global one, such as ovum surface, the accurate definition of which is defined in differential geometry, but there is no algebra determining method for the convexity. B. Q. Su[12] defined the local convex surface by Gauss curvature $K > 0$, also, by the non-negativity (or non-positive) of normal curvature K_n, Koras and Kaklis[5] defined the local convex surface, thus, the necessary and sufficient condition for the local convexity is the second fundamental form of curved surface $\phi_2 \le 0 (\ge 0)$. Since both Gauss curvature and normal curvature describe only the local properties of curved surface, for some point on which, the Gauss curvature $K > 0$ means that, in the vicinity of this point, all normal section lines passing through this point bend to the same direction. The nonnegative (or non-positive) of the normal curvature at some point means that when the curvatures of the normal section lines which pass this point are not null, the bending directions of normal section lines are the same, on the contrary, for zero curvature, the bending directions cannot be determined. Thus, with Gauss curvature $K > 0$,

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it is not comprehensive to define the local convex surface, and so does for normal curvature. On the other hand, if the geometry definition of local convex curve is generalized to convex surface, it will be too rigor. Based on the above, we present a reasonable geometrical definition for local convex surface, based on this definition and with normal curvature, we can connect the local convexity of parameterized surface with the second foundational form of curved surface together, then the necessary and sufficient condition for local convexity of parameterized surfaces, which is a algebraic inequality determined by the second foundational form of curved surface, is derived. For binary functional surface, as a special parameterized one, based on the determining condition of local convexity, it is easy to get the necessary and sufficient expressions of functional convex surfaces. Meanwhile, determining conditions for local convexity (concavity) for parameterized surfaces are established, and a necessary and sufficient condition is presented for the global convexity of parameterized surfaces. Finally, we prove that the local parameterized closed convex surface is a global convex surface.

II. Definition of convex curve

Definition2.1 A plane parameterized curve $\Gamma:\mathbf{r}=\mathbf{r}(t)$, $a\le t\le b$ is an ordered set in Euclidean plane R^2, when its direction is from $t=a$ to $t=b$.

If the direction of the curve is anticlockwise, then with the transformation $t=a+b-t$, the curve direction can be changed to the clockwise one, therefore, in this paper, the direction of parameterized curve can be assumed as clockwise one.

Definition2.2 For parameterized curve $\Gamma:\mathbf{r}=\mathbf{r}(t), a\le t\le b$, we call Γ is a regular curve, if $\mathbf{r}'(t)\ne 0$.

In this paper, only the regular curves in R^2 are considered, and for every point of the curve, the second derivative exists.

Since the positive direction of the tangent vector at the parameterized curve is coincident with the increment of parameter t, we define the tangent vector direction as that of tangent line'. For plane curve, the tangent line divides the plane which the curve lies in into two half plane, and along the tangent direction, the half surface that on the night of the curve is called the right surface, and the other surface is called the left one. The half surface which contains the tangent line is called as the closed half-plane.

From the view of global convexity, K. Wilhelml [11] gave the definition of global convex curves as follows:

Definition2.3 Let $\mathbf{P}$ be an arbitrary point on Γ, we call Γ a global convex curve, if it lies in the right closed half-plane (left closed half-plane) of the tangent line at $\mathbf{P}$.

Straight lines are special global convex curves. However, the definition of local convex curve is:

Definition2.4 Let $\mathbf{P}$ be an arbitrary point on the regular plane curve Γ, if there exists a neighborhood of $\mathbf{P}$, and in which the segment corresponding to Γ is located in the right closed half-plane of the tangent line at point $\mathbf{P}$, then Γ is called a local convex curve.

It can be discerned that global convex curves are also local convex ones. The following lemma is a discriminating theorem for local convex curves.

Lemma2.1[1] A parameterized curve Γ is local convex, if and only if its relative curvature κ_r are unchanged.

For simplicity, we assume that the arc-length parameterized curve of plane parameterized curve Γ is

$$\Gamma:\mathbf{r}=\mathbf{r}(s) \tag{2.1}$$

We set the plane which curve Γ lies in is a directional one, and its direction vector is $\mathbf{k}$. Let $\boldsymbol{\alpha}$ be the unit tangent vector, and write $\mathbf{N}=\mathbf{k}\times\boldsymbol{\alpha}$ the major normal vector of plane curve Γ, then the Frenet Formula of curve Γ is:

$$\begin{cases} d\boldsymbol{\alpha}/ds=\kappa_r(s)\mathbf{N} \\ d\mathbf{N}/ds=-\kappa_r(s)\boldsymbol{\alpha} \end{cases} \tag{2.2}$$

where $\kappa_r(s)$ is the relative curvature of the plane curve.

Theorem2.1 Let Γ be a plane curve in C^2, and if the curvature of Γ at point $\mathbf{P}$ (corresponding to parameter t) is $\kappa_r>0(\kappa_r<0)$, then there exists a neighborhood of t, and the segments corresponding to which lie in the left closed half-plane (right closed half-plane) of the tangent line at point $\mathbf{P}$.

Proof: For simplicity, we just discuss the arc parameterized curve. By the Taylor expansion at $\mathbf{P}$ (corresponding to parameter s), we have

$$[\mathbf{r}(s+\Delta s)-\mathbf{r}(s)]=\dot{\mathbf{r}}\Delta s+\frac{1}{2}(\ddot{\mathbf{r}}+\boldsymbol{\varepsilon})\Delta s^2 \tag{2.3}$$

Where $\boldsymbol{\varepsilon}=\varepsilon_1\boldsymbol{\alpha}+\varepsilon_2\mathbf{N}$, and $\lim\limits_{\Delta s\to o}\boldsymbol{\varepsilon}=\mathbf{0}$.

With Frenet Formulae, obtain $\dot{\mathbf{r}}=\boldsymbol{\alpha}$, $\ddot{\mathbf{r}}=\kappa_r\mathbf{N}$, combing with equation(2.3), we have

$$[\mathbf{r}(s+\Delta s)-\mathbf{r}(s)]=\Delta s(1+\frac{1}{2}\varepsilon_1\Delta s)\boldsymbol{\alpha}+\frac{1}{2}(\kappa_r+\varepsilon_2)\mathbf{N}(\Delta s)^2 \tag{2.4}$$

Hence,

$$[\mathbf{r}(s+\Delta s)-\mathbf{r}(s)]\cdot\mathbf{N}=\frac{1}{2}(\kappa_r+\varepsilon_2)(\Delta s)^2 \tag{2.5}$$

If the curvature $\kappa_r>0(\kappa_r<0)$, then by (2.5), for Δs s-mall enough, we have $(\kappa_r+\varepsilon_2)>0(<0)$

The above formula shows that there is a neighborhood of t, such that the corresponding curve segment locates in the left closed half-plane (right closed half-plane) of the tangent at $\mathbf{P}$. This finished the proof of theorem 2.1.

The following theorem can be easily deduced by theorem 2.1 and the definition of global convex curve.

Theorem2.2 Let Γ be a local convex curve in C^2, and the curvature of which is $\kappa_r\ge 0(\kappa_r\le 0)$, then for any $\mathbf{P}$ of Γ, there exists a neighborhood such that the curve segments corresponding to which locate in the left closed half-plane (right closed half-plane) of the tangent line.

By theorem 2.2, and the definition of global convex curve, we have

Theorem2.3 Let Γ be a local convex curve in C^2, and the curvature of which is $\kappa_r\ge 0(\kappa_r\le 0)$, then for any $\mathbf{P}$

of Γ , Γ lies in the left closed half-plane (right closed half-plane) of the tangent line at $\mathbf{P}$.

By (2.4), we have

$$[\mathbf{r}(s+\Delta s)-\mathbf{r}(s)]\cdot\boldsymbol{\alpha}=\Delta s(1+\frac{1}{2}\varepsilon_1\Delta s)$$

then, for $\Delta s>0(<0)$ small enough,

$$[\mathbf{r}(s+\Delta s)-\mathbf{r}(s)]\cdot\boldsymbol{\alpha}=\Delta s(1+\frac{1}{2}\varepsilon_1\Delta s)>0(<0)$$

Consequently, we can derive the following theorem:

Theorem2.4 Let Γ be a local convex curve in C^2, and $\mathbf{P}$ (corresponding to parameter t) be any point of Γ ,then in the neighborhood of $\mathbf{P}$, the segments of Γ corresponding to $t+\Delta t$ lie on the right or left of the major normal according to $\Delta t>0$ or $\Delta t<0$.

III.DETERMINING CONDITION FOR LOCAL CONVEX SURFACE

Before discussing the convex surfaces, several conceptions are presented. For parameterized surface

$$\sum:\quad \mathbf{r}=\mathbf{r}(u,v)$$

we write

$$\mathbf{r}_u=\frac{\partial\mathbf{r}(u,v)}{\partial u},\quad \mathbf{r}_v=\frac{\partial\mathbf{r}(u,v)}{\partial v},\quad \mathbf{r}_{uu}=\frac{\partial^2\mathbf{r}(u,v)}{\partial u\partial u},$$

$$\mathbf{r}_{uv}=\frac{\partial^2\mathbf{r}(u,v)}{\partial u\partial v},\quad \mathbf{r}_{vv}=\frac{\partial^2\mathbf{r}(u,v)}{\partial v\partial v}$$

and call $$\mathbf{n}=\frac{\mathbf{r}_u\times\mathbf{r}_v}{|\mathbf{r}_u\times\mathbf{r}_v|}$$

the normal tangent, and the positive direction of normal vector $\mathbf{n}$ the positive side of Σ . Also, we call

$$E=\mathbf{r}_u\cdot\mathbf{r}_u,\quad F=\mathbf{r}_u\cdot\mathbf{r}_v,\quad G=\mathbf{r}_v\cdot\mathbf{r}_v$$

the first fundamental form of a surface, and

$$L=\mathbf{n}\cdot\mathbf{r}_{uu},\quad M=\mathbf{n}\cdot\mathbf{r}_{uv},\quad N=\mathbf{n}\cdot\mathbf{r}_{vv}$$

the second fundamental one.

For any point $\mathbf{P}$ on Σ , the tangent plane at which is π , the semi space which the normal vector directs to is the upper semi-space, the other one is the lower semi-space. Also, the semi-space which contains π is called the upper closed half-space.

From the view of global convexity, K.wilhelm [11] presented the primeval geometrical definition of the convex surface.

Definition3.1 For parameterized surface

$$\sum:\mathbf{r}(u,v),\quad (u,v)\subset D\subset R^2$$

If $\mathbf{r}_u\times\mathbf{r}_v\neq 0$, then Σ is called a regular surface.

Definition3.2 For regular a surface

$$\sum:\mathbf{r}(u,v),\quad (u,v)\subset D\subset R^2$$

If Σ lies totally in the upper closed semi-space (lower closed semi-space) of the tangent plane at $\mathbf{P}=\mathbf{r}(u,v)$, then Σ is global convex.

Planes are special global convex surfaces. It is difficult to discriminate the global convexity of a surface, we can only prove it with the primeval geometrical definition, but there is no algebra method till now. However, B. Q. Su[12] presented the local algebra definition of a convex surface (local convex surface actually) with Gauss curvature.

Definition3.3 Given $\sum:\mathbf{r}(u,v)$, $(u,v)\subset D$, for $\forall\mathbf{P}\in\Sigma$, if Gauss curvature $K>0$, then Σ is a convex surface.

For the Gauss curvature in definition 3.3, the author assumed $K>0$, rather than $K\geq 0$. The following is an example.

Let $C:\mathbf{r}(u)=\{x(u),y(u),0\}$ be a global convex surface on plane XY, take C as the traverse, and the unit assistant vector $\boldsymbol{\gamma}(u)=\{0,0,1\}$ of C as the direction vector of the straight line generatrix , then we can engender the developable surface

$$\sum:\mathbf{p}(u,v)=\mathbf{r}(u)+\boldsymbol{\gamma}(u)v\qquad (u,v)\subset D$$

For $\forall\mathbf{p}_0(u_0,v_0)\in\mathrm{D}$, $\mathbf{n}_0$ is the normal vector of Σ at $\mathbf{p}_0$, if $\mathbf{p}(u,v)\neq\mathbf{p}_0(u_0,v_0)$, then

$$\begin{aligned}(\mathbf{p}(u,v)-\mathbf{p}(u_0,v_0))\cdot\mathbf{n}_0&=[\mathbf{r}(u)-\mathbf{r}(u_\mathbf{0})+\boldsymbol{\gamma}(u)v-\boldsymbol{\gamma}(u_0)v_0)]\cdot\mathbf{n}_0\\&=(\mathbf{r}(u)-\mathbf{r}(u_\mathbf{0}))\cdot\mathbf{n}_0\end{aligned}$$

Due to the definition of the global convex curve, we have

$$(\mathbf{p}(u,v)-\mathbf{p}(u_0,v_0))\cdot\mathbf{n}\geq 0(\leq 0)$$

thus, Σ is global convex.

Above example explains that there exists a surface (developable surface) whose Gauss curvature k is 0 , but it is global convex. Then, it is not comprehensive to define a convex surface with Gauss curvature $K>0$.

If the definition of local convex curve is generalized to the global one directly, i.e., for any point $\mathbf{P}$ on the regular Σ , there exists a neighborhood of $\mathbf{P}$, such that for any point of which,

$$(0\geq)0\leq\delta=(\mathbf{r}-\mathbf{p})\cdot\mathbf{n}=\frac{1}{2}(\mathbf{n}\cdot\ddot{\mathbf{r}}+\mathbf{n}\cdot\boldsymbol{\varepsilon})(ds)^2$$

Suppose that Σ is local convex, if $\mathbf{n}\cdot\ddot{\mathbf{r}}\neq 0$, then $\varphi_2=\mathbf{n}\cdot\ddot{\mathbf{r}}ds^2$ is the principal part of δ , and its sign is the same with δ ; if $\mathbf{n}\cdot\ddot{\mathbf{r}}=0$ implies $\varphi_2=0$, then, $\varphi_2\geq 0$ (≤ 0)is the necessary condition of local convex. On the contrary, if $\varphi_2=0$ implies $\mathbf{n}\cdot\ddot{\mathbf{r}}=0$, then the positive or negative sign is uncertain, consequently, for the local convex surface, δ and ϕ_2 are not necessary to posse the same positive or negative sign. Thus, it is too rigorous to define the local convex surface with the geometrical method, and also it is hard to find out an algebraic discriminating method.

Koras& Kaklis[5] defined the local convex surface with normal curvature directly.

Definition3.4 Let $\sum:\mathbf{r}(u,v)$, $(u,v)\subset D\subset R^2$ be a regular surface, and $\mathbf{P}$ be any point of Σ , if along any direction at $\mathbf{P}$, the normal curvatures $k_n\geq 0(\leq 0)$, then Σ is a local convex surface.

In differential geometry terminology, for any $\mathbf{P}$ of Σ , the geometric meaning of normal curvature $k_n\geq 0(\leq 0)$ is that, when the curvatures of normal section lines at $\mathbf{P}$ is not null, then directions of the bending direction of normal section lines are all the same. However, when it is null, the bending directions of normal section can not be determined. On the other hand, because the signs of ϕ_2 and normal curvature K_n are the same, we can not ensure that K_n and δ possess the same sign, hence, if the local convex surface is defined with normal curvature $k_n\geq 0(\leq 0)$, it will not be consistent with the geometry definition.

A reasonable geometric definition for local convex surface is given as follows.

Definition3.5

Let $\sum:\mathbf{r}(u,v)$, $(u,v)\subset D\subset R^2$ be a regular surface, $\mathbf{P}$ be any point of Σ , if for any normal section line which crosses

$\mathbf{P}$, there exists some neighborhood of $\mathbf{P}$, in which the segment corresponding to the normal section line lies totally in the upper or lower half-space of the tangent space at $\mathbf{P}$, then Σ is local convex surface.

Obviously, the global convex surface is a local convex one. Now, we will provide a necessary condition for the local convex surface by algebraic analysis method.

Theorem3.1

The necessary condition for a regular surface $\sum:\mathbf{r}(u,v),(u,v)\subset D\subset R^2$ to be local convex is that for any point on Σ, the Gauss curvature $K\geq 0$.

Proof: Since Σ is a regular surface, the Gauss curvature exists at any point on Σ. Suppose that the Gauss curvature $K_p<0$ at $\mathbf{P}$, i.e., $\mathbf{P}$ is a hyperbolic point, at which there exists two asymptotic directions, they form two pairs of vertical angles, and in which the normal section lines at $\mathbf{P}$ bend to the opposite side of the tangent plane respectively, which is contradicted to the definition of local convex surface, therefore, $K\geq 0$. This finishes the proof of Theorem 3.1.

Before the discussion of the discriminating condition of convexity, conception of positive semi-definite matrix is stated.

Definition3.6 Let $f(x_1,x_2,\cdots,x_n)=\mathbf{X'AX}$, where $\mathbf{A}$ is real symmetric matrix of order-N, if for any group of real number $c_1,c_2,\cdots,c_n$ that are not all zeros, $f(c_1,\cdots,c_n)\geq 0(\leq 0)$, then $f(c_1,\cdots,c_n)$ is called positive (negative) semi-definite, and $\mathbf{A}$ is called a positive (negative) semi-definite matrix.

Lemma 3.1[13] A real symmetric matrices $\mathbf{A}$ is semi-positive define if and only if all the principal minor larger or equal to 0.

Now we fix a point $\mathbf{p}(u,v)$ on the regular surface Σ, the unit normal vector of Σ on $\mathbf{P}$ is $\mathbf{n}$, and write it Γ_λ the normal sections through $\mathbf{P}$,

$$\Gamma_\lambda:\quad \mathbf{r}_\lambda(s_\lambda)=\mathbf{r}_\lambda(u(s_\lambda),v(s_\lambda)) \qquad (3.1)$$

where $\mathbf{P}=\mathbf{r}_\lambda(\bar{s}_\lambda)$, s_λ is the arc length parametric of Γ_λ. The unit tangent vector of normal vector Γ_λ is $\boldsymbol{\alpha}_\lambda$, in particular, the unit tangent vector at $\mathbf{P}$ is $\bar{\boldsymbol{\alpha}}_\lambda$. We assume the plane which the normal section lies in is a directional plane, and the directional vector is $\mathbf{k}_\lambda=\bar{\boldsymbol{\alpha}}_\lambda\times\mathbf{n}$, write $\mathbf{N}_\lambda=\mathbf{k}_\lambda\times\boldsymbol{\alpha}_\lambda$ the principle normal vector of Γ_λ, then $\mathbf{N}_\lambda$ directs to the left semi-plane of the tangent line, and the principle normal vector of Γ_λ at $\mathbf{P}$ is

$$\bar{\mathbf{N}}_\lambda=(\bar{\boldsymbol{\alpha}}_\lambda\times\mathbf{n})\times\bar{\boldsymbol{\alpha}}_\lambda=(\bar{\boldsymbol{\alpha}}_\lambda\cdot\bar{\boldsymbol{\alpha}}_\lambda)\mathbf{n}-(\mathbf{n}\cdot\bar{\boldsymbol{\alpha}}_\lambda)\bar{\boldsymbol{\alpha}}_\lambda=\mathbf{n} \qquad (3.2)$$

by Lemma 3.1, we have the following lemma.

Lemma 3.2 The second basic form of a surface Σ

$$Ldu^2+2Mdudv+Ndv^2\geq 0(\leq 0)$$

if and only if for $\forall(u,v)\in \mathrm{D}$,

$$K(u,v)=\begin{bmatrix} L & M\\ M & N\end{bmatrix} \qquad (3.3)$$

is a semi-positive(semi-negative) define matrix.

Theorem 3.1

(The criterion of local convex surface)A regular surface $\sum:\mathbf{r}(u,v)$ $(\mathbf{r}(u,v)\in C^2(D),D\in R^2)$ is local convex if and only if for $\forall(u,v)\in\mathrm{D}$, $K(u,v)$ is a semi-positive(semi negative) define matrix.

Proof: For a fixed point $\mathbf{P}(u,v)$ on Σ, let $\mathbf{n}$ be the unit normal vector of Σ at $\mathbf{P}$, and the positive direction of the tangent plane π of Σ at $\mathbf{P}$ is determined by $\mathbf{n}$, then π is a directed plane. By the Talor' expansion, for all normal sections Γ_λ which passes through $\mathbf{P}$, we have

$$\mathbf{r}_\lambda(s_\lambda)-\mathbf{r}_\lambda(\bar{s}_\lambda)=\dot{\mathbf{r}}_\lambda\Delta s+\frac{1}{2!}\ddot{\mathbf{r}}_\lambda\Delta s^2+\boldsymbol{\varepsilon}\Delta s^2$$

where $\Delta s_\lambda=s_\lambda-\bar{s}_\lambda$, $\lim\limits_{\Delta s\to o}\boldsymbol{\varepsilon}=\mathbf{0}$, then the directed distance between Γ_λ and tangent plane π is

$$\delta_\lambda=[\mathbf{r}_\lambda(s_\lambda)-\mathbf{r}_\lambda(\bar{s}_\lambda)]\mathbf{n}=\frac{1}{2!}\ddot{\mathbf{r}}_\lambda\mathbf{n}\Delta s^2+\boldsymbol{\varepsilon}\mathbf{n}\Delta s^2 \qquad (3.4)$$

by differential geometry[10] theory, we have

$$\ddot{\mathbf{r}}_\lambda\mathbf{n}\Delta s^2=\ddot{\mathbf{r}}_\lambda\mathbf{n}ds^2=Ldu^2+2Mdudv+Ndv^2 \qquad (3.5)$$

which is the second basic form of a surface.

Substituting $\ddot{\mathbf{r}}_\lambda=\dot{\boldsymbol{\alpha}}_\lambda=\kappa^\lambda{}_r\mathbf{N}_\lambda$ to equation (3.5), we have

$$\kappa^\lambda{}_r\mathbf{N}_\lambda\cdot\mathbf{n}ds^2=Ldu^2+2Mdudv+Ndv^2 \qquad (3.6)$$

Write $f_\lambda(s_\lambda)=\mathbf{N}_\lambda\cdot\mathbf{n}$, observing the equation (3.2), for $\mathbf{P}$, we have $f_\lambda(\bar{s}_\lambda)=\bar{\mathbf{N}}_\lambda\mathbf{n}=\mathbf{nn}=1$.

By the properties of continued function, there exists a neighborhood $\delta_\lambda(\bar{s}_\lambda)$ of $\bar{s}_\lambda$, such that for any point in $\delta_\lambda(\bar{s}_\lambda)$, have $0<f_\lambda(s_\lambda)\leq 1$. Hence by lemma 3.2, for the neighborhood $\delta_\lambda(\bar{s}_\lambda)$, the relative curvature $\kappa^\lambda{}_r\geq 0(\leq 0)$ of Γ_λ, if and only if

$$K(u,v)=Ldu^2+2Mdudv+Ndv^2\geq 0(\leq 0)$$

is a semi-positive (semi-negative) define matrix.

In view of theorem 2.2, for the normal section segment Γ_λ at $\mathbf{P}$, when $\kappa^\lambda{}_r\geq 0(\leq 0)$, there exists a neighborhood $\delta_\lambda(\bar{s}_\lambda)$ such that in which Γ_λ lies in left closed semi plane (right closed semi-plane) of the tangent line at $\mathbf{P}$, i.e. Γ_λ lies in the upper closed semi-plane(lower closed semi-plane) of the tangent line at $\mathbf{P}$ entirely, since $\mathbf{P}$ is arbitrary, $\sum$ is a local convex surface.

Combining theorem 3.1 and lemma 3.1, we can get the following theorem.

Theorem 3.2

A regular surface $\sum:\mathbf{r}(u,v)(\mathbf{r}(u,v)\in C^2(D),D\in R^2)$ is local convex if and only if

$$\mathrm{L}\geq 0,\quad \mathrm{N}\geq 0,\ \ \mathrm{LN\text{-}M}^2\geq 0 \qquad (3.7)$$

or

$$\mathrm{L}\leq 0,\quad \mathrm{N}\leq 0,\ \ \mathrm{LN\text{-}M}^2\geq 0 \qquad (3.8)$$

Corrolary3.1 Let Σ be a regular local convex surface, then $K(u,v)$ is semi-positive (semi-negative) define if and only if for any point of Σ, the tangent vector directs to the concave side (convex side) of Σ.

Proof: For the fixed point $\mathbf{P}(u,v)$ of Σ, let $\mathbf{n}$ be the unit normal vector of Σ at $\mathbf{P}$, and the positive direction of the tangent plane π of Σ at $\mathbf{P}$ is determined by $\mathbf{n}$ (the positive direction of $\mathbf{n}$ is relative to the selection of parametric),.by (3.6) and the proof of theorem 3.2, in the neighborhood of $\mathbf{P}$, all the normal sections Γ_λ through $\mathbf{P}$ bend to the upper closed semi-space(lower semi-space) of the tangent plane π, hence, for any point on Σ, normal vector direct to the concave(convex) side of the surface.

For a closed convex surface Σ, since the unit normal vector $\mathbf{n}$ is a continuous vector function, we have

Corollary 3.2

Let Σ be a regular local closed convex surface, then $K(u,v)$ is semi-positive define (semi-negative define) if and only if for any point on Σ, the normal vector of which direct to the interior (exterior) side.

Let $z = f(x, y)$ be a binary function defined on D, and the surface which it corresponds to is Σ, and the parametric form of which is $\Sigma: \mathbf{r}(x,y) = \{x, y, f(x,y)\}$, a simple calculation shows that

$$L = \frac{f_{xx}}{\sqrt{1+f_x^2+f_y^2}}, \quad M = \frac{f_{xy}}{\sqrt{1+f_x^2+f_y^2}},$$

$$N = \frac{f_{yy}}{\sqrt{1+f_x^2+f_y^2}}$$

by theorem 3.2 and corollary 3.1, we have theorem3.3 Binary function surface Σ: $z = f(x,y)$ is a convex function if and only if

$$f_{xx} \le 0, \quad f_{yy} \le 0, \quad f_{xx}f_{yy} - f_{xy}^2 \ge 0 \tag{3.9}$$

And Σ is the lower convex function if and only if

$$f_{xx} \ge 0, \quad f_{yy} \ge 0, \quad f_{xx}f_{yy} - f_{xy}^2 \ge 0 \tag{3.10}$$

Ⅳ. THE NECESSARY CONDITION FOR THE GLOBAL CONVEX SURFACE

Firstly, we present the following lemma.

Lemma 4.1 Let $\mathbf{M}$ be any point on the regular global convex surface Σ, Γ be an arbitrary normal section through $\mathbf{M}$ on Σ, and Γ lies in π, then the tangent plane of any point on Γ can not coincide with π.

Proof; If the tangent plane of Γ at $\mathbf{P}$ is also π, by the global convexity of Σ, we can assume that Σ lies in the lower semi-space of the tangent π. Then, we can draw another normal section $\overline{\Gamma}$ at $\mathbf{M}$ through some other direction (rather than the direction of the tangent line), and the plane which $\overline{\Gamma}$ lies in is $\overline{\pi}$, such that $\overline{\pi}$ and π intersects but not coincides with each other. By theorem 2.4, in the neighborhood of $\mathbf{M}$, $\overline{\Gamma}$ lies on two sides of the normal line with normal vector $\mathbf{n}$, then there exists some point on $\overline{\Gamma}$ lies in the upper semi-space of tangent plane π of Σ, which is contradicted with the fact that Σ lies in the lower semi-space of the tangent plane π.

Theorem 4.1

(The necessary condition for global convex)A regular surface $\Sigma: \mathbf{r}(u,v)$ $(\mathbf{r}(u,v) \in C^2(D), D \in R^2)$ is global convex if and only if the normal section through any point of the surface is a global convex curve.

Proof: Let $\mathbf{M}$ be any point of the regular convex surface Σ, Γ is the normal section of Σ through $\mathbf{M}$, and Γ lies in π, then for $\mathbf{M}$, by the global convexity of Σ, Γ lies on some side of the tangent line through $\mathbf{M}$. For any point $\mathbf{P}$ on Γ, we can draw the tangent plane $\overline{\pi}$ of Σ through $\mathbf{P}$, by lemma 4.1, π intersects with $\overline{\pi}$, and denote L the intersection of the planes above. Since L lies in both π and $\overline{\pi}$, then L is the tangent line of Γ at $\mathbf{P}$. By global convexity, we can assume Σ lies in the lower semi-space of the tangent plane π, i.e. Γ lies totally on some side of the tangent line L, thus Γ is a global convex curve.

Ⅴ. THE CRITERION CONDITION FOR THE GLOBAL CLOSED CONVEX SURFACE

Lemma 5.1[10]

A simple and regular closed curve of a plane is global convex if and only if the relative curvature $\kappa_r(s)$ keeps the same sign.

Lemma 5.2

The regular closed curve Γ of a plane is global convex if and only if it is a local convex one and does not contain double points.

Proof: Sufficiency: Suppose that the regular closed curve Γ in a plane is local convex, i.e. $\kappa(s)$ keeps sign. Since for each point on the regular curve $\Gamma: \mathbf{r} = \mathbf{r}(s)$, there exists a neighborhood, such that in which the vector function $\mathbf{r}(s)$ corresponding to the curve is one to one, and there is no double points on the curve, then the vector function corresponding to the regular curve $\Gamma: \mathbf{r} = \mathbf{r}(s)$ is one to one, i.e. Γ is a simple curve. Hence, by lemma 5.1, Γ is a global convex curve, which finishes the proof of sufficiency.

Necessity: Let $\mathbf{P}$ be a double point of the global convex curve, and the corresponding two parameters are t_1 and t_2, then $\overline{\Gamma}$ corresponding to $[t_1, t_2]$ is a piece of regular closed curve of Γ. On the other hand, if there exists a point Q belongs to Γ but not to $\overline{\Gamma}$, then write O the nearest point from Q to $\overline{\Gamma}$,and the tangent line through O is T, thus $\overrightarrow{OQ} \perp T$, i.e. $\overline{OQ}$ coincides with the principal vector N at O. By theorem 2.4, there exists a point of $\overline{\Gamma}$ lies in both sides of the principal vector N, and N passes through $\overline{\Gamma}$ inevitably. Assume that $\overline{\Gamma}$ lies in the right semi-plane of the tangent line T, then Q lies in the left semi plane of the tangent line T, and there exists points of Γ on both sides of the left and right semi plane of the tangent line T, which is contradicted with the fact that Γ is global convex, hence there does not exist any point of Γ outside $\overline{\Gamma}$.

Simultaneously, if there exists Q on Γ lies in $\overline{\Gamma}$, then the tangent line through Q inevitably passes through $\overline{\Gamma}$, which is contradicted with the global convexity of Γ, thus, there does not exist any points of $\overline{\Gamma}$ on $\overline{\Gamma}$. By above facts, there does not exist double point on $\overline{\Gamma}$. This finishes the proof of the lemma.

Lemma 5.3

Let Σ be some regular local closed convex surface, $\overline{\Gamma}$ be a normal section of Σ at some point, and the plane which $\overline{\Gamma}$ lies in is π, then the tangent plane at any point of Γ can not be the plane π.

Proof: If the tangent plane of Γ at $\mathbf{P}$ is also π, since π divides Σ into two open surfaces, we can write Σ_1 and Σ_2 the upper semi-open surface and the lower one. Draw the normal section $\overline{\Gamma}$ of any direction at $\mathbf{P}$, the parts of $\overline{\Gamma}$ on Σ_1 and Σ_2 is written as $\overline{\Gamma}_1$ and $\overline{\Gamma}_2$ respectively. Then, there exists a neighborhood of $\mathbf{P}$, such that in which the curve segment corresponds to $\bar{\Gamma}_1$ and $\bar{\Gamma}_2$ in the upper and lower semi space respectively, which is contradicted with the local convexity of Σ.

Lemma 5.4[10] Let θ be the angel between the tangent vector and positive direction of x axis, then

$$\frac{d\theta(t)}{dt} = \kappa(s)\left|\frac{d\mathbf{r}}{dt}\right|$$

i.e., θ is monotone decreasing when $\kappa(s) \le 0$, and monotone increasing when $\kappa(s) \ge 0$.

We assume that $\Gamma: \mathbf{r} = \mathbf{r}(t), a \le t \le b$ is a piecewise regular plane curve, that is, there exists parametric series

$$a = a_0 < a_1 < \cdots a_{n-1} < a_n = b$$

such that the curves is regular in the each intervals $(a_j, a_{j+1})(j = 0,1,\cdots,n-1)$, and $\mathbf{r}(a_j)$ is the corner point.

For the piecewise closed plane curve, the rotation number is defined as [10]

$$n_c = \frac{1}{2\pi}\sum_{j=0}^{n-1}[\theta(a_{j+1}) - \theta(a_j)] + \frac{1}{2\pi}\sum_{j=0}^{n-1}\beta_j$$

where β_j is the exterior angle of Γ on the corner point $\mathbf{r}(a_j)$, namely, the direction angel is from the tangent direction $\dot{\mathbf{r}}(a_j-)$ to $\dot{\mathbf{r}}(a_j+)$, and $-\pi \le \beta_j \le \pi$.

Particularly, the above formula is

$$n_c = \frac{1}{2\pi}[\theta(b) - \theta(a) + \alpha_0]$$

and when the plan curve Γ contains only one corner, the tangent line rotates

$$\theta(b) - \theta(a) = 2\pi n_c - \beta_0$$

Lemma 5.5[10]

(The rotation number theorem) If the plane curve Γ is piecewise regular, simple and closed, then the rotation number is $n_c = \pm 1$,

Lemma 5.6

Let Σ be a local regular closed convex surface, and Γ be any normal section on Σ through M, then Γ is global convex.

Proof: Since Σ is closed, and so does for Γ. Let $\mathbf{P}$ be any point on Σ, and the unit normal vector of Σ at $\mathbf{P}$ is $\mathbf{n}$, the positive direction of tangent plane $\bar{\pi}$ of Σ at $\mathbf{P}$ is determined by that of $\mathbf{n}$, then $\bar{\pi}$ is a directional plane. For simplicity, we can assume that $\mathbf{n}$ directs to the interior of Σ, by corollary 3.2, the matrix $K(u,v)$ corresponding to Σ is positive semi-definite.

Assume that π which Γ lies in is a directional one, the tangent vector of Γ at each point is $\boldsymbol{\alpha}$, and the principal normal vector is $\mathbf{N}$.

For any point $\mathbf{P}$ on Γ, write $\theta(\mathbf{P})$ the angel between $\mathbf{N}$ and $\mathbf{n}$, then $0 \le \theta(\mathbf{P}) \le \pi$, Write

$$F(\mathbf{P}) = \mathbf{Nn}$$

If there exits $\mathbf{P}^*$ on Γ, such that $\theta(\mathbf{P}^*) = \pi/2$, then $F(\mathbf{P}^*) = 0$, i.e., the tangent plane $\bar{\pi}$ of Γ at $\mathbf{P}^*$ coincides with the plane π which Γ lie in, which is contradicted with lemma 5.4, thus, $\theta(\mathbf{P}) \ne \frac{\pi}{2}$.

Suppose that there exists $\bar{\mathbf{P}}$ on $\bar{\Gamma}$ such that

$$0 < F(\bar{\mathbf{P}}) = \bar{\mathbf{N}} \cdot \bar{\mathbf{n}} < 1$$

then

$$0 \le \theta(\bar{\mathbf{P}}) < \pi/2$$

Since the $\theta(\mathbf{P})$ is continuous, then $0 \le \theta(\bar{\mathbf{P}}) < \pi/2$, i.e. $0 < F(\mathbf{P}) = \mathbf{Nn} < 1$, By equation(3.6), we have

$$\kappa_r \mathbf{Nn} ds^2 = \kappa_r F(\mathbf{P}) ds^2 = Ldu^2 + 2Mdudv + Ndv^2 \ge 0 \tag{3.11}$$

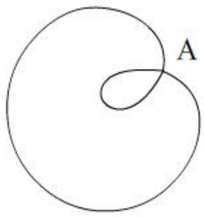

Figure 1. Closed curve with double point.

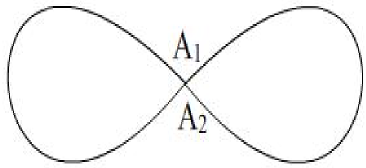

Figure 2. Closed curve with double point.

i.e., for each point on $\bar{\Gamma}$, the relative curvature $\kappa_r \ge 0$, thus $\bar{\Gamma}$ is local convex. If there exists some double point on $\bar{\Gamma}$ (Figure 1), since for normal section $\bar{\Gamma}$, there does not exist intersecting curve, $\bar{\Gamma}$ are two independent closed curves, and there is only one common point, and there is a hole in the curved surface body which Σ contains in, which is impossible for a closed curve. If there exists a double point on $\bar{\Gamma}$ (figure 2), simultaneously, there exist two coinciding points $\mathbf{A}_1, \mathbf{A}_2$ on $\bar{\Gamma}$. Suppose that $\mathbf{T}_1, \mathbf{T}_2$ is the tangent line corresponding to $\mathbf{A}_1, \mathbf{A}_2$, since Γ is local convex, there exists a neighborhood of $\mathbf{A}_1$ ($\mathbf{A}_2$), such that the locals the convex curve segment Γ_1 (Γ_2)lies in the left semi plane of $\mathbf{T}_1$ ($\mathbf{T}_2$).

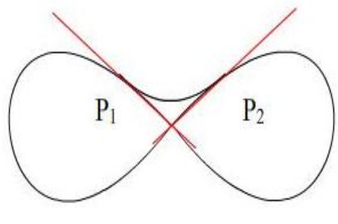

Figure 3. .Local convex Bezier curve three times the structure.

Two cases will be considered:

① For the neighboring of $\mathbf{A}_2$, if there does not exist the neighborhood of $\mathbf{A}_1$ of Γ_1 lies in the left semi space of the tangent $\mathbf{T}_2$, then there are points of Γ lie in both the left and right semi space of $\mathbf{T}_2$.

② For the neighboring of $\mathbf{A}_2$, if there exists the neighborhood of $\mathbf{A}_1$ of Γ_1 lies in the left semi space of the tangent $\mathbf{T}_2$, then $\mathbf{T}_2$ is the tangent through $\mathbf{A}_1$, and $\mathbf{T}_1, \mathbf{T}_2$

coincide each other, on the other hand, by $\kappa_r \geq 0$, $\mathbf{T}_1, \mathbf{T}_2$ possess the same direction.

For case①, we take $\mathbf{T}_1$ as x axis, choose two points $\mathbf{P}_1, \mathbf{P}_2$ on Γ_1, and the corresponding tangent line is $\mathbf{T}^*_1, \mathbf{T}^*_2$ respectively, the angel between $\mathbf{T}^*_1, \mathbf{T}_1, \mathbf{T}^*_2$ and the positive direction of x is $\theta_1, \mathbf{0}, \theta_2$ respectively, which are all monotone increasing. Figure 5.3 is a local convex cubic Bézier curve Γ^*, the start point and the end one of which is $\mathbf{P}_1, \mathbf{P}_2$ respectively, and is tangent to $\mathbf{T}^*_1, \mathbf{T}^*_2$, then for Γ, by substituting curve $P_1A_1P_2$ with Γ^*, we can get a closed curve $\bar{\Gamma}$, which is a regular, simple closed curve. By the construction of Γ^*, we know that for Γ^*, the relative curvature $\kappa^*_r \geq 0$, and so does for $\bar{\Gamma}$, by lemma 2.1, $\bar{\Gamma}$ is global convex, which is contradicted with case ①.

For case ②, by lemma 5.5, the rotation numbers of $\bar{\Gamma}$ is $n_c = 1$, i.e., as $\mathbf{T}^*_1$ rotates a period along the parametric direction of $\bar{\Gamma}$, $\mathbf{T}^*_1$ rotates 2π also. Also, as $\mathbf{T}^*_1$ rotates to $\mathbf{T}^*_2$ along Γ_1 and $P_1A_1P_2$, the rotation angel is the same, then as $\mathbf{T}^*_1$ rotates a period along the parametric direction of Γ, so does for $\mathbf{T}^*_1$.

Along the parametric direction, if we divide Γ into two regular and simple closed curves at $\mathbf{A}_1$ ($\mathbf{A}_2$), i.e., $A_1P_2A_2$ (denoted by Γ_3) and $A_2P_1A_1$ (denoted by Γ_4), then by Lemma 5.5, as the tangent line $\mathbf{T}_1$ of Γ_3 rotates to $\mathbf{T}_2$ along the parametric direction, $\mathbf{T}_1$ rotates to $\theta_1 = 2\pi - \mathbf{0}$, where $\mathbf{0}$ is the exterior of the corner $\mathbf{r}(\mathbf{A}_1)$ of Γ_1, namely, the direction angel from tangent direction $\mathbf{r}(\mathbf{A}_1-)$ to $\mathbf{r}(\mathbf{A}_1+)$, and $-\pi \leq \mathbf{0} \leq \pi$.

Similarly, as the tangent line $\mathbf{T}_2$ of Γ_4 rotates to $\mathbf{T}_1$ along the parametric direction, the rotation angel of $\mathbf{T}_1$ is $\theta_1 = 2\pi - \mathbf{0}$. Simultaneously, as the tangent line $\mathbf{T}_2$ of Γ_4 rotates to $\mathbf{T}_1$ along the parametric direction again, the rotation angel of $\mathbf{T}_1$ is also

$$\theta_1 = 2\pi - \mathbf{0}$$

Since $\bar{\Gamma}$ is local convex, by Lemma5.4, the angel between the tangent vector and positive direction of x axis θ is monotone increasing, and as $\mathbf{T}_1$ rotates to $\mathbf{T}_1$ along the parametric direction of $\bar{\Gamma}$, the total rotation angel of $\mathbf{T}_1$ is $\theta = \theta_1 + \theta_2 = 4\pi$, which is contradicted with case ②, thus, $\bar{\Gamma}$ is a global convex curve.

Theorem 5.1 The regular closed surface $\sum$ global convex if and only if $\sum$ is local convex.

Proof: The necessity can be deduced by the definition of global convex surface.

Sufficiency: Let M be a point on $\sum$, $\bar{\mathbf{n}}$ is the unit normal vector of Σ at M, and the positive direction of the tangent plane $\bar{\pi}$ of $\sum$ at M, then the tangent plane is a directional one. For simplicity, we can assume the normal vector $\mathbf{n}$ directs to the interior of Σ, by corollary 3.2, the matrix $K(u,v)$ corresponding to Σ is negative semi-definite.

Now, we prove that Σ lies in the lower half-space. Let $\bar{M}$ be any point on Σ, then for M, we can draw a plane through $\mathbf{n}$ and $\bar{M}$ which intersects with Σ, and the intersection Γ is the normal section of Σ at M, by lemma 5.4, Γ is a global closed convex curve. Assume that the plane π which Γ lies in is a directional one, for any $\mathbf{P}$ on Γ, $\boldsymbol{\alpha}$ is the corresponding tangent vector, and the tangent line is $\mathbf{T}$, then for π, we can choose unit direction vector $\mathbf{k}$ such that the principal normal vector of $\bar{\Gamma}$ at M is $\bar{\mathbf{N}} = \bar{\mathbf{n}}$. Assume that the principal normal vector of Γ at $\mathbf{P}$ is $\mathbf{N}$, the unit normal of Σ along Γ at $\mathbf{P}$ is $\mathbf{n}$, by the proof process of lemma 5.4, the angel between $\mathbf{N}$ and $\mathbf{n}$ is, $0 \leq \theta(\mathbf{P}) < \pi/2$ i.e. $0 < F(\mathbf{P}) = \mathbf{N} \cdot \mathbf{n} \leq 1$, by equation(3.6), we have

$$\kappa_r \mathbf{N}\mathbf{n} ds^2 = \kappa_r F(\mathbf{P}) ds^2 = Ldu^2 + 2Mdudv + Ndv^2 \leq 0$$

then for any point on $\bar{\Gamma}$, the relative curvature $\kappa_r \leq 0$, by theorem 2.3, for any $\mathbf{P} \in \Gamma$, $\bar{\Gamma}$ lies in the closed right semi-plane of the tangent line at $\mathbf{P}$, i.e. Γ lies in the closed lower semi-space of the tangent plane of Σ at $\mathbf{P}$, as a consequence, $\bar{M}$ lies in the closed lower semi-space of the tangent plane $\bar{\pi}$. This finishes the proof of the theorem.

VI. Conclusion

For local convexity of any point on the parametric surface, if it is defined by the local convexity of each normal section passing through this point, then the local convexity of parametric surface and the second basic quantity of the surface can be connected, and the algebra expression of the necessary and sufficient condition for the determination of local convexity surface is derived, which solves the determination method of the local convexity of parametric surface well. In this paper, the necessary and sufficient condition [7] of discriminate the convexity of binary function is a special case. Also, the discriminate condition based on local convexity surface can be applied for determining the convexity and concavity of a parametric surface easily. Also, we show that, for the local parametric closed convex surface, the local convexity is consistent with the global convexity, which means that the local parametric closed convex surface is just a global one. Since the determination of the global convexity of a parametric surface is very difficult, in this paper, only a necessary condition is presented for the determination of such surface, and the algebra method is still unknown, which is a subject which is should be further researched.

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