A Self-adaptive Optimization Removal Algorithm for Continuum Evolutionary Structural Topology

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Abstract—This paper presents a new self-adaptive optimization removal algorithm for evolutionary structural optimization (ESO) method. The elements rejection based on this algorithm is determined by their own attributes (statistical distribution of element sensitivity) rather than comparing the sensitivities to manual selective parameters in the traditional removal process. Thus, it avoids the choices and adjustments of a series of parameters, involves fewer subjective factors, and makes the removal process more adaptive. Furthermore, a self-adaptive coefficient is introduced to make the element removal vary with the iteration process and the calculation more stable. Besides, this algorithm is modified by the local secondary statistical distribution of sensitivity to depress the influence made by sensitivity concentration and improve the effectiveness of element removal. The numerous studies of structure cases based on this modified removal algorithm show that: the final results are reliable, and the optimization period is much shorter, the adaptability and robustness for different model are also well.

Keywords—removal algorithm, adaptive optimization, evolutionary structural topology

I. INTRODUCTION

Topology optimization technology which has made significant development in recent years is a branch of structure optimization, and the aim of topology optimization is to get the optimum shape and material layout under the initial design space and constrain conditions. At present, the main methods concerning continuum structure topology optimization contain homogenization method, thickness-varying method, variable density method, levelsetmethod method and evolutionary structural optimization (ESO) method [1-5]. Many topology optimization methods abide by the parametric way, namely that the shape and layout of structure are defined by a set of parameters and the objective in the premise of constrains is achieved by adjusting parameters. Yet many studies over the last thirty years have proven it is difficult and limited by

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mathematical programming method to accomplish this task [6]. However, the ESO method, proposed by Xie and Steven [5], provides a new approach based upon the concept of gradually removing redundant material to achieve an optimal design. And it overcomes numerous problems of conventional optimization technology. In addition, unlike many other FEA based topology optimization techniques, the ESO method does not need to regenerate new finite element meshes even when the final structure is quite different from the initial design. This is another major merit of the ESO method [7]. Moreover, it owns the characteristics such as the strong universality, the ease of integrating with FEM analysis software. Due to these advantages, the ESO method has been used widespreadly for structural optimization in the past few years.

The validity of topology optimization by ESO method has been also obviously and broadly demonstrated in the literatures [5-17]. But little research has been conducted in the element removal algorithm used by this method. In Section 2 are discussed the notable shortcomings in the element removal algorithm and evolutionary process of ESO method. In responding to these drawbacks, a selfadaptive optimization removal algorithm and its mathematical model for ESO method are presented in Section3. Additionally, this algorithm is modified to improve its effectiveness. Then Section4 covers a series of typical examples of 2D and 3D continuum structures with different boundary conditions to verify the proposed algorithm.

II. THE ESO METHOD AND DRAWBACKS IN ELEMENT REMOVAL ALGORITHM AND EVOLUTIONARY PROCESS

A. The ESO Method

The essence behind the ESO method is the gradual removal of inefficient material from a structure in an iterative process. And the key point of this method is to work out a characterization factor for assessing the contribution of each element to the specified behavior (response) of the structure, as well as an appropriate rejection algorithm to remove the inefficient elements subsequently. Practically, the contribution of each element, which is characterized with sensitivity numbers (α), is referred to as the sensitivity analysis and this has been given in detail in literature [7].

The evolutionary procedure for the ESO method is as follows:

Step1: A region, which is large enough to cover the area/volume of the final design, is divided into a mesh of finite elements.

Step2: Analyze the structure with the given loads and boundary conditions.

Step3: Calculate the sensitivity number for each element.

Step4: Delete a number of inefficient elements which satisfy the element removal algorithm (1):

$$\frac{\alpha_i}{\alpha_{\max}} \le RR_k \,, \tag{1}$$

where α_i is the sensitive number of the *i* th element, α_{\max} is the maximum sensitive number of all elements, RR_k is the current removal rate.

Step5: If a steady state is reached when no more elements are deleted with current removal rate, an evolutionary rate (ER) is introduced and added to the rejection ratio, i.e.

$$RR_{k+1} = RR_k + ER, \qquad (2)$$

and step 4 is repeated.

Step6: Repeat Step 2-5 until one of the constraints reaches its given limit.

The ESO method can be summarized with the flow chart of Fig.1, which shows a series of logical steps above.



Figure 1. Flow chart description of ESO method

B. Shortcomings in Element Removal Algorithm and Evolutionary Process

The evolutionary procedure and element removal algorithm of ESO method are developed above. It is easy to find that the rejection process involves a number of selections like the choices of rejection ratio and evolutionary rate. And the element deletion, determined by comparing the ratio α_i / α_{max} to the rejection rate, depends on these choices to a large extent. In practice, the element removal is influenced by some subjective factors due to the need of these manual selections. Research has demonstrated that the selections of rejection ratio and evolutionary rate make a great influence on calculation stability in evolutionary process and excellence of final results [5-9]. Thus, to avoid the bad situation, the

parameters for one model should be specified by repetitive adjustment. Likewise, the different model requires reselection of the optimization parameters. In addition, the evolutionary rate is constant in the iterative procedure. Consequently, the rejection ratio and the element removal can't be adjusted accompanying with the structure change. In view of the discussions above, the flexibilities of element rejection algorithm and removal process are not very well conclusively. These will directly lead to the difficult parameter choice and longer optimization period. Also, the unsuitable parameters setting may give rise to unstable calculating process and optimization failure.

The literature [9] has put forward one method to construct the rejection rate in polynomial form. And it is depicted as

$$RR = a_0 + a_1 * SS + a_2 * SS^2 + a_3 * SS^3 \dots$$
(3)

where a_0 , a_1 are coefficients determined by experience with ESO method, SS is the steady state number (an integer counter $0 < SS < \infty$). Yet much more coefficients require specifying in this polynomial, and this method does not solve the issues mentioned above basically. So in allusion to these drawbacks, a new, simple and effective rejection algorithm is developed in the next section.

III. SELF-ADAPTIVE OPTIMIZATION REMOVAL ALGORITHM FOR EVOLUTIONARY STRUCTURAL TOPOLOGY

A. Self-Adaptive Optimization Removal Algorithm

The generalized mathematical representation of ESO method is then:

Minimize
$$M(\omega) = \sum_{i=1}^{n} (\omega_i m_i)$$
, (4)

Subject to $C \le C^*$, (5)

$$\boldsymbol{\omega} = (\boldsymbol{\omega}_1, \boldsymbol{\omega}_2 \dots \boldsymbol{\omega}_n), \qquad (6)$$

$$\omega_i = \begin{cases} 0: if \text{ the element } i \text{ is deleted} \\ 1: if \text{ the element } i \text{ is not deleted} \end{cases}, i = 1 \cdots n, (7)$$

where *M* is the objective function of minimizing structure weight, C^* is the prescribed limit for *C*, m_i is the mass of the *i*th element, ω_i is the ESO multiplier and denotes the state of the *i*th element (ω_i equals to 0 if the element is deleted and ω_i equals to 1 if the element is not deleted), *n* is the number of elements in the structure.

In general, the ideal optimal structure is about full stress design [7]. Likewise, the full sensitivity design is hoped in final structure by ESO method. That is to say, the sensitivity number of each element in ultimate structure is approximately same with each other and closes to the allowable maximum. Also, along with the ideal evolutionary procedure, the overall sensitivity numbers of current elements should grow continuously and converge toward the same. And the convergence should be gradual so as to ensure the computational stability at each iteration.

From the above analysis, we propose one new optimization removal algorithm based on the statistics principle. And the element rejection by this algorithm depends on the self-distribution of sensitivity number. The removal algorithm is depicted as follows:

$$\overline{\alpha}_{\lambda} = \frac{1}{k} \sum_{i=1}^{k} \alpha_{i\lambda} , \qquad (8)$$

$$\overline{\sigma}_{\lambda}(\alpha) = \sqrt{\frac{1}{k} \sum_{i=1}^{k} (\alpha_{i\lambda} - \overline{\alpha}_{\lambda})^2} , \qquad (9)$$

$$\zeta = \zeta(\bar{\alpha}_{\lambda}, \bar{\alpha}_{\lambda-1}, \bar{\alpha}_{\lambda-2}, \dots), \qquad (10)$$

$$\overline{\alpha}_{\lambda} - \alpha_{\lambda} \geq \zeta \overline{\sigma}_{\lambda}(\alpha) , \qquad (11)$$

where k is the number of current elements in the structure at the λ th iteration, $\alpha_{i\lambda}$ is the sensitivity number of element *i* at the λ th iteration, $\overline{\alpha}_{\lambda}$ is sensitivity mean at the λ th iteration, $\overline{\sigma}_{\lambda}(\alpha)$ is the standard deviation of sensitivity numbers at the λ th iteration, ζ is the adaptation coefficient which is functioned by sensitivity numbers. At the λ th iteration, all the elements which satisfy the condition (11) are deleted from the model for its lower sensitivity number.

Evidently, the deletion of elements in above algorithm is determined by their own attributes (the self-distribution of element sensitivity) rather than comparing the ratio α_i / α_{max} to the manual selective parameters. Moreover, this algorithm does not require a series of choices like the rejection ratio, evolutionary rate, and involves fewer subjective factors brought by the manual choice. Hence, it avoids the troubles of the adjustment and reselection of parameters, and makes the element removal process more flexible. In addition, based on the thought of feedforward control, the self-adaptation coefficient (ζ) is introduced, which is functioned by sensitivity numbers in previous iterations to control element removal in latter iteration. And it makes the element removal vary with iteration process and the calculation more stable. In brief, this removal algorithm is much more flexible and adaptive compared to the traditional algorithm, and its principle is also simple.

B. Modification of Self-Adaptive Optimization Removal Algorithm

The variation of sensitivity numbers about case 1(given in detail in section 4) by the ESO method (see section 2) is plotted in Fig.2.



Figure 2. Variation of sensitivity numbers with the evolutionary process

And in this figure, $\sigma(\alpha)$ which can be referred to (9) is the standard deviation of sensitivity number, $\max(\alpha)$ is the maximum sensitivity number, $\overline{\alpha}$ which can be referred to (8) is the sensitivity mean. As shown in Fig.2, the mean and standard deviation of sensitivity numbers all increase along with the evolutionary procedure but the increase of the former is more significant. Consequently, we can infer that the whole sensitivity level of current elements tends to rise along with the iterations, as is expected in the analysis above (see section 3.1). In addition, it is worthwhile to note that the maximum of sensitivity number is much bigger than the mean, especially at the beginning of the revolutionary process, which indicates that the sensitivity concentration exists in the iteration process. In other words, there are some elements whose sensitivity numbers are much bigger compared with other elements. And the value of the mean ($\overline{\alpha}$) and the standard deviation ($\sigma(\alpha)$) become great due to these elements. Therefore, the element removal by the above self-adaptive algorithm is influenced by the sensitivity concentration, which is depicted in Fig.3. The elements with lower sensitivity may not be deleted completely.

For this reason, the self-adaptive removal algorithm is modified to depress the influence made by sensitivity concentration and improve the validity of element removal. The modification is based on the local secondary statistical distribution of sensitivities. And this statistics is about elements whose sensitivity numbers are less than first mean ($\overline{\alpha}_{\lambda}$). The modified self-adaptive optimization removal algorithm is presented as in (12)-(16):

$$\overline{\alpha}_{\lambda} = \frac{1}{k} \sum_{i=1}^{k} \alpha_{i\lambda} , \qquad (12)$$

$$\stackrel{=}{\alpha}_{\lambda} = \frac{1}{p} \sum_{i=1}^{p} \alpha_{i\lambda} , \left\{ \alpha_{i\lambda} \mid \alpha_{i\lambda} \leq \overline{\alpha}_{\lambda} \right\}, \qquad (13)$$

$$= \sigma_{\lambda}(\alpha) = \sqrt{\frac{1}{p} \sum_{i=1}^{p} (\alpha_{i\lambda} - \alpha_{\lambda})^{2}}, \left\{ \alpha_{i\lambda} \mid \alpha_{i\lambda} \leq \overline{\alpha}_{\lambda} \right\}, (14)$$

$$\zeta = \prod \left(\frac{\overline{\alpha}_{\lambda}}{(\overline{\alpha}_{\lambda-1} + \overline{\alpha}_{\lambda-2} + \overline{\alpha}_{\lambda-3})/3} \right), \quad (15)$$

$$\stackrel{=}{\alpha}_{\lambda} - \alpha_{i\lambda} \geq \zeta \stackrel{=}{\sigma}_{\lambda}(\alpha), \qquad (16)$$

where k is the number of current elements in the structure at the λ th iteration, $\alpha_{i\lambda}$ is the sensitivity number of element *i* at the λ th iteration, $\overline{\alpha}_{\lambda}$ is the first sensitivity mean at the λ th iteration, *p* is the number of elements whose sensitivities are less than first mean($\overline{\alpha}_{\lambda}$), $\overline{\alpha}_{\lambda}$ is the second sensitivity mean with elements whose sensitivities are less than first mean($\overline{\alpha}_{\lambda}$), likewise $\overline{\sigma}_{\lambda}(\alpha)$ is the second standard deviation at the λ th iteration, ζ is the adaptation coefficient. While the elements which satisfy (16) are removed from the model.

The procedure for the ESO method based on the selfadaptive optimization removal algorithm is as follows (step 1-3 are the same with the traditional ESO method):

Step1: A region, which is large enough to cover the area/volume of the final design, is divided into a mesh of finite elements.

Step2: Analyze the structure with the given loads and boundary conditions.

Step3: Calculate the sensitivity number for each element.

Step4: Carry out the first statistical distribution of sensitivity number ($\overline{\alpha}, \overline{\sigma}(\alpha)$), and the local secondary

statistical distribution of sensitivity (α , $\sigma(\alpha)$).

Step5: Calculate the self-adaptive coefficient (ζ).

Step6: Remove the element with lower sensitivity which satisfies (16).

Step7: Repeat Step 6 until no more elements are deleted.

Step8: If a steady state is reached when no more elements are deleted with the current statistical distribution parameters and adaptive coefficient, update these parameters in the next iteration.

Step9: Repeat Step 2-8 until one of the constraints reaches its given limit.

Fig.4 shows the flow chart of software implementation for the ESO method based on the self-adaptive optimization removal algorithm.



Figure 3. Influence of sensitivity concentration on element removal



Figure 4. Flow chart description of ESO method based on the self-adaptive optimization removal algorithm

IV. INVESTIGATION OF 2D AND 3D STRUCTURE CASES BASED ON THE SELF-ADAPTIVE REMOVAL ALGORITHM

The structural topology program based on the modified self-adaptive removal algorithm is constituted and several typical 2D and 3D continuum structure cases are given as follows. Young's modulus E = 100GPa and Poisson's ratio v = 0.3 are assumed in these cases.

Case1: the system shown in Fig.5(a) is one typical structural optimization case-the truss structure bearing single load. Supposing the connection between the components is hinged, it can be derived easily that the optimum height *H* equals to 2L. Fig.5(b) shows the rectangular design domain of the size $10m \times 24m$ (larger than $L \times 2L$). The thickness of the plate is 1mm and the shear stress applied on the structure equals to 1MPa.



Figure 5. (a) The truss structure bearing single load; (b) design domain and boundary conditions (case 1)



Figure 6. (a) Traditional removal algorithm solution; (b) adaptive removal algorithm solution (case1).

Case2: one of the Michell structures is presented in Fig.7(a). It is the minimum weight truss under the vertical load *F* acting in the middle of two fixed supports as shown in Fig.7(b). Fig.7(b) shows the rectangular design region of the size $2H \times H$, (H = 5m), is divided into 25×50 quadrilateral elements. The thickness of the plate is 0.1m. The vertical load *F* equals to 1000N.



Figure 7. (a) One typical Michell structure; (b) design domain and boundary conditions (case 2).



Figure 8. (a) Traditional removal algorithm solution; (b) adaptive removal algorithm solution (case2).

Case3: the 3D continuum structures problems with different boundary conditions are took into consideration. The design domain of case3 is a cube with simply supports at the four corners of the bottom face, and an upward vertical force is applied on the center of the top face. The structure in Fig.9(a) is the least weight continuum structure for the initial design domain of Fig.9(b). The size of design domain is $H \times H \times H$, (H = 5m) and the initial volume is then $125m^3$. The design domain is divided into $10 \times 10 \times 10$ hexahedral elements. The vertical load *F* is equal to 1000N.

Case4: the structure problem and boundary conditions are similar with case3. But the simply supports at the three corners of the bottom face have been changed into the rollers. Fig.11(b) shows the initial design domain which is also divided into $10 \times 10 \times 10$ hexahedral elements. Fig.11(a) shows the minimum weight continuum structure for this structure case.

Figs.6(a),8(a),10(a),12(a) show the topology results based on the conventional removal algorithm with desirable optimization parameter specified by repetitive adjustments and attempts. And the topology results based on the self-adaptive removal algorithm are presented in Figs.6(b),8(b),10(b),12(b).Evidently, the results of two solutions are nearly the same and both similar to the theoretical minimum weight continuum structure. This implies the good accuracy and reliability of self-adaptive optimization removal algorithm. Also, Table I presents the volume of final structure and the iteration number based on the two algorithms. A point worth emphasizing is that the iteration number based on conventional removal algorithm in Table I is only about the evolutionary procedure with desirable optimization parameter, and it does not count other iteration procedures used for parameter selection and adjustment. Though the iterative number of self-adaptive solution is some more than the conventional solution with desirable optimization parameter, the former optimization period is much shorter considering the iteration process consumed by repeated parameter adjustment in traditional method. The extensive 2D and 3D continuum structure topology cases have been carried out and there is no space here to go into detail on all these cases but give several typical cases above. In general, the optimum structural based on this algorithm is reliable and the optimization period is shorter. Also, the adaptability and robustness of this removal algorithm for different model are well. It overcomes the drawbacks in traditional removal algorithm and evolutionary process mentioned above to a large extent.



Figure 9. (a) 3D continuum structure problem; (b) design domain and boundary conditions (case 3).



Figure 10. (a) Traditional removal algorithm solution; (b) adaptive removal algorithm solution (case3).



Figure 11. (a) 3D continuum structure problem; (b) design domain and boundary conditions (case 4).



Figure 12. (a) Traditional removal algorithm solution; (b) adaptive removal algorithm solution (case4).

 TABLE I.

 COMPARISON BETWEEN CONVENTIONAL REMOVAL ALGORITHM (CRA) AND SELF-ADAPTIVE REMOVAL ALGORITHM (SARA)

	Case 1		Case 2		Case 3		Case 4	
	CRA	SARA	CRA	SARA	CRA	SARA	CRA	SARA
Volume (%) (the ratio to initial volume)	16.7%	24.1%	24.8%	19.0%	13.6%	14.4%	18.0%	18.8%
Iteration number	60	57	63	143	31	73	30	72

V. CONCLUSIONS

In allusion to the shortcomings in traditional removal algorithm and evolutionary procedure, a new selfadaptive removal algorithm for ESO method is proposed. The element rejection by this algorithm is determined by the self-distribution of element sensitivity. Moreover, a self-adaptive coefficient is introduced based on the thought of feedforward control. It makes the element deletion vary with iteration process and the calculation more stable. In addition, this algorithm is modified by the local secondary statistical distribution of element sensitivity to depress the influence brought by sensitivity concentration and improve the effectiveness of element removal. Comparing to the traditional deletion solution, the rejection of elements by this algorithm depends much on their own attributes (sensitivity distribution) and involves fewer subjective factors. The removal process is more flexible and adaptive. The numerous studies of structure cases based on this modified removal algorithm show that: the final results are reliable and the optimization period is shorter, the adaptability and robustness of this removal algorithm for different model are also well.

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