Multiple-Attribute Decision Making Based on Attribute Preference

Houxing Tang
Nanchang Institute of Technology, Nanchang, 330099, China
Email: tanghouxing@yahoo.com.cn

Abstract—**Multiple-attribute decision making in the presence of independence between attributes, the shortage of quantitative value of one attribute can be compensated with that of other attributes. But, many a times, this is not true. On another side, the quantitative value of one attribute is treated as equally important in different intervals. However, this violates the law of diminishing marginal rate of substitution and destroys the additive of attributes. So, the valuations of alternatives based on weighed aggregation operator are failure. In this paper, two styles of attribute substitution are proposed, named by, complete-substitution and incomplete-substitution. For the complete-substitution, firstly, based on the law of diminishing marginal rate of substitution, the quantitative value of attributes is divided into many intervals with different weight levels. On the same level, the substitution rate is 100% and the additive of different attributes is effective. Secondly, the multiple-attribute decision making is transformed into single-attribute decision making and the valuations of alternatives are calculated based on single attribute with different levels. Thirdly, the comprehensive value of each alternative is calculated through by accumulating different levels and all alternatives are ranked by the comprehensive value of alternative. For the incomplete substitution, with the help of shortest path method and one-vote-down system, it is transformed into complete-substitution and calculated.**

Index Terms—**Multiple-attributes decision making, Complete-incomplete substitution, Substitution effect, Attribute preference, Substitution matrix**

I. INTRODUCTION

At present, the multiple-attribute decision making is the hot topic in decision-making science and system engineering. In most literatures, the focus is put on the weight of attributes, the normalization and order of interval and linguistics, and their application in daily life. Some researchers such as Yager R.P, Xu Zeshui and Xu Jiuping have made a systematic study on that [1-5]. These fundamental researches also have yielded substantial fruit in reality but not far from the theories themselves. As we all know, the fundamental hypothesis of these multiple-attribute decision making is the independence and additive between attributes. Therefore, it can get the comprehensive value of attributes by weighting summation and rank these alternatives. Such method is often called multiple-attribute decision making in the presence of independence (IMADM).

However, there are often correlations between attributes and these attributes are not necessary to be substituted with each other. Taking the entrance examination of master graduate student in China as an example, the students not only are required to pass the total scores, but also pass the score of each subject. Well, the university is treated as the decision maker. The students are treated as alternatives. The total scores are treated as comprehensive value. The score of each subject is treated as threshold value and the subjects are treated as attributes. Now, there are two students A and B(alternatives A and B).If the A and B both pass the threshold value of each subject, different subjects can balance the scores between them, for instance, the higher score of subject English can compensate the lower score of subject Math(That means the attributes can substitute each other). So, the university (decision maker) will choose the optimal student (alternative) based on the total scores (comprehensive value). However, if the A doesn't pass the threshold value of English, then his score of Math can not compensate English and he will not be matriculated even though his total scores are higher than B. Here, it means that there be no substitution between attributes or the substitution rate is zero.

On another side, the score has different marginal-value at different interval. For example, it will be more valuable for the score of subject English from 80 to 90 than that of from 70 to 80, although they both increase 10. Because the more difficult the increasing of score, the higher the score. In another words, if attribute U_1 wants to substitute attribute U_2, then U_1 will pay more with the increasing of the substitution as the law of diminishing marginal substitution rate. It realizes that the substation rate of attributes is not necessary fixed.

All above analysis will show that the hypothesis of independence and fixed substitution rate between attributes lead to instability and inconsistent in many decision making methods, such as the rank reversal of AHP. In another words, this hypothesis destroys the

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additive of attributes. So, the valuations of alternatives based on weighed aggregation operator are failure.

At present, there are two ways to solve this problem. One way is to avoid to weighting sum when constructing decision making model such as Electre method and Fromethee method. In Electre method, for alternatives x_j and x_k, it sets two comparative parameters c_{jk}, d_{jk}. The c_{jk} is the sum of weight of some attributes and the d_{jk} is the comprehensive value based on the same attribute for x_j and x_k. So, the two comparative parameters do not relate to weighting sum between different attributes. In Fromethee method, for alternatives x_j and x_k, it does not rank the alternatives directly by the comprehensive value of attributes. It compares the alternatives under the same attribute and translates the better or worse into additive preference level [6]. The other way is the multiple-attribute decision making in the presence of relationship (RMADM). This method considers the correlation between attributes and tries to measure the importance of attributes and attributes sets properly. Christer and Fuller (1995) gave an operator and transformed the RMADM problem into a single-attribute decision making. And then, by constructing a correlation matrix, the RMADM will be transformed into IMADM [7]. Zhang Lin and Zhou Dequn(2008) gave a Literature summary about RMADM[8].

According to the law of diminishing marginal substitution rate, we think that the pure weighting sum can not reflect the value of score of attributes. In another words, the attributes can not substitute each other completely and perfectly. So, in this paper, we propose two styles of substitution, complete-substitution and incomplete-substitution. For the complete-substitution, firstly, based on the law of diminishing marginal rate of substitution, the quantitative value of attributes is divided into many intervals with different weight levels. On the same level, the substitution rate is 100% and the additive of different attributes is effective. Secondly, the multiple-attribute decision making is transformed into single-attribute decision making and the valuations of alternatives are calculated based on single attribute with different levels. Thirdly, the comprehensive value of each alternative is calculated through by accumulating different levels and all alternatives are ranked by the comprehensive value of alternative. For the incomplete substitution, with the help of shortest path method and one-vote-down system, it is transformed into complete-substitution and calculated.

II. BASIC DEFINITIONS AND HYPOTHESIZES

Let $X = \{x_1, x_2, \cdots, x_m\}$ be the set of alternatives, $U = \{u_1, u_2, \cdots, u_n\}$ be the set of attributes, $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ be the weight vector of attributes, Ω be the set of weights that are given, $\omega \in \Omega$. $A = (a_{ij})_{m\times n}$ be the decision matrix, and a_{ij} be the jth attribute value corresponding to the ith alternative. $M = \{1, 2, \cdots, m\}$, $N = \{1, 2, \cdots, n\}$, $R = (r_{ij})_{m\times n}$ be the decision matrix normalized of matrix A.

Hypothesis 1: there are different substitution rate between attributes and there are also different importance for the different interval value of the same attribute.

That is to say, the importance of different attributes is not equal. Usually, we endow these attributes with different weights. On another side, based on the law of diminishing marginal rate of substitution, in the different interval of attribute value, the importance is also different. So, in this paper, we also endow different interval with different weight level.

Definition 1: Let $T(\bullet)$ be transformation function of attributes. If $u_i = T(u_j)$, that is to say, the value of attribute u_j can be transformed into value of attribute u_i, then it is called to be *substitution* between attributes u_i and u_j.

Definition 2: If $\forall i, j \in N$, we get $u_i = T(u_j)$, then it is called to be *complete substitution* between attributes for alternatives X.

Definition 3: if $\exists i, j \in N$, we get $u_i \neq T(u_j)$, then it is called to be incomplete substitution between attributes for alternatives X.

Definition 4: The interval $[0,1]$ is divided into *k* intervals, the endpoint of each small interval is called *threshold of attribute*.

Let λ be the threshold sets and $\lambda = \{\lambda_0, \ \lambda_1, \cdots \lambda_k\}$, and $\lambda_i > \lambda_j$ $i > j$, $i, j \in K, K = \{0, 1, \cdots, k\}$. The λ_i is called the *i*th level threshold of attribute, where $\lambda_0 = 0, \lambda_k = 1$. In order to avoid the complexity because of different threshold sets of attribute, we can firstly normalize the attribute value into interval [0, 1]. So, all attributes can set the same threshold.

Definition 5: The same attribute has different weight level under different threshold, so, the weight of attributes is a matrix rather than a vector. Such matrix is called *incremental weight matrix* as follow.

$$\begin{array}{ccccc} & u_1 & u_2 & \cdots & u_n \\ \lambda_1 & \omega_{11} & \omega_{12} & \cdots & \omega_{1n} \\ \lambda_2 & \omega_{21} & \omega_{22} & \cdots & \omega_{2n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \lambda_k & \omega_{k1} & \omega_{k2} & \cdots & \omega_{kn} \end{array}$$

$$\Rightarrow \begin{array}{ccccc} & u_1 & u_2 & \cdots & u_n \\ \lambda_1 & \alpha_1\omega_1 & \alpha_1\omega_2 & \cdots & \alpha_1\omega_n \\ \lambda_2 & \alpha_2\omega_1 & \alpha_2\omega_2 & \cdots & \alpha_2\omega_n \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \lambda_k & \alpha_k\omega_1 & \alpha_k\omega_2 & \cdots & \alpha_k\omega_n \end{array}$$

Where, $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_k)$ is the weight vector of level, and $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$ is the weight vector of attributes. The $\omega_{pj} = \alpha_p \omega_j (p \in K, j \in N)$ represents the weight of attribute u_j under *p*th level of threshold. Because the resource will be scarcity with the increasing of level, the attribute will be more important, and then it is $\omega_{pj} < \omega_{qj} (p < q)$.

In another words, the weight of attribute will be higher with the increasing of level of threshold. This increasing is reflected by the weight vector of level, named by α. According to the incremental tax rate rule, the weight vector of level should increase incrementally too. So, we suppose that $\alpha_k = f(k), k \in K$ and $f^{'}(k) > 0, f^{"}(k) > 0$.

Definition 6: Let

$$s_{ij}^{pq} = S(\frac{u_i^p}{u_j^q}) = \frac{\omega_{pi}}{\omega_{qj}}$$

This function represents the substation ratio of attribute u_i under *p*th level of threshold to attribute u_j under *q*th level of threshold. And S(*) is called *substation function*.

The properties of substitution function:

(1) Transitivity

$$s_{il}^{ph} \bullet s_{lj}^{hq} = s_{ij}^{pq}$$

proof: $\because s_{il}^{ph} = \frac{\omega_{pi}}{\omega_{hl}}, s_{lj}^{hq} = \frac{\omega_{hl}}{\omega_{qj}},$

$$\therefore s_{il}^{ph} \bullet s_{lj}^{hq}. = \frac{\omega_{pi}}{\omega_{hl}} \bullet \frac{\omega_{hl}}{\omega_{qj}} = \frac{\omega_{pi}}{\omega_{qj}} = s_{ij}^{pq}$$

(2) Reciprocity

$$s_{ij}^{pq} \bullet s_{ji}^{qp} = 1$$

proof: $\because s_{ij}^{pq} = \frac{\omega_{pi}}{\omega_{qj}}, s_{ji}^{qp} = \frac{\omega_{qj}}{\omega_{pi}},$

$$\therefore s_{ij}^{pq} \bullet s_{ji}^{qp} = \frac{\omega_{pi}}{\omega_{qj}} \bullet \frac{\omega_{qj}}{\omega_{pi}} = 1$$

(3) Reflexivity

$$s_{jj}^{pp} = 1$$

Definition 7: Let

$$S_q^p = \begin{bmatrix} s_{11}^{pq} & s_{12}^{pq} & \cdots & s_{1n}^{pq} \\ s_{21}^{pq} & s_{22}^{pq} & \cdots & s_{2n}^{pq} \\ \vdots & \vdots & \cdots & \vdots \\ s_{n1}^{pq} & s_{n2}^{pq} & \cdots & s_{nn}^{pq} \end{bmatrix},$$

$p, q \in K$

Where S_q^p represents the substitution ratio of attributes under *p*th level to attributes under *q*th level. It is called the *substitution matrix* which reflects the substitution ratio between different attributes under different weight of level. There are total $k \times k$ matrices.

III. Decision Making Method and Procedures

A. Complete substitution style

For a multiple-attributes decision making problem, the weight of attributes is given. Now, Let $X = \{x_1, x_2, \cdots, x_m\}$ be the set of alternatives, $U = \{u_1, u_2, \cdots, u_n\}$ be the set of attributes, $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ be the weight vector of attributes, Ω be the set of weights that are given, $\omega \in \Omega$. $A = (a_{ij})_{m \times n}$ be the decision matrix, and a_{ij} be the jth attribute value corresponding to the ith alternative. $M = \{1, 2, \cdots, m\}$, $N = \{1, 2, \cdots, n\}$. The procedures are as follows.

Step 1: Normalizing the decision making matrix $A = (a_{ij})_{n \times m}$ into $R = (r_{ij})_{n \times m}$ according to formula (1) and (2) which focus on the cost style and profit style of attributes respectively [4].

$$r_{ij} = \frac{a_{ij}}{\max_i(a_{ij})}, i \in M, j \in N \quad (1)$$

$$r_{ij} = \frac{\min_i(a_{ij})}{a_{ij}}, i \in M, j \in N \quad (2)$$

Step 2: According to practical condition, the decision maker gives the vector of threshold $\lambda = (\lambda_0, \lambda_1, \cdots, \lambda_k)$. And then, based on the function $\alpha_k = f(k), k \in K$

with $f^{'}(k)>0, f^{"}(k)>0$, its concrete pattern is set. Therefore, the weight vector of level $\alpha=(\alpha_1,\alpha_2,\cdots,\alpha_k)$ can be calculated.

Step 3: Dividing the normalized matrix R into *k* sub-matrices based on threshold.

$$R^1=\begin{bmatrix} r_{11}^1 & r_{12}^1 & \cdots & r_{1n}^1 \\ r_{21}^1 & r_{22}^1 & \cdots & r_{2n}^1 \\ \vdots & \vdots & \cdots & \vdots \\ r_{m1}^1 & r_{m2}^1 & \cdots & r_{mn}^1 \end{bmatrix}$$

$$R^2=\begin{bmatrix} r_{11}^2 & r_{12}^2 & \cdots & r_{1n}^2 \\ r_{21}^2 & r_{22}^2 & \cdots & r_{2n}^2 \\ \vdots & \vdots & \cdots & \vdots \\ r_{m1}^2 & r_{m2}^2 & \cdots & r_{mn}^2 \end{bmatrix}, \quad \cdots, $$

$$R^k=\begin{bmatrix} r_{11}^k & r_{12}^k & \cdots & r_{1n}^k \\ r_{21}^k & r_{22}^k & \cdots & r_{2n}^k \\ \vdots & \vdots & \cdots & \vdots \\ r_{m1}^k & r_{m2}^k & \cdots & r_{mn}^k \end{bmatrix}$$

Where, $R^i, i\in k$ represents the normalized decision making matrix under *i*th level of threshold. If $\lambda^{h-1}\le r_{ij}<\lambda^h$, $h\in K$, then $r_{ij}^{h+1}=\cdots=r_{ij}^k=0$ and

$$r_{ij}^1=\lambda_1-\lambda_0, r_{ij}^2=\lambda_2-\lambda_1,\cdots,r_{ij}^{h-1}=\lambda_h-\lambda_{h-1}, r_{ij}^h=r_{ij}-\lambda_h$$

Step 4: According to definition 6 and 7, constructing the substitution matrix

$$S_q^p=\begin{bmatrix} s_{11}^{pq} & s_{12}^{pq} & \cdots & s_{1n}^{pq} \\ s_{21}^{pq} & s_{22}^{pq} & \cdots & s_{2n}^{pq} \\ \vdots & \vdots & \cdots & \vdots \\ s_{n1}^{pq} & s_{n2}^{pq} & \cdots & s_{nn}^{pq} \end{bmatrix},$$

$p,q\in K$

Step 5: multiplying the *k* sub-matrices with corresponding substitution matrix and weight vector, and then turn attributions and level into the first and the *i*th level respectively. So, we can rank the alternatives through the adjusted comprehensive value.

$$Z_p^1=R^1S_1^p\omega=\begin{bmatrix} r_{11}^1 & r_{12}^1 & \cdots & r_{1n}^1 \\ r_{21}^1 & r_{22}^1 & \cdots & r_{2n}^1 \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1}^1 & r_{m2}^1 & \cdots & r_{mn}^1 \end{bmatrix}\times \begin{bmatrix} s_{11}^{1p} & s_{12}^{1p} & \cdots & s_{1n}^{1p} \\ s_{21}^{1p} & s_{22}^{1p} & \cdots & s_{2n}^{1p} \\ \vdots & \vdots & \ddots & \vdots \\ s_{n1}^{1p} & s_{n2}^{1p} & \cdots & s_{nn}^{1p} \end{bmatrix}\times\begin{bmatrix}\omega_1\\ \omega_2\\ \vdots\\ \omega_n\end{bmatrix}$$

$$=\begin{bmatrix} z_{p1}^1 & z_{p2}^1 & \cdots & z_{pm}^1\end{bmatrix}^T$$

$$Z_p^2=R^2S_2^p\omega=\begin{bmatrix} r_{11}^2 & r_{12}^2 & \cdots & r_{1n}^2 \\ r_{21}^2 & r_{22}^2 & \cdots & r_{2n}^2 \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1}^2 & r_{m2}^2 & \cdots & r_{mn}^2 \end{bmatrix}\times \begin{bmatrix} s_{11}^{2p} & s_{12}^{2p} & \cdots & s_{1n}^{2p} \\ s_{21}^{2p} & s_{22}^{2p} & \cdots & s_{2n}^{2p} \\ \vdots & \vdots & \ddots & \vdots \\ s_{n1}^{2p} & s_{n2}^{2p} & \cdots & s_{nn}^{2p} \end{bmatrix}\times\begin{bmatrix}\omega_1\\ \omega_2\\ \vdots\\ \omega_n\end{bmatrix}$$

$$=\begin{bmatrix} z_{p1}^2 & z_{p2}^2 & \cdots & z_{pm}^2\end{bmatrix}^T$$

$$\cdots$$

$$Z_p^k=R^kS_k^p\omega=\begin{bmatrix} r_{11}^k & r_{12}^k & \cdots & r_{1n}^k \\ r_{21}^k & r_{22}^k & \cdots & r_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1}^k & r_{m2}^k & \cdots & r_{mn}^k \end{bmatrix}\times \begin{bmatrix} s_{11}^{kp} & s_{12}^{kp} & \cdots & s_{1n}^{kp} \\ s_{21}^{kp} & s_{22}^{kp} & \cdots & s_{2n}^{kp} \\ \vdots & \vdots & \ddots & \vdots \\ s_{n1}^{kp} & s_{n2}^{kp} & \cdots & s_{nn}^{kp} \end{bmatrix}\times\begin{bmatrix}\omega_1\\ \omega_2\\ \vdots\\ \omega_n\end{bmatrix}$$

$$=\begin{bmatrix} z_{p1}^k & z_{p2}^k & \cdots & z_{pm}^k\end{bmatrix}^T$$

Then, the adjusted comprehensive value of attributes is as follow

$$Z^*=\sum_{i=1}^k Z_p^i=\left[\sum_{i=1}^k z_{p1}^i,\sum_{i=1}^k z_{p2}^i,\cdots,\sum_{i=1}^k z_{pm}^i\right]^T$$

B Incomplete substitution style

Step 1: Normalizing the decision making matrix $A=(a_{ij})_{n\times m}$ into $R=(r_{ij})_{n\times m}$ according to formula (3) and (4) which focus on the cost style and profit style of attributes respectively [4].

$$r_{ij}=\frac{a_{ij}}{\max_i(a_{ij})}, i\in M, j\in N \tag{3}$$

$$r_{ij}=\frac{\min_i(a_{ij})}{a_{ij}}, i\in M, j\in N \tag{4}$$

Step 2: According to practical condition, the decision maker gives the vector of threshold $\lambda=(\lambda_0,\lambda_1,\cdots,\lambda_k)$. And then, based on the function $\alpha_k=f(k), k\in K$ with $f^{'}(k)>0, f^{"}(k)>0$, its concrete pattern is set. Therefore, the weight vector of level $\alpha=(\alpha_1,\alpha_2,\cdots,\alpha_k)$ can be calculated.

Step 3: Dividing the normalized matrix R into *k* sub-matrices based on threshold.

$$R^1=\begin{bmatrix} r_{11}^1 & r_{12}^1 & \cdots & r_{1n}^1 \\ r_{21}^1 & r_{22}^1 & \cdots & r_{2n}^1 \\ \vdots & \vdots & \cdots & \vdots \\ r_{m1}^1 & r_{m2}^1 & \cdots & r_{mn}^1 \end{bmatrix}$$

$$R^2=\begin{bmatrix} r_{11}^2 & r_{12}^2 & \cdots & r_{1n}^2 \\ r_{21}^2 & r_{22}^2 & \cdots & r_{2n}^2 \\ \vdots & \vdots & \cdots & \vdots \\ r_{m1}^2 & r_{m2}^2 & \cdots & r_{mn}^2 \end{bmatrix}$$

$$\cdots,\quad R^k=\begin{bmatrix} r_{11}^k & r_{12}^k & \cdots & r_{1n}^k \\ r_{21}^k & r_{22}^k & \cdots & r_{2n}^k \\ \vdots & \vdots & \cdots & \vdots \\ r_{m1}^k & r_{m2}^k & \cdots & r_{mn}^k \end{bmatrix}$$

Where, $R^i, i\in k$ represents the normalized decision making matrix under *i*th level of threshold. If $\lambda^{h-1}\le r_{ij}<\lambda^h$, $h\in K$, then $r_{ij}^{h+1}=\cdots=r_{ij}^k=0$ and

$$r_{ij}^1=\lambda_1-\lambda_0, r_{ij}^2=\lambda_2-\lambda_1,\cdots, r_{ij}^{h-1}=\lambda_h-\lambda_{h-1}, r_{ij}^h=r_{ij}-\lambda_h$$

Step 4: According to definition 6 and 7, constructing the substitution matrix

$$S_q^p=\begin{bmatrix} s_{11}^{pq} & s_{12}^{pq} & \cdots & s_{1n}^{pq} \\ s_{21}^{pq} & s_{22}^{pq} & \cdots & s_{2n}^{pq} \\ \vdots & \vdots & \cdots & \vdots \\ s_{n1}^{pq} & s_{n2}^{pq} & \cdots & s_{nn}^{pq} \end{bmatrix},\quad p,q\in K$$

Step 5: Transforming substation matrix S_q^p based on different substitution condition. Firstly, we need judge that the alternatives X is complete substitution or incomplete substitution by substitution matrix S_q^p. We just consider the substitution rather its quantity. So, we can select the same threshold interval, that is p=q. Therefore, the substitution matrix is $S_p^p, p\in K$. If attribute u_i and u_j is not substituted, then the substitution rate $s_{ij}^{pp}=0$ and $p\in K; i,j\in N$.

Step 6: as the above analysis, there are three cases as follows:

(1) If $\forall i,j\in N$, $s_{ij}^{pp}\neq 0$, then the attributes are substituted each other, so, we follow the complete substitution method and turn to step 4 in section 3.1.

(2) If $\forall j\in N, j\neq i$, $s_{ij}^{pp}=0$, then the attribute u_i can not be substituted by other attributes. It is absolutely incomplete substitution. So, we can use one-vote-down system to rank these alternatives X. If the score of u_j is equal in many alternatives $r_{ik}=r_{jk}; i,j\in M; k\in N$, then we can eliminate the attribute u_k.

Step 6-1: reconstructing the substitution matrix, we get

$$\hat{S}_q^p=\begin{bmatrix} s_{11}^{pq} & s_{12}^{pq} & \cdots & s_{1,k-1}^{pq} & s_{1,k+1}^{pq} & \cdots & s_{1n}^{pq} \\ s_{21}^{pq} & s_{22}^{pq} & \cdots & s_{2,k-1}^{pq} & s_{2,k+1}^{pq} & \cdots & s_{2n}^{pq} \\ \vdots & & \ddots & & & & \vdots \\ s_{k-1,1}^{pq} & s_{k-1,2}^{pq} & \cdots & s_{k-1,k-1}^{pq} & s_{k-1,k+1}^{pq} & \cdots & s_{k-1,n}^{pq} \\ s_{k+1,1}^{pq} & s_{k+1,2}^{pq} & \cdots & s_{k+1,k-1}^{pq} & s_{k+1,k+1}^{pq} & \cdots & s_{k+1,n}^{pq} \\ \vdots & & \ddots & & & & \vdots \\ s_{n1}^{pq} & s_{n2}^{pq} & \cdots & s_{n,k-1}^{pq} & s_{n,k+1}^{pq} & \cdots & s_{nn}^{pq} \end{bmatrix}$$

Step 6-2: As the complete substitution style, multiplying the k sub-matrices with corresponding substitution matrix and weight vector, and then turn attributions and level into the first and the *i*th level respectively. So, we can rank the alternatives through the adjusted comprehensive value.

$$Z_p^h=R^h\hat{S}_h^p\omega=$$

$$\begin{bmatrix} r_{11}^h & r_{12}^h & \cdots & r_{1,k-1}^h & r_{1,k+1}^h & \cdots & r_{1n}^h \\ r_{21}^h & r_{22}^h & \cdots & r_{2,k-1}^h & r_{2,k+1}^h & \cdots & r_{2n}^h \\ \vdots & & \ddots & & & & \vdots \\ r_{k-1,1}^h & r_{k-1,2}^h & \cdots & r_{k-1,k-1}^h & r_{k-1,k+1}^h & \cdots & r_{k-1,n}^h \\ r_{k+1,1}^h & r_{k+1,2}^h & \cdots & r_{k+1,k-1}^h & r_{k+1,k+1}^h & \cdots & r_{k+1,n}^n \\ \vdots & & \ddots & & & & \ddots \\ r_{n1}^h & r_{n2}^h & \cdots & r_{n,k-1}^h & r_{n,k+1}^h & \cdots & r_{nn}^h \end{bmatrix}\times$$

$$\begin{bmatrix} s_{11}^{ph} & s_{12}^{ph} & \cdots & s_{1,k-1}^{ph} & s_{1,k+1}^{ph} & \cdots & s_{1n}^{ph} \\ s_{21}^{ph} & s_{22}^{ph} & \cdots & s_{2,k-1}^{ph} & s_{2,k+1}^{ph} & \cdots & s_{2n}^{ph} \\ \vdots & & \ddots & & & & \vdots \\ s_{k-1,1}^{ph} & s_{k-1,2}^{ph} & \cdots & s_{k-1,k-1}^{ph} & s_{k-1,k+1}^{ph} & \cdots & s_{k-1,n}^{ph} \\ s_{k+1,1}^{ph} & s_{k+1,2}^{ph} & \cdots & s_{k+1,k-1}^{ph} & s_{k+1,k+1}^{ph} & \cdots & s_{k+1,n}^{ph} \\ \vdots & & \ddots & & & & \vdots \\ s_{n1}^{ph} & s_{n2}^{ph} & \cdots & s_{n,k-1}^{ph} & s_{n,k+1}^{ph} & \cdots & s_{nn}^{ph} \end{bmatrix}\times\begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_{k-1} \\ \omega_{k+1} \\ \vdots \\ \omega_n \end{bmatrix}$$

$$=\begin{bmatrix} z_{p1}^h & z_{p2}^h & \cdots & z_{pm}^h \end{bmatrix}^T$$

Then, the adjusted comprehensive value of attributes is as follow

$$Z^*=\sum_{i=1}^h Z_p^i=\left[\sum_{i=1}^h z_{p1}^i, \sum_{i=1}^h z_{p2}^i, \cdots, \sum_{i=1}^h z_{pm}^i\right]^T$$

(3) If $\exists j \in N, j \neq i$, $s_{ij}^{pp} = 0$, then it is relative incomplete substation. For example, $s_{13}^{pp} = 0$, and $s_{12}^{pp} \neq 0, s_{23}^{pp} \neq 0$, according to the transitivity of substation function, we get $u_1 = T(u_3)$. From the viewpoint of grapy theory, the attributes are treated as nodes, the attributes u_1 and u_3 are connected with chain rather than arc.

Step 6-1': Now, we will transform this incomplete substitution into complete substitution. The first, we change the substitution matrix S_p^p. The elements 0 all remain unchanged which means the two codes of attributes are connected indirectly. The non-zero elements all changed into 1 which means the two codes of attributes are connected directly.

Step 6-2': We use the *Floyd shortest path method* to get the shortest path such as $u_i \to u_{k_1} \to u_{k_2} \to \cdots \to u_{k_l} \to u_j$. Therefore, the substitution rate between u_i and u_j is calculated with $\hat{s}_{ij}^{pp} = s_{ik_1}^{pp} \times s_{k_1 k_2}^{pp} \times \cdots \times s_{k_l j}^{pp}$. Well, we get a new matrix with complete substation as follow.

$$\hat{S}_q^p = \begin{bmatrix} \hat{s}_{11}^{pq} & \hat{s}_{12}^{pq} & \cdots & \hat{s}_{1n}^{pq} \\ \hat{s}_{21}^{pq} & \hat{s}_{22}^{pq} & \cdots & \hat{s}_{2n}^{pq} \\ \vdots & \vdots & \cdots & \vdots \\ \hat{s}_{n1}^{pq} & \hat{s}_{n2}^{pq} & \cdots & \hat{s}_{nn}^{pq} \end{bmatrix}, \quad p, q \in K$$

Step 6-3':

$$Z_p^h = R^h \hat{S}_h^p \omega = \begin{bmatrix} r_{11}^h & r_{12}^h & \cdots & r_{1n}^h \\ r_{21}^h & r_{22}^h & \cdots & r_{2n}^h \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1}^h & r_{m2}^h & \cdots & r_{mn}^h \end{bmatrix} \times \begin{bmatrix} \hat{s}_{11}^{ph} & \hat{s}_{12}^{ph} & \cdots & \hat{s}_{1n}^{ph} \\ \hat{s}_{21}^{ph} & \hat{s}_{22}^{ph} & \cdots & \hat{s}_{2n}^{ph} \\ \vdots & \vdots & \cdots & \vdots \\ \hat{s}_{n1}^{ph} & \hat{s}_{n2}^{ph} & \cdots & \hat{s}_{nn}^{ph} \end{bmatrix} \times \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{bmatrix} = \begin{bmatrix} z_{p1}^h & z_{p2}^h & \cdots & z_{pm}^h \end{bmatrix}^T$$

Then, the adjusted comprehensive value of attributes is as follow

$$Z^* = \sum_{i=1}^{h} Z_p^i = \left[\sum_{i=1}^{h} z_{p1}^i, \sum_{i=1}^{h} z_{p2}^i, \cdots, \sum_{i=1}^{h} z_{pm}^i \right]^T$$

IV. AN EXAMPLE

Now, we consider a decision making problem. Suppose that X={x1,x2,x3,x4}, U={u1,u2,u3,u4}, w={0.25,0.25,0.25,0.25}, k=10,f(k)=k^2+k. the Table I is a normalized decision matrix.

TABLE I TEST DATA

	u_1	u_2	u_3	u_4
x_1	1.0000	0.7000	0.5000	0.6000
x_2	0.9500	0.7500	0.5000	0.6000
x_3	0.8000	0.9000	0.8000	0.7000
x_4	0.7000	1.0000	1.0000	1.0000

According to our method, we get the comprehensive value of all alternatives as follow:

z_1=39.5000,z_2=38.5500,z_3= 48.9000;z_4=74.4000

The rank of alternatives is $x_4 \succ x_3 \succ x_1 \succ x_2$

According to the weighting summation method, the comprehensive value of all alternatives as follow:

z_1=0.7,z_2=0.7,z_3=0.8,z_4=0.925

The rank of alternatives is $x_4 \succ x_3 \succ x_2 \sim x_1$

To know intuitively, the best alternative is x_4, we pay no attention on it. Contrary to, we compare the alternatives x_1 and x_2 based on above two decision-making methods. According to the weighting summation, the value of one attribute is compensated completely and perfectly by the value of other attributes. So, the x_1 equals to x_2. However, which do the decision-makers will choose when they face the problem in practice? If the decision-makers have preference for the value of attributes, for example, it is much more difficult to increase the scores from 90 to 100 than that from 50 to 60. In another words, the higher the value of attribute, the better. So, we choose the alternative x_1 rather than x_2. Fortunately, our method can just reflect this preference.

Addition to, we calculate another two tests for k=5 and k=3 respectively. The results are as follows:

k=5,f(k)=k^2+k,

z1=13.4000,z2=13.1500,z3=16.5000,z4=24.0000;

k=4 f(k)=k^2+k,

z1=9.8000,z2= 9.6000,z3=12.2000,z4=17.2000.

The results show that the value of k does not affect the rank of alternatives. On another side, it shows that our method has a good stability. From the perspective of calculation, the complexity of our method is higher than weighting summation. But it could be neglected with the help of computer programming.

V. CONCLUSION

For multiple-attribute decision making in the presence of independence (IMADM), it requires the independence and fixed substitution rate between attributes. However, we give some examples to show that it is not the case for all conditions and leads to failure for decision making. Based on the law of diminishing marginal substitution rate, we propose the hypothesis which the substation between attributes is incomplete. So, the value of attribute is divided into different levels and a substitution matrix is constructed in order to make the different attributes additive. This method will solve the problem that the weighed aggregation operator is failure to decision making. By an example, we test the weighting summation and our method. This results shows that our method can reflect the attribute preference of decision-makers. The weighting summation method could not do it. Addition to, our method has better consistency and stability. Although its complexity is much higher, it is neglected with the help of computer programming.

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Houxing Tang Anhui Province, China. Birthdate: July, 1982, is Management Science and Engineering Ph.D., graduated from Dept. Information and Management Jiangxi University of Finance and Economics. And research interests on decision-making analysis and knowledge management.

He is a lecturer of Dept. Business Administration, Nanchang Institute of Technology.