Match Planar Curve Based on Affine Invariant

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Abstract— **In order to recognize the curve with the same feature points but the curvature between the two adjacent feature points is different; a novel method for matching curve is proposed under the affine transformation. First feature points are matched both on model contour and object contour, then the affine region is divided for obtaining a series of sub-region's centroids by using feature points and the whole contour's centroid. Because area ratio of triangles, which constituted by centroid of the whole contour region and sub-region's centroids, is invariant under affine transformation. We introduce area ratio of triangles to construct affine invariant, and normalized it too. And then a new recognition vector matrix is defined, and a different measure function is defined by us too. Finally the different measure value between model curve and object curve is calculated, which can be employed to judge whether the model and object are matched. Theory analysis and experiment results show the method is feasible, which can not only match the different curve but also distinguish the object with the similar shape according to the different measure value.**

Index Terms—**Affine invariant, match, planar curve, feature points**

I. Introduction

The planar curve match has been found wide applications in computer vision and pattern recognition, such as object recognition and image retrieval based on contour curve match. Contour is a typical feature of object, it can provide more direct information for distinguishing all kind of object, so the method to represent object contour is important in object recognition, and the representation method affects the recognition effect directionally. Zhang[1] construct invariant based on area ratio, and describe object using texture invariant, but construct texture invariant must have better image segmentation effect. Chen[2] proposed the conception named extended centroid, and used the triangle area ratio constructing invariant, the triangle is constituted by the extended centroid. The method is suit for affine transform, but when three extended centroid points are approximately co-linearity, the area of the triangle become close to zero. When this area is taken as denominator, the invariant becomes infinity. So this invariant is much sensitive to the errors of image data or the errors of feature points extracting, which affect the differentiation of invariant greatly. Carlsson[3] presented a method to recognize planar object, they partitioned curve based on the invariant points under perspective transformation, and approximated each sub-curve with ellipse arc, from the two ellipse arc two group invariants (the conics trace) can be calculated, in terms of the conics trace two curves can be recognized. Xu[4] also proposed a method to recognize planar curve, they used invariant points to partition curves, described each sub-curve use conics. However, in their methods there is the same problem that sub-curve approximated by conics is very complex and low accuracy. M. Cui[5] presented a curve matching framework for planar open curves under similarity transform based on a new scale invariant signature. But this method is only suit for similarity transform. Zhang[6] presented a global to Local partial matching algorithm for planar curve, however the method is only suit for rigid transform. We[7] proposed the method for matching planar curve under affine transform, unfortunately, the algorithm would be invalid when the number of feature points conserved under transform is less than three.

This paper proposed a new method for matching planar curve under affine transform.

II. Affine geometry Characteristics

Imaging process of camera is decrypted by perspective projection. But it can also be decrypted approximately by affine projection when the distance between camera and object is much farther than the size of object. The general equation of a 2D affine transform can be written as

$$\begin{cases} x' = a_{11}x + a_{12}y + a_{13} \\ y' = a_{21}x + a_{22}y + a_{23} \end{cases} \quad \text{where} \quad \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0 ,$$

$A(x, y)$ is a point in scene, $A'(x', y')$ is a projection of A, or $A(x, y)$ and $A'(x', y')$ are two images of a point in scene.

Affine transform is a parallel projection chain. The characteristics preserved under affine transform have been named affine invariant. The affine invariants are as follows:

Characteristic 1: The project of line is still line.

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Characteristic 2: The projects of parallel lines are still parallel lines.

Characteristic 3: The two triangle area ratio keeps invariant under affine transform.

The proof the characteristic 3 is as follow:

Assume under affine coordinate system, no-collinear three points denoted by $P_i(x_i, y_i)$, $(i = 1,2,3)$, the triangle area is as follow:

$$S_{\Delta p_1 p_2 p_3} = \frac{1}{2} abs(\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}) \quad (1)$$

where, abs( ) denoted absolute value. The affine transform corresponding points are $P_i'(x_i, y_i)$, $(i = 1,2,3)$ respectively, the area constituted by $P_i'(x_i, y_i)$, $(i = 1,2,3)$ is denoted by $S'_{\Delta P_1'P_2'P_3'}$. It is noted from formula (1), $S'_{\Delta P_1'P_2'P_3'}$ must meet the following relation, such as:

$$S'_{\Delta p_1' p_2' p_3'} = \frac{1}{2} abs(\begin{vmatrix} x_1' & y_1' & 1 \\ x_2' & y_2' & 1 \\ x_3' & y_3' & 1 \end{vmatrix})$$

$$= \frac{1}{2} abs(\begin{vmatrix} a_{11}x_1 + a_{12}y_1 + a_{13} & a_{21}x_1 + a_{22}y_1 + a_{23} & 1 \\ a_{11}x_2 + a_{12}y_2 + a_{13} & a_{21}x_2 + a_{22}y_2 + a_{23} & 1 \\ a_{11}x_3 + a_{12}y_3 + a_{13} & a_{21}x_3 + a_{22}y_3 + a_{23} & 1 \end{vmatrix}) \quad (2)$$

We can obtain from the above formulas:

$$S'_{\Delta p_1' p_2' p_3'} = |\det(A)| S_{\Delta p_1 p_2 p_3} \quad (3)$$

Where A is affine transform matrix, it is observed:

$$\frac{S'_{\Delta p_1' p_2' p_3'}}{S'_{\Delta p_1' p_2' p_3'}} = |\det(A)| \quad (4)$$

It is noted that area ratio is equal to $|\det(\boldsymbol{A})|$, it is irrespective to the vertices coordinate of triangle, and is only relational to the affine transformation matrix. This is desirable for us.

Characteristics 3 has the following deduction:

Deduction 1: If the arbitrary region of one planar is denoted by D, the affine project of the region is denoted by D', the two region areas ratio is denoted by: $\frac{S'_D}{S_D} = |\det(A)|$.

Affine invariant can be constructed by using characteristics 3 and deduction 1.

III. CONSTRUCT AFFINE INVARIANT

It is noted that, the key of constructing affine invariant is choosing the region. Triangle region is a more simple region, and the calculation its area is more simple too. Suppose the two triangles of image are V_1 and V_2 respectively and the corresponding areas are denoted by $S(V_1)$ and $S(V_2)$, the two triangle areas ratio is denoted by f, then

$$f = \frac{S(V_1)}{S(V_2)} \quad (5)$$

Suppose the two triangle areas ratio after affine transform is denoted by $f' = \frac{S(V_1')}{S(V_2')}$, From the deduction 1 we can know, $S(V_1') = |\det(A)| S(V_1)$, $S(V_2') = |\det(A)| S(V_2)$

then:

$$f' = \frac{S(V_1')}{S(V_2')} = \frac{|\det(A)| S(V_1)}{|\det(A)| S(V_2)} = \frac{S(V_1)}{S(V_2)} = f \quad (6)$$

From formula(6), it is noted that the two triangle areas ratio keeps invariant under affine transform, that is $f = f'$. So we can construct affine invariant by using two triangle areas ratio. But there are two main problems to be overcome:

First, it is observed that the triangle areas ratios are invariant under affine transform, but if they are used to describe object directionally, which will result in the problems, such as: When three extended centroid points are approximately co-linearity, the area of the triangle become close to zero. When this area is taken as denominator, the invariant becomes infinity. So this invariant I is much sensitive to the errors of image data or the errors of feature points extracting, which affect the differentiation of invariant greatly.

In order to overcome this problem, we employ the function $I' = \frac{e^I - 1}{e^I + 1}$.

I is an affine invariant, so I' is also an affine invariant. Noted that I' increased as I increased monotony. When I is approximately to infinity, I' become close to parameter 1, when I is close to negative infinity, I' become close to parameter -1, i, e, $I \in (-\infty, +\infty)$, $I' \in (-1, +1)$.

So we introduced the new invariant to describe curve shape.

The second is how to construct two or more different triangles, and find the corresponding triangles in image after transformation.

We introduce the feature points of contour curve and the cetroid to construct triangle. Following we prove the invariant property of cetroid first, and then discuss how to construct affine invariant using affine geometry property.

A. The affine invariant of cetroid

Let image is denoted by $I(x)$, affine transform is denoted by $T : R^2 \to R^2$, which make $T(x) = x'$, $x, x' \in R^2$, affine transform is denoted using matrix: $x' = T(x) = Ax + b$, where A is a nonsingular matrix, the image after affine transform is $I'(x)$, which satisfy $I'(x) = I(x') = I(Ax + b)$, the cetroid of image $I(x)$ can be denoted by:

$$C(x) = \frac{\int_{R^2} xI(x)dx}{\int_{R^2} I(x)dx} \quad (7)$$

Then the cetroid of image $I'(x)$ must satisfy the equation $C'(x) = A \cdot C(x) + b$.

Proof:

The cetroid of image $I'(x)$ is doted by:

$$C'(x) = \frac{\int_{R^2} xI'(x)dx}{\int_{R^2} I'(x)dx} \quad (8)$$

because $x' = T(x) = Ax + b$, then:

$$C'(x) = \frac{\int_{R^2} (Ax+b)I(x)|J|dx}{\int_{R^2} I(x)|J|dx} \quad (9)$$

Where $|J|$ is the Jacobin determinant and $|J| = |A|$, matrix A is a nonsingular matrix, then

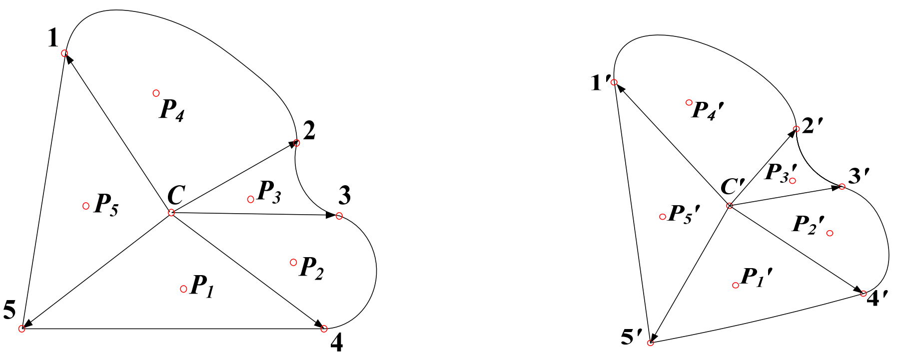
$$C'(x) = \frac{\int_{R^2} (Ax+b)I(x)|J|dx}{\int_{R^2} I(x)|J|dx} = A \cdot \frac{\int_{R^2} xI(x)dx}{\int_{R^2} I(x)dx} + b = A \cdot C(x) + b \quad (10)$$

B. Affine region divide

The contour curve's feature point is invariant under perspective transform, obviously, it is invariant point under affine transform, and the cetroid is invariant under affine transform too. So cetroid point and contour curve's feature points can be considered as inherent points of image and can be employed to describe contour. Feature points and cetroid can determine one line. From affine geometry characteristics, we can know, this line is the inherent line of image too, its inherent property is, the line and its affine project have the same corresponding relation, that is, this corresponding relation is same as that of the image.

As shown in Fig.1(a) , point C is the cetroid of contour curve, and point 1, 2⋯, 5 is the contour curve's feature point respectively, the line determined by cetroid point C and feature point divided contour curve into some sub-region, suppose point $P_1, P_2 \ldots P_5$ is the cetroid of each sub-region, as shown in Fig. 1(b), point C' is the cetroid of the contour curve after affine transform, point $P_1', P_2' \ldots P_5'$ is the corresponding sub-region's cetroid. Because point C 、1, 2⋯, 5and point C' 、1', 2'⋯5' are all inherent point of the whole region, From affine geometry characteristics, it is known, the line determined by cetroid and feature point divided the whole contour curve into some sub-region, and each sub-region has the same affine corresponding relation.

The sub-region's cetroid and the whole contour's cetroid provide the triangle vertex set for constructing affine invariant. We will describe planar curve more using this vertex set.



(a)before affine transform (b)after affine transform

Figure1. Affine region divide

C. Construct affine invariant

It is noted that the affine invariant can be constructed by using two or more triangle areas ratio in image. Based on cetroid and feature point, the whole contour can be divided into some sub-regions, and a series of sub-region's cetroids can be obtained too. The triangle area ratio keep invariant under affine transform, and the triangle vertices set is constituted by the whole curve's cetroid and the sub-region's cetroid. So they are employed to construct affine invariant. Finally affine invariant recognition vector is set up to match planar curve.

The approach for constructing affine invariant is as follows:

Step 1: Extracting centroid and feature points of contour curve, the line through the feature points and centroid divided the contour curve into some sub-regions.

Step 2: extracting the sub-region's centroids P_n and order them as the sequence as the feature points[7], it is denoted as $P = \{P_1, P_2 \ldots P_n\}$.

Step 3: take the centroid point C as triangle's inherent vertex, and the two adjacency sub-region's centroids constitute triangle, that is, $\Delta CP_1P_2, \Delta CP_2P_3, \ldots, \Delta CP_{n-1}P_{n-2}, \Delta CP_nP_1$, we can obtain some triangles, denoted as $V = \{V_1, V_2 \ldots, V_n\}$, where n is the number of triangles.

Step 4: calculate each triangle's area, obtain the area order, denoted as $\{S(V_1), S(V_2) \ldots, S(V_n)\}$.

Step 5: construct the affine invariant, as follows:

$$f_1 = \frac{S(V_1)}{S(V_2)}, f_2 = \frac{S(V_2)}{S(V_3)}, \ldots, f_{n-1} = \frac{S(V_{n-1})}{S(V_n)}, f_n = \frac{S(V_n)}{S(V_1)} \tag{11}$$

Step 6: normalize the above affine invariant as formula(8), we can obtain the satisfied affine invariant.

$$I_1 = \frac{e^{f_1} - 1}{e^{f_1} + 1}, \quad I_2 = \frac{e^{f_2} - 1}{e^{f2} + 1}, \ldots, \ I_n = \frac{e^{f_n} - 1}{e^{f_n} + 1} \tag{12}$$

In order to using this affine invariant to match planar curve, we should define recognition vector and difference measure function.

Definition 1: Recognition Vector: Suppose the extracted affine invariant is I_i, $i = 1,2 \ldots n$, define recognition vector F as:

$$F = \{I_1, I_2, \ldots, I_n\} \tag{13}$$

For the contour curve after affine transform, extract its affine invariant, and calculate the corresponding recognition vector F', in theory, there is $F = F'$, however, because of various factor effect, which make the affine invariant has lesser difference before affine transform and after affine transform. In this paper, a difference measure function is defined to recognize planar curve.

Definition 2: Difference Measure Function: Suppose the affine recognition vectors of two original match curves are denoted by F_1 and F_2 respectively, the definition of their difference is as follows

$$D(F_1, F_2) = \frac{\sum_{i=1}^{n} | I_i^{F_1} - I_i^{F_2} |}{n^2} \tag{14}$$

Where n is the dimension of recognition vector, the model corresponding to the least difference value is match with the object to be recognized

IV. THE MATCH ALGORITHM OF CURVE

The matching algorithm is as follows:

Step1: Extract the feature points of object's contour.

Step2: calculate NRLCTI[7] code of object contour, and compare NRLCTI code of model with object, if they are not equal, do not match feature points and exclude the model; if they are equal, turn to step 3.

Step3: calculate the centroid point C, and divide the whole contour curve into some sub-region by using inherent line which is determined by the centroid point C and feature points.

Step4: by using the inherent line, calculate the sub-region's centroid $P_i, (i = 1,2,\ldots,n)$, where n is the number of sub-region.

Step5: order sub-region's centroid P_i accord to the sequence of feature points.

Step6: construct a series of triangles based on point C and points $P_i, (i = 1,2,\ldots,n)$, and calculate their areas respectively, then construct affine invariant based on formula(11)and formula(12).

Step7: calculate the model contour curve's recognition vector and that of the object contour curve based on definition 1.

Step8: calculate the difference measure between model contour and object contour based on definition 2. If the different measure is less than the given threshold, the object curve and model are matched.

V. EXPERIMENT RESULT AND DISCUSSION

Experiment 1:

Input: see Fig.2 (a)-(e). Where Fig.2 (b)-(f) are model contour, Fig.2 (a) is the object contour to be recognized.

Output: recognize which model is corresponding to Fig.2 (a) and match them.

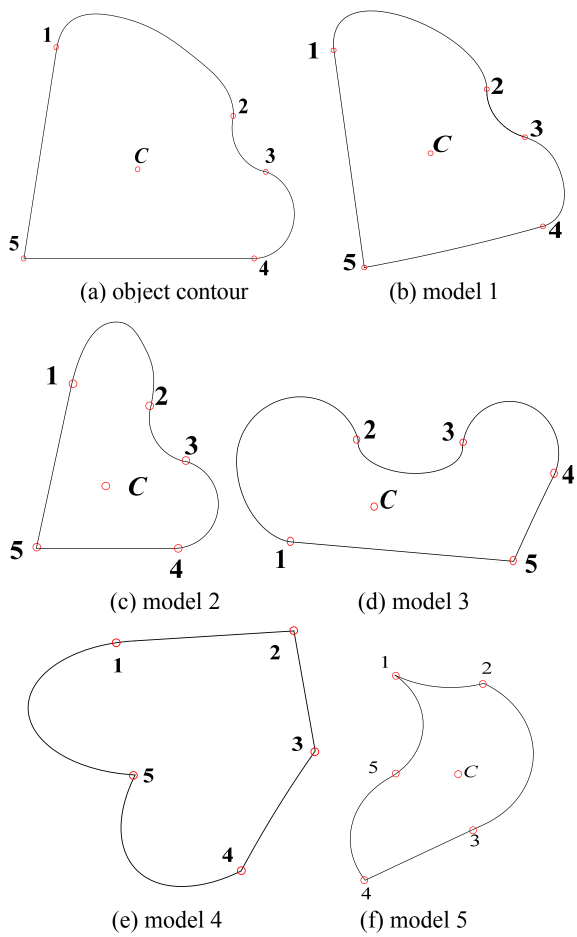


Figure 2. experiment 1

First, we extract the feature points both on model contour and object contour, calculate their NRLCTI code. The code of object contour is 112(*)1, the codes of Fig.2(b), Fig.2(c) and Fig.2(d) are 112(*)1 too. But the code of Fig.2(e) and Fig.2(f) are 41, 2111(*)respectively, so we exclude the model 4 and model 5.

Then we use recognition vector to match the other three models contours with object contour. The experiment data are as shown inTab.1 and Tab.2 .

TABLE1 the eigenvector of different contour

	Recognition vector F				
	f_1	f_2	f_3	f_4	f_5
(a) object contour	0.6648	0.3252	0.4412	0.6495	0.3539
(b) model 1	0.6811	0.3347	0.4151	0.5889	0.3467
(c) model 2	0.5803	0.3815	0.3781	0.8054	0.2588
(d) model 3	0.9070	0.5868	0.4481	0.2487	0.2459

TABLE 2 the eigenvector different measure of model and object

	(b) model1	(c) model 2	(d) model 3
(a) object contour	0.0048	0.2182	0.2408

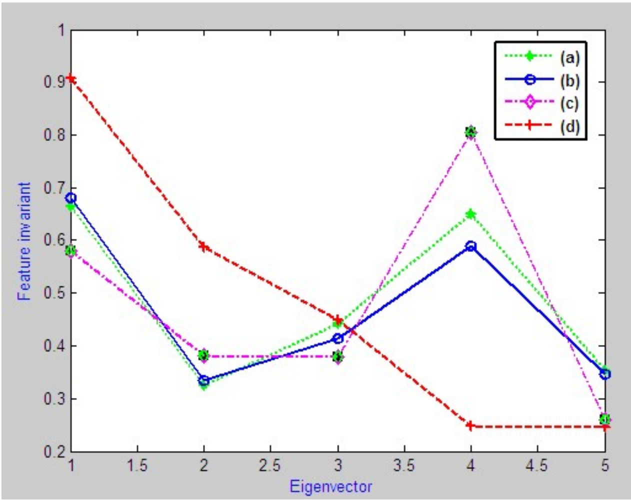


Figure 3. Eigenvector curves of model and object

It is noted from Tab.2, the difference measure between object and model 1 is the least, and that is, the object is matched with model 1.

In order to show the relation between the feature vectors, we express the feature vector as curve, as shown in Fig.3, where different curve is labeled with different line type and symbol, and each curve represent one model or object to be recognized. It is observed that, the object and model 1 is the most similar, and the object and model 3 is the least similar. So the proposed method for curve describing and matching is feasible, which can not only match the different curve but also distinguish the object with the similar shape according to the different measure value.

Experiment 2: In this example, we choose two tool models from toolbox randomly, as shown in Fig. 4(a) and Fig.4 (b). The image to be recognized is obtained by shooting two models from three different viewpoints, the camera is above wrench, and the distance from camera to wrench is much larger 10 times to the height of wrench. The image is as shown in Fig.5.

（a）Sp1

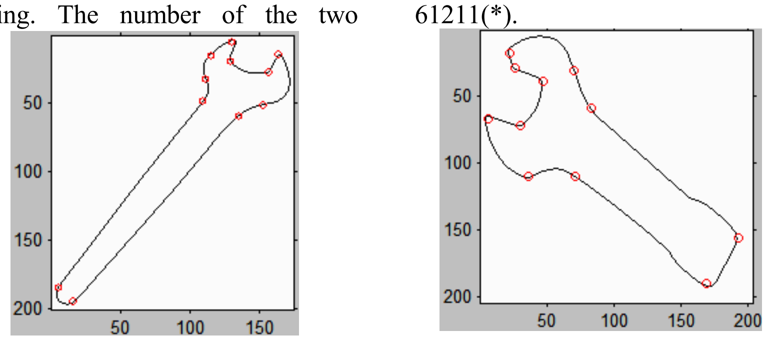
(b)Sp2

Figure 4. tool model

Figure 5. The images shot from three different views

Test samples contain two different wrenches, labeled as"Sp1(2)". The image is labeled as "Spx_a(b)(c)", where "x" is wrench kind number .

Fig.6 and Fig.7 is the result of edge extracting and feature points extracting. The number of the two wrenches feature points both are eleven, but their NRLCTI codes are not same, the NRLCTI codes of "Sp1" and the corresponding images are 41(*)41(*)1, and those of "Sp2" and the corresponding images are 61211(*).



(a)Sp1 (b)Sp2

Figure 6. Model contour and their feature points

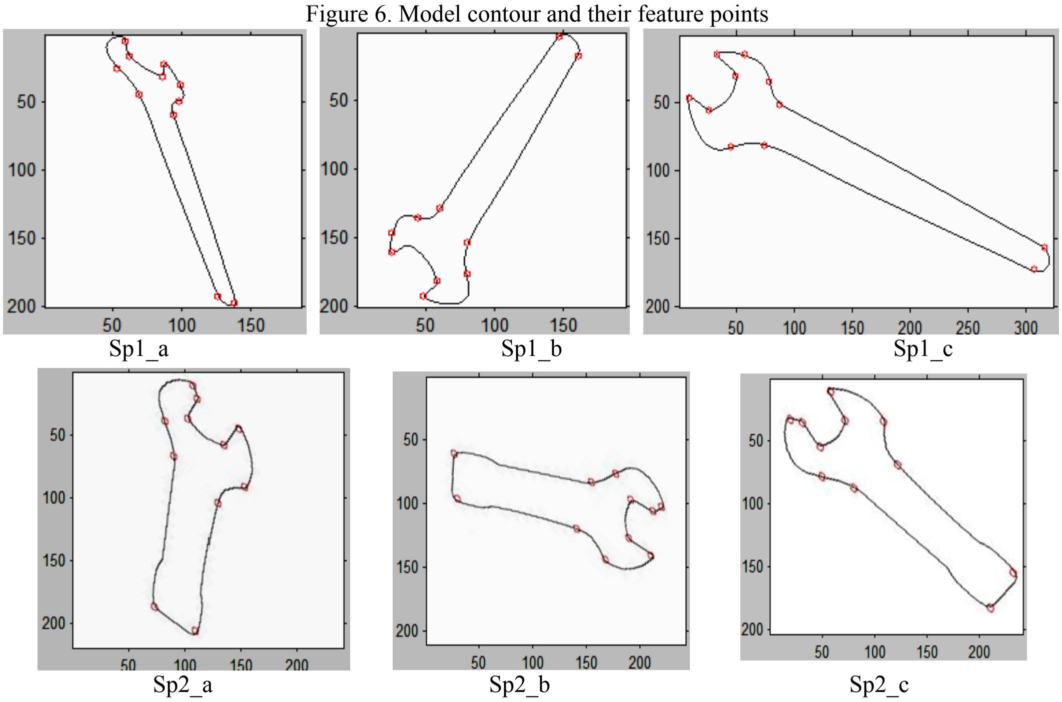
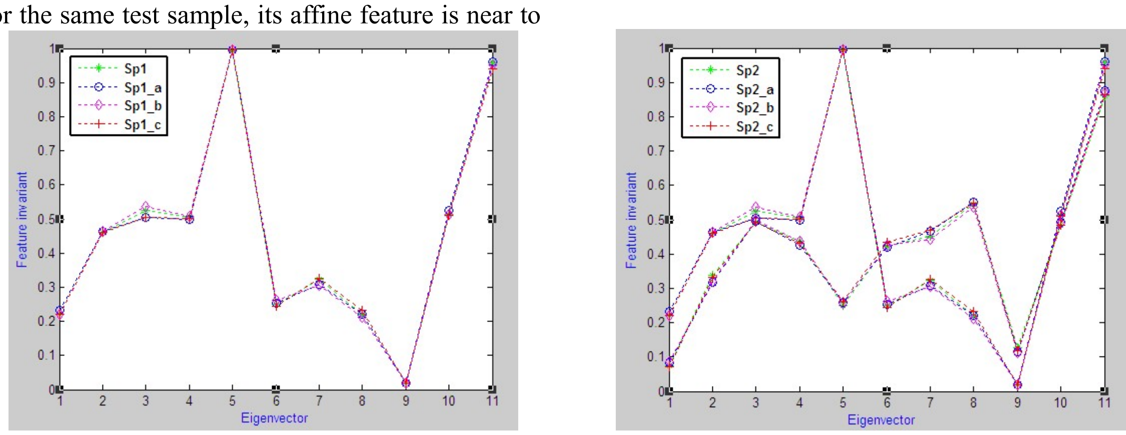


Figure 7. The contours and their feature points of image shot from three different views

Based on the method proposed in this paper, first we match the feature points both on model and object to be recognized, and then extract their invariant. the affine invariant is as shown in Tab.3. The different measure is as shown in Tab.4. It is noted, for the same test sample, the model and images shot from different views, the different measure values of their affine feature are all less than 0.02. In order to show the relation between the feature vectors, we express the feature vector as curve, as shown in Fig.8, where different curve is labeled with different line type and symbol, and each curve represent one test sample or its corresponding image. It is observed that, for the same test sample, its affine feature is near to that of its corresponding, and for different test sample, the different measure becomes larger. It is shown that the affine invariant constructed by us is desirable, which can distinguish object better.

The eigenvector error between the same model and its test samples comes from the following facts: 1. the image's digitalization error; 2. the extracting error of feature points; 3. the rounding error of centriod, and so on. We can see from the figures that the eigenvectors are quite different on the shape of curve between two different tools, so as to obtain a better result of classification.



(a)Sp1 (b)Sp2 Feature invariant

Figure 7. Eigenvector curves of two image shot from different views

TABLE 3 The eigenvector of real model and that of their affine transform samples

Recognition vector ***F***	Sp1				Sp2			
	Sp1	Sp1_a	Sp1_b	Sp1_c	Sp2	Sp2_a	Sp2_b	Sp2_c
I_1	0.2194	0.2319	0.2194	0.2205	0.0752	0.0819	0.0859	0.0723
I_2	0.4603	0.4648	0.4639	0.4615	0.3370	0.3178	0.3214	0.3326
I_3	0.5236	0.5050	0.5359	0.5044	0.4931	0.4952	0.4997	0.4932
I_4	0.5031	0.4980	0.5068	0.5021	0.4368	0.4246	0.4378	0.4327
I_5	0.9937	0.9962	0.9957	0.9948	0.2497	0.2574	0.2565	0.2613
I_6	0.2504	0.2526	0.2605	0.2433	0.4233	0.4194	0.4256	0.4347
I_7	0.3244	0.3072	0.3049	0.3272	0.4520	0.4665	0.4423	0.4687
I_8	0.2220	0.2197	0.2128	0.2328	0.5466	0.5514	0.5369	0.5490
I_9	0.0195	0.0198	0.0190	0.0201	0.1267	0.1146	0.1118	0.1180
I_{10}	0.5119	0.5244	0.5129	0.5096	0.4834	0.4919	0.4892	0.4835
I_{11}	0.9577	0.9612	0.9514	0.9412	0.8625	0.8764	0.8693	0.8653

TABLE 4 The eigenvector different measure of two tool models and that of their affine transform samples

	Sp1_a	Sp1_b	Sp1_c			Sp2_a	Sp2_b	Sp2_c
Sp1	0.0016	0.0010	0.0025		Sp2	0.0017	0.0018	0.0016

VI. CONCLUSION

This paper proposed a novel algorithm for matching planar curve based on affine invariant. We used the method from rough to fine to match the planar curve. Firstly, we initially matched the feature points of the object contour and that of model contour based NRLCIT code. And based on the matching of the feature points, affine region is divided for extracting the whole invariant points effectively by using the whole contour's centroid and feature points. According to the properties of affine geometry, we constructed the affine invariant feature and normalized it. Then we solved the problem that the constructed affine feature is sensitive to the image's digitalization error and the error of feature points extracting when the three vertices of triangular are approximately co-linearity. A new recognition vector matrix and a different measure function are defined based on the invariant feature. Comparing different measure and pre-threshold, we can recognize and match planar curve.

We used synthesize and real images to perform simulation experiment. The experiment results show that the proposed method for curve describing and matching is feasible, which can not only match the different curve but also distinguish the object with the similar shape according to the different measure value.

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REFERENCES

[1] Jianguo Zhang, Tieniu Tan. Affine Invariant Texture Analysis Based on Structural properties. Proceedings of the Fifth Asian Conference on Computer Vision(ACCV 2002), 2002, 216-221.
[2] Chen Tao, Li Yi, Jiang Yongmei and Tang Tao. Affine invariant feature extraction based on affine geometry [J]. Chinese Journal .of Image and Graphics, 2007, 12(9): 1633-1641.
[3] S. Carlsson, Projectively invariant decomposition and recognition of planar shapes [J], International Journal of Computer Vision, 1996, 17(2): 193-209.
[4] Xu Zhengwei, Wu Chengke. Perspective invariant decompose of spatial planar curve [J], Chinese Computer Journal, 1996, 19(8): 631-635.
[5] M. Cui , J. Femiani, J. Hu, P. Wonka, A. Razdan, Curve matching for open 2D curves, Pattern Recognition Letters, Pattern Recognition Letters 30 (2009) 1-10.
[6] Zhang Chunying, Pan Rongjiang. A global to local partial matching algorithm for planar curves [J]. Chinese Journal .of Computer Aided Design and Computer Graphics, Chinese, 2008, 20(7): 894-899.
[7] Zhang Guimei, Gao Mantun, Zeng Jiexian Recognizing Planar Curve based on NRLCTI and Match Sub-curve[J], Chinese Journal of Image and Graphics, 2009, 14 (2): 353-358.

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