Competitive Multi-attribute On-line Auction

Lili Ding
School of Economics and Management
Shandong University of Science and Technology, Qingdao, China
Email: dinglili0220@gmail.com

Xinmin Liu
Shandong University of Science and Technology, Qingdao, China
Email: liu xin-min@163.com

Wanglin Kang
Shandong University of Science and Technology, Qingdao, China
Email: wlkang@163.com

***Abstract*—The emergence of auction that supports bids characterized by several attributes is one of the most recent evolutions within auction theory. This paper introduces the multi-attribute and transaction cost into the on-line auction, proposed by Lavi and Nisan. Different from the traditional on-line auction, the buyer makes decision only based on the bids, when each bidder comes one by one. We propose a novel multi-attribute on-line auction mechanism for a limited bidder base. The mechanism consists of two parts: first, evaluation of bidders' utility with an idea of a scoring rule designed by a buyer and second, the winner determination model. Furthermore, we present the price and utility algorithm, together with its competitive analysis, which is designed for the buyer to decide the trading price and quantities. Finally, we also perform numerical examples showing our results.**

***Index Terms*—multi-attribute, on-line auction, transaction cost, competitive ratio**

I. INTRODUCTION

In recent years, the Internet has become a fertile ground for the development of auctions. A lot of web sites like eBay and Yahoo have also demonstrated the great potentialities of these mechanisms applied to e-commerce [1]. The nature of the Internet has also modified the classic auctions: new rules and behaviors have emerged. Considerable interest in such mechanisms that do not know the preferences of bidders who will arrive in the future and it must decide whether to satisfy each bid as the requests are received in real time. This is different from the traditional assumption that all participants are willing to wait for some amount of time before performing any trade. Such auction is called on-line auction, which has quickly gained prominence in computer science [2].

Goldberg et al. first attempt to analyze the digital goods auction in an unlimited supply by using the competitive analysis. They consider that the use of a randomized sampling auction instead of a deterministic one can achieve competitiveness for the simple and multi-priced auctions in [3]. Lavi and Nisan [4] extend the work to analyze the on-line limited supply auctions. They point out that the deterministic one has constant competitive ratio when they use the offline Vickrey auction as their benchmark for comparing their on-line auction. Ding et al.[5,6] take the seller risk into the on-line limited supply auction and design the optimal risk auction strategies. For the limited supply auction, they describe such cases that k=1 and k=n. Awerbuch et al. and Hajiaghayi et al. [7,8] describe an online truth telling mechanism base on the adversary model or the offline Vickrey model. Gallien [9] considers such auction that the identical goods are sold to self-interested, time sensitive buyers with unit demand. About the competitive auction, Blum and Hartline [10] simply define the notion of an attribute one for modeling the problem of selling items to buyers who are not a priori indistinguishable. However, in some case, e.g., more valuable and less standardized items, the buyer might be interested in purchasing a higher quality product at higher price rather than a lower quality product at lower price. To our perspective, auction mechanisms that support bids characterized by several attributes seem to be one of the last frontiers to the generalization of auctions. Thus we consider the multi-attribute of goods into the on-line auction.

In numerous successful websites, such as Free Markets, buyers and sellers involved in auctions always incorporate multiple transaction attributes, e.g. quality, delivery time, warranty and payment conditions to bid or sell. Such mechanisms are usually referred to as multi-attribute, multiple issue or multi-dimensional auctions. These auctions are mostly deployed in monopsony situations such as governmental or corporate procurement where a buyer takes on auctions about goods or services he wants to buy. Such mechanisms are usually referred to as multi-attribute, multiple issue or multi dimensional auctions[11]. Laffont and Tirole[12] describe some issues involve in procurement negotiations. They set up a tender to evaluating the bids about the cost in such a process. They also mention the need that auction theory must be generalized to multi-attribute bidding. For example, in procurement auctions the bidders often provide very different kinds of goods and services in their bids. Kameshwarana et al. [13] consider there are large

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numbers of the suppliers in the market consisting of large companies as well as a large number of small- and medium-sized enterprises. On the contrary, the buyers are a small number of large food retailers who aggregate the demand and distribute it to the end consumer. How to buy the goods? The buyers have their own preferences for product quality, price, terms of payment and delivery and they are looking for the offer that best satisfies these preferences. The overall utility of a deal for the buyer not only contains the price of the item but a combination of the different attributes. The works of Che [14] and their extension by Branco [15] are among the first considering multi-dimensional auctions for government procurement when technical and quality factors extremely matter. Che studies design competition in government procurement. He establishes a two-dimensional auctions model where firms bid on price and quality. He focuses on an optimal mechanism in cases where bids are evaluated by a scoring rule designed by the procurer. Branco considers such conditions that the bidding firms' costs are correlated, but the initial information of firms is independent. Thus an optimal auction mechanism for the case can be derived. This is somehow equivalent to the common value approach in classic auction theory [16]. Most extended researches are base on their works.

This paper is at the intersection of on-line auction between the transaction cost and multi-attribute spheres. Our purpose is to underline some basics of what we call multi-attribute on-line auctions. Different from the traditional analysis, our attention will be more precisely paid on the management of incomparability between bids, which is a cornerstone of this multi-attribute auction. Firstly, we extend the on-line auction mechanism by a novel application of a scoring function to incorporate the buyer's preferences across attributes. Secondly, we describe the price and utility algorithm with transaction cost, which helps the buyer decide when and how many goods he should buy. Finally, we also perform numerical examples showing our results from two cases. One is the on-line auction has no transaction cost, the other is has the transaction cost.

II. THE ON-LINE AUCTION

A. Problem Statement

We extend the traditional auctions to the multi-attribute framework. We consider the case of a reverse auction: m sellers are competing to sell identical goods to a unique buyer in the time $[0,T]$. Let M denote the unique buyer with initial wealth E. Let $A=\{a_1,a_2,...,a_n\}$ be a set of n attributes defined by the buyer. Without loss of generality, we assume that all the attributes are public to the sellers. Let $S=\{s_1,s_2,...,s_m\}$ be the set of sellers involved in the auction and $B=\{b_1,b_2,...,b_m\}$ be the set of their corresponding bid; b_i is the bid associated to seller s_i. The bids b_i proposed by each seller s_i are constrained by his attribute valuation, i.e., $b_i(a_{ij}, w_{ij})$. a_{ij} denotes the value of the jth attribute of the ith bid and w_{ij} denotes the weight of the jth attribute of the ith bid. Each seller s_i has a valuation $v_i(a_{ij}, w_{ij})$ for the auctioned good, which is known only to himself and stays fixed throughout the auction. His aim is to maximize his utility u_i, i.e., $u_i = v_i - b_i$. In the remainder of this paper, we consider only truthful auctions. As bidding v_i is a dominant strategy for bidder s_i in a truthful auction, we assume that $v_i = b_i$. Thus, before we obtain some results, some definitions must be declared.

Definition 2.1 The on-line auction (OA) is defined as follows:

(1) m sellers arrive one by one in a sequence during the time of $[0,T]$. For simplicity, we assume that each b_i (a_{ij}, w_{ij}) is a real number in the interval $[\underline{b},\overline{b}]$, where $\underline{b}$ is also the seller's reservation price.

(2) A buyer has initial wealth E and is given a job to determine whether to buy the goods from bidder s_i and if so, at what price p_i and quantity q_i according to the assessment of a_{ij} and w_{ij} before open the next bid. We also assume that each p_i (a_{ij}, w_{ij}) is a real number in the interval $[\underline{p},\overline{p}]$, where $\overline{p}$ is the buyer's reservation price. For example, the reservation price is the highest preference price for the buyers.

(3) There is the transaction cost c during the each trading period.

(4) The game ends when the last bidder announce his bid during the time of $[0,T]$ and the buyer maximize his profit.

B. Supply Curve for the On-line Auctions

Intuitively, if a price a bidder is offered in the on-line auction is independent of the bidder's bid value, the auction is truthful. These famous consequences can be found in the Merson' works. In [4], Lavi and Nisan point out that the on-line auction (many buyers and one seller) is competitive based on the non-decreasing supply curve without considering the multi-attribute. In this paper we assume that the on-line auction is truthful, when the auctions are based on demand curves. Although the buyer makes decision according to multi-attributes of the goods, the final decisions variables are only price and quantity. Thus the trading price is independent of the multi-attribute of the goods. For example, in Fig.1 the buyer has non-increasing demand curve p_i (a_{ij}, w_{ij}). The character of such curve is slope down and the price and quantity relationship is negative. Therefore, the quantity q_i sold to the bidder s_i is the quantity q that maximizes the sum of the utility of the buyers

$$\int_i^q [b_i(q) - p_i(q)]dq \tag{1}$$

The price paid by the buyer is

$$\int_i^{q_i} p_i(q)dq \tag{2}$$

For the bidders, they have the non-decreasing marginal valuation. Considering the marginal valuation functions may increase significantly the complexity of presenting the valuation function to the buyer. This problem can be solved by using the following modification. The buyer can present his demand curve to all bidders before the auction, instead of receiving valuation functions. More simply put, each bid is a price and quantity relationship on the demand curve. This simply relationship can be found in the auction mechanism from the economics scope.

Definition 2.2 An on-line auction is truthful if and only if it is based on the no-increasing demand curves.

To see that the on-line auction based on demand function is incentive compatible. This means that the total price of the buyer is determined uniquely by the total quantity sold to him and by previous bids.

C. Competitive Analysis

For an on-line auction, we focus on defining optimal strategies for the buyer from an online algorithmic standpoint. The motivation is that many buyers at auction websites do not use good pricing models and thus lose revenue as a result. In [2], they present the definition of competitive analysis, which uses the competitive ratio for the evaluation of online algorithm. An online algorithm is said to be r-competitive($r \geq 1$), if, given any instance of the problem denoted by σ, the cost of the solution given by the online algorithm is no more than r multiplied by that of an optimal offline algorithm. Namely, an auction mechanism is r-competitive with respect to offline auction mechanism, such as Vickrey auction. Further, for any bidding sequence B, the online algorithm of the online auction can generate a revenue value that is at least $1/r$ of the revenue produced by the offline auction using the same input B. The competitive analysis framework is the following equation:

$$R_A(B) \geq R_{OPT}(B)/r \tag{3}$$

where $R_{OPT}(B)$ denotes the profit of optimal offline auction (the buyer has known all bids before the auction). An online algorithm is said to be best-possible if there does not exist another online algorithm with a strictly smaller competitive ratio.

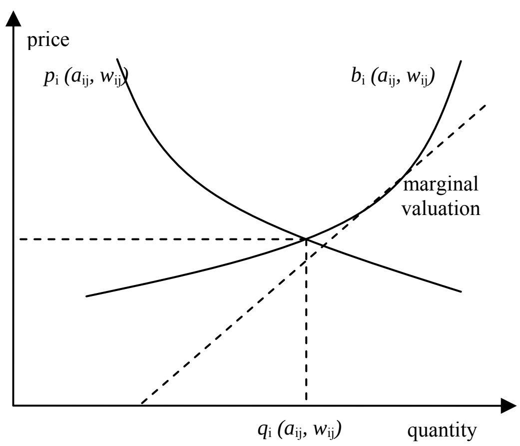

Figure 1. Demand curve for the on-line auction

III. MULTI-ATTRIBUTE BASED ON-LINE AUCTION

Conversion of the non-price attributes to a monetary equivalent for evaluation of the bids is a difficult task. The buyer may use the valuation function as the preferential independence over the non-price attributes. We consider a valuation function similar to the one defined by Branco [15] as follows:

$$S_i(A) = \sum_{j=1}^{n} w_{ij} s_i(a_{ij}) \tag{4}$$

where $S_i(A)$ is the overall utility for a bid b_i and $s_i(a_{ij})$ is a scoring function. We define the scoring function as the valuation on the jth attribute of the goods. The weight w_{ij} of the jth attribute satisfies

$$\sum_{j=1}^{n} w_{ij} = 1,\ 0 \leq w_{ij} \leq 1 \tag{5}$$

Using the above utility function, the buyer declares a scoring rule for the evaluation of the bids and computes scores for each bidder s_i. Note that the scoring function consists of bid-price and the non-price attributes. Here, we normalize the different attributes to achieve the final scores l_{ij} according to the rule of [15]. The buyer makes decision based on the computing scores. There are six attribute types as follows. For the profit type, the score l_{ij} is

$$l_{ij} = a_{ij} \big/ \max a_{ij} \tag{6}$$

For the cost type, we compute the score according to the reverse of the profit type.

$$l_{ij} = \min a_{ij} \big/ a_{ij} \tag{7}$$

For the fixed type, the buyer has the fixed value standard. Thus, the score is

$$l_{ij} = \begin{cases} \dfrac{a_{ij}}{F_j} & a_{ij} \in [\min a_{ij}, F_j] \\ 1 + \dfrac{F_j}{\max a_{ij}} - \dfrac{a_{ij}}{\max a_{ij}} & a_{ij} \in [F_j, \max a_{ij}] \end{cases} \tag{8}$$

where F_j is the fixed attributes value. The interval type is similar to the fixed type. For this type the buyer has the valuation scope. The score can be normalized as the following equation:

$$l_{ij} = \begin{cases} \dfrac{a_{ij}}{\underline{F}_j} & a_{ij} \in [\min a_{ij}, \underline{F}_j] \\ 1 & a_{ij} \in [\underline{F}_j, \overline{F}_j] \\ 1 + \dfrac{\overline{F}_j}{\max a_{ij}} - \dfrac{a_{ij}}{\max a_{ij}} & a_{ij} \in [\overline{F}_j, \max a_{ij}] \end{cases} \tag{9}$$

where $\underline{F}_j$ is the lower bound of the interval attributes value and $\overline{F}_j$ is the upper bound. More complexity, there are deviation types and the score value is

$$l_{ij} = \begin{cases} 0 & a_{ij} \neq F_j \\ \dfrac{|F_j - a_{ij}|}{\max[F_j - \min a_{ij}, \max a_{ij} - F_j]} & a_{ij} = F_j \end{cases} \tag{10}$$

The deviation interval type can be computed by the following equation:

$$l_{ij} = \begin{cases} 0 & a_{ij} \notin [\overline{A}_j, \underline{A}_j] \\ \dfrac{\max[\overline{A}_j - a_{ij}, a_{ij} - \underline{A}_j]}{\max[\underline{A}_j - \min a_{ij}, \max a_{ij} - \overline{A}_j]} & a_{ij} \in [\overline{A}_j, \underline{A}_j] \end{cases} \tag{11}$$

Different from the above, $\underline{A}_j$ is the lower bound of the deviation interval attributes value and $\overline{A}_j$ is the upper bound. For the identical goods, we can achieve such relation that

$$w_{1j} = w_{2j} = \cdots = w_{nj} = \overline{w}_j \tag{12}$$

We assume that the utility function is a linear function. Thus,

$$S_i(A) = \sum_{j=1}^{n} l_{ij}\overline{w}_j,\ i = 1, 2, \cdots, m \tag{13}$$

For the on-line auction, the buyer hopes to make a tradeoff between price and non-price attributes, i.e. getting the best possible attributes with the lowest possible price. It is clear that offline auction can achieve this balance. However, the buyer has no information about the future bid price and its additive attributes. For the buyer, there is a reservation price $\overline{p}$ taken as the basic preferable price. According to $\overline{p}$ and the other attributes, the buyer also can compute his reservation utility $\overline{S}$. When a bidder presents his b_i, the buyer has to make his decision based on the total utility of the all attributes. If the bidder's bid satisfies the following rule, then the buyer may decide to buy the goods. However, how does the buyer decide to trade goods at what price and quantity will be discussed in the next section.

$$S_{i+1}(A) \geq S_i(A) \geq \overline{S},\ i = 1, 2, \cdots, m \tag{14}$$

IV. Online Algorithms Based Multi-Attribute On-Line Auction

For the multi-attribute on-line auction, we focus on defining optimal algorithm for the buyer from an online algorithmic standpoint. We use the definition of competitive analysis, which uses the competitive ratio for the evaluation of online algorithm. Different from [4] and [5], we introduce the transaction cost into the on-line auction. In reality, the third providers would collect fees, i.e. commission, from the buyers or sellers during the internet trading. Thus, our studies focus on whether the transaction cost considered into the original model will influence the buyers' behavior. For simplicity, we only consider the fixed transaction cost c instead of the scaled transaction cost. For this problem, we propose the price and utility algorithm as follows.

Step 1: Initialize m, T, E, c, $\underline{p}$, $\overline{p}$, $\overline{S}$.

Step 2: The bidder s_i, $i = 1, 2, \cdots, m$, comes and presents his bid b_i , together with other attributes, such as quality and transportation.

Step 3: The buyer computes the total utility of the bidder s_i, according to the definition of (13). When the bid reaches a new low, i.e., $\underline{b} \leq b_i < \min_{i>j} b_j$ and the utility hits a new high, i.e., $S_i \geq \max_{i>j} S_j \geq \underline{S}$, the bidder s_i succeed. If not, then go to step 3.

Step 4: The buyer decides the trading price p_i based on his non-increasing demand curve.

Step 5: The buyer decides the trading quantity q_i for the bidder s_i. Let r^* be the optimal competitive ratio that can be attained by the price and utility algorithm. Here, q_i satisfies

$$q_i = \frac{(p_{i-1} - p_i)\overline{p}E}{(\overline{p} - p_i)p_i r^*} + \frac{p_i \overline{p} cE}{(\overline{p} - p_i)p_i} \tag{15}$$

Step 6: When the time T will end or there are no bidders, the buyer has to buy goods at the market price, i.e., the reserved price $\overline{p}$. If he does not buy any goods, it means that he has not any profit.

A. Computing q_i

We can compute the cost E_i when a new bid b_i is given together with the other attributes by considering the threat of the bid up to $\overline{p}$. Based on the definition of competitive analysis, the buyer has to avoid the worst case, i.e., next bid is the worse. For the buyer, if he knows all the bid sequences and attributes sequences, then he can decide the best choice and achieve the maximum return. According to step 3, if $b_i = \min_{j \in m} b_j$ and $S_i = \max_{j \in m} S_j$, then the offline return R_{off} is

$$R_{off} = \frac{E - c}{p_i} \tag{16}$$

According to the scoring rules, the on-line buyer, who has no information about future bid sequences, accepts the bid or rejects the bid. If the price and utility algorithm wish to achieve a competitive ratio of r , then it must ensure that the on-line algorithm return R_{on} is

$$R_{on} = \sum_{j=1}^{i} \frac{E_i - c}{p_j} + \frac{E - c - \sum E_i}{\overline{p}} \tag{17}$$

The optimality of the price and utility algorithm can be measured by the competitive ratio, which has been described in (2). Thus,

$$R_{on} = \frac{R_{off}}{r} \tag{18}$$

For the online buyer, he wants to design the online algorithm A achieve the minimum competitive ratio r^* , i.e., $r^* = \min(r)$. Notice that (18) can be divided into the following equation

$$\frac{E-c}{p_i} = r^* \cdot \left(\sum_{j=1}^{i} \frac{E_i - c}{p_j} + \frac{E - c - \sum E_i}{\overline{p}}\right) \qquad (19)$$

We also divide the (19) into the following equation:

$$\begin{aligned} &\sum_{j=1}^{i}(E_i - c)/p_j + (E - c - \sum E_i)/\overline{p} \\ &= \sum_{j=1}^{i-1}(E_i - c)/p_j + (E_i - c)/p_j \\ &\quad + (E - c - \sum E_{i-1})/\overline{p} - (E_i - c)/\overline{p} \\ &= \sum_{j=1}^{i-1}(E_i - c)/p_j + + (E - c - \sum E_{i-1})/\overline{p} \end{aligned} \qquad (20)$$

Substitute this relation into the (18) and get E_i. Therefore, we can achieve the quantity q_i as follows.

$$q_i = \frac{E_i}{p_i} = \frac{(p_{i-1} - p_i)\overline{p}E}{(\overline{p} - p_i)p_{i-1}p_i r^*} + \frac{\overline{p}c}{\overline{p} - p_i} \qquad (21)$$

B. Competitive Ratio

We first show the competitive ratio of *RUS* in [5], not considering the transaction cost. The buyer buys goods in such sequence $\overline{B}$ that the current bid is lower than the earlier bid. However, in the multi-attribute on-line auction the rule can delete some bids that are less than the earlier bids for their lower utility. Therefore, the buyer trades in the special sequence of bids B' when the bids satisfy the rule one. It is clear that $B' \in \overline{B}$. It is obtained the competitive ratio r_1 based on the competitive ratio in [4]. Thus,

$$r_1 = \ln \frac{(\overline{p}/\underline{p}) - 1}{r_1 - 1} \qquad (22)$$

Furthermore, we present the competitive ratio of the price and utility algorithm. Since the transaction cost exists, the buyer has to buy more goods to offset the opportunity loss at each trading. As the same as the analysis in [5], we use the scoring function of (4) to help the buyer to make decision whether to trade, or not. When the buyer decides the price and quantity at each bid b_i, he has to face the following condition.

$$E_1 + E_2 + \cdots + E_m = E,\ 0 \le E_i \le E \qquad (23)$$

Substitute (21) into (23). We can achieve the deviation result of $\frac{\partial r}{\partial p_i}$. For the buyer, the less competitive ratio is, the better performance of the online algorithm is. Namely, he wants to find the special input sequence of bids, only if in which the buyer also can get a better performance based on his online algorithm. Thus, we should compute the result of $\frac{\partial^2 r}{\partial p_i^2}$ and achieve that $\frac{\partial^2 r}{\partial p_i^2} < 0$. From the above the analysis, such special input sequences can be designed that the competitive ratio of the price and utility algorithm is

$$r^* = (1 + c \cdot r^*) \ln \frac{(\overline{p}/\underline{p}) - 1}{(r^* - 1) + (c/\underline{p})} \qquad (24)$$

Next, we show that the price and utility algorithm is optimal. It is proved that the competitive ratio r^* is less than the competitive ratio r_1. According to the above rules of the price and utility algorithm, we will find that this algorithm can achieve more goods than the reservation utility strategy.

V. NUMERICAL EXAMPLES

In this section we discuss simulation results of the multi-attribute on-line auction model and provide some interesting managerial insights. Specifically, the goal of this section is to analyze how parameters within the score function of the multi-attribute affect the buyer's decision. In here, we report results only on two attributes, i.e., price and quality, since it seems to have a greater impact on the buyer's decision. We discuss the following two kinds of the multi-attribute on-line auction with transaction cost or without it. For simulation purposes and without loss of generality, we assume that the buyer's ideal price and quality is [10, 160] and reservation price and quality is [23, 120]. Furthermore, let the range for the number of the bidders n is 5.

For the multi-attribute on-line auction without the transaction cost, we can achieve the competitive ratio of the reservation utility strategy, i.e., $r_1 = 1.0674$, according to the (22). For simplicity, let the buyer has original wealth of 1000 and w_1=70%. When these five bidders bid one by one, the buyer has to compute each bidder's score based on the (4). After computing the total score, the buyer utilizes the reservation utility strategy to make decision, i.e., whether to buy or not, how much he should buy. In Tab.1, we found that the fourth bid is reject for the lower score. The other four bids are accepted and the buyer only trades certain quantity to maintain the less competitive ratio. Even if the buyer runs into the worst state, i.e., no lower bids appear, he also can make some profit based on the reservation utility strategy. Different from the results of [4], the optimal auction strategy designed by Lavi and Nisan is to accept all the bids. The multi-attribute introduced into the on-line auction helps the buyers to make more complex situations about the goods.

Table 1 Two-dimensional attribute on-line auction [5]

S	Attribute (price and quality)	Score	Quantity	Decision
S_1	[20,140]	0.9081	43	accept
S_2	[18,130]	0.9084	24	accept
S_3	[16,160]	0.9222	21	accept
S_4	[15,140]	0.9182		reject
S_5	[15,180]	0.9561	11	accept

For the multi-attribute on-line auction with the transaction cost, we also can obtain the competitive ratio of the price and utility algorithm, i.e., $r^* = 1.0434$, according to the (24). The transaction cost is introduced into the original model to maintain the less competitive ratio. Let the transaction cost is c=1.2. As the same as the before, let the buyer has original wealth of 1000 and w_1=75%. Here, the weight of price is higher with the transaction cost than the weight without the transaction cost. These five bidders bid one by one and the buyer has to compute each bidder's score based on the (4). Only if the buyer computes the total score, he refers to these scoring value to make decision, i.e., whether to buy or not, how much he should buy. In Tab.2, it is found that the third, the fourth and the fifth bids are all rejected for the lower score. Since the transaction cost exists, the buyer would like the less price and the better quantities. Although the same scoring rule of these two attributes, the expected total utility is higher than the original model in [4] and [5]. At the trading period, we also find that the trading quantities are more than the ones without the transaction cost.

Table 2 Two-dimensional attribute on-line auction with transaction cost

S	Attribute (price and quality)	Score	Quantity	Decision
S_1	[20,140]	0.9021	45	accept
S_2	[18,130]	0.9250	31	accept
S_3	[16,160]	0.9166		reject
S_4	[15,140]	0.8750		reject
S_5	[15,180]	0.8750		reject

VI. CONCLUSIONS

We have described the theoretical foundations of a multi-attribute on-line auction to aid procurement professionals in purchasing multiple units of a good. Features of the mechanism include the suggested price and the sold quantities to a coming bidder. In the spirit of many recently developed auction/negotiation sites, we consider not only price and quality attributes combinations, but also other relevant aspects, such as bidder attributes. We conclude with several suggestions for future research.

While some theoretical results have been outlined, there is still a huge amount of work to do, including: an extension of multi-attribute on-line auctions to the risk-reward framework. The classical competitive analysis is the most fundamental and important approach to study on-line auction problems. But it is not very flexible since it avoids making assumptions about future inputs. In the future, we expect to find some better ways to deal with this problem. We want to provide an online risk algorithm, which allows the sellers to manage their risk and utilize their forecasts. Furthermore, there are some research issues we will focus in the future. But how to improve the performance of the competitive algorithm by other methods, such as probability statistics, is another direction.

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Lili Ding received the BS and MS degrees from Northeastern University in July 2001 and Ocean University of China in July 2004, respectively. In November 2007, she received the Ph.D. degree in management science and engineering from Xi'an Jiaotong University.

Since December 2007, she serves as an associate professor at Shandong University of Science and Technology. Her major interests include combinatorial optimization, online algorithm and online decision-making and pricing theory.

Xinmin Liu is a professor of Shandong University of Science and Technology. His major interests include combinatorial optimization, enterprise decision-making and technical economy.

Wanglin Kang is a Ph.D. student at Shandong University of Science and Technology. His major interests include combinatorial optimization and technical economy.