

Lateral Control for Autonomous Parking System with Fractional Order Controller

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Abstract—Lateral control in autonomous parking system, which is always at low speed, simultaneously requiring very small following error, is a difficult problem to resolve. To address this problem, this paper introduces the Fractional order $PI^\lambda D^\mu$ controller. What's more, to address the parameters tuning problem in design of the controller, an improved genetic algorithm is adopted. With the proposed method, simulation and real vehicle experiments are carried out. Results show that fractional order $PI^\lambda D^\mu$ controller can manage the vehicle to follow the planned trajectory precisely and that stably and robustly in autonomous parking process, which is at low speed.

Index Terms—Autonomous Parking; Fractional Order $PI^\lambda D^\mu$; Genetic Algorithm; Lateral Control

I. INTRODUCTION

With the development of society, vehicle is becoming more and more popular in people's life and making people's living conveniently. However, as more and more people owning private vehicle, lacking of parking space is a fatal problem. What's more, statistics in 2002 indicates that more than 21% of the vehicle accidents result from parking. To resolve this problem, much research about the autonomous parking system has been done for a long decade. Paromtehic and Florencio present the continuous control in column parking system[1,2]. At the same time, many automobile manufacturers also spend much money and time on autonomous parking system, and there are many useful autonomous parking systems such as: Citroen's C3 City Park autonomous parking system, Valeo's Park4U autonomous parking system, Volkswagen's PAV (Park Assist Vision) autonomous parking system, Lexus LS460L autonomous parking system and so on. However, research on autonomous parking system is inchoate in China. Many researchers

just focus on simulation of the autonomous parking process and most of them use fuzzy controller to control the vehicle[3,4,5].

Concept of fractional order calculus has been proposed for 300 years. Recently, with the development of fractional order calculus, more and more researchers have considered describing the mechanical system by fractional order state equations. The idea of using fractional order controllers for dynamic systems comes from Oustaloup, who developed so-called CRONE controller[7]. Now, many other fractional order controllers have been proposed, such as $PI^\lambda D^\mu$ controller[6], TID controller[12]. The fractional order controller is remarkably insensitive to change of the parameters and can behave robustly[13]. Even when parameters of the controller or the plant change in a special range, the fractional order controller can still get good performance. In this paper, the autonomous vehicle is a nonlinear system, and accurate dynamic model of the vehicle is complicated and not useful. Simple model is always adopted in real vehicle control, which results in some error in the dynamic model. With this case, traditional integer order controller can not satisfy the high-precision needs in the autonomous parking process. So this paper introduces the use of fractional order $PI^\lambda D^\mu$ controller for the lateral control of the autonomous parking system, and discusses the parameters tuning problem in design of the controller based on an improved genetic algorithm.

This paper is organized as follows. In section 2, first we introduce the Fractional order $PI^\lambda D^\mu$ controller and how to calculate fractional order differential equation. Section 3 describes the design of the $PI^\lambda D^\mu$ controller in autonomous parking system, especially emphasizing the parameters tuning process based on an improved genetic algorithm. Some simulation results are given, to verify the feasibility of the controller. In section 4, real vehicle experiments are carried out on an autonomous vehicle in kinds of situations, and results show the expected performance of the controller. Finally, a conclusion is given in section 5.

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II. FRACTIONAL ORDER $PI^\lambda D^\mu$ CONTROLLER AND FRACTIONAL ORDER CALCULUS

First, consider the fractional order calculus given as following[6]:

$${}_a D_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha} & \text{re}(\alpha) > 0 \\ 1 & \text{re}(\alpha) = 0 \\ \int_a^t (d\tau)^{-\alpha} & \text{re}(\alpha) < 0 \end{cases} \quad (1)$$

Among above, α is a complex number, a represents the lower limit and t represents the upper limit.

Fractional order calculus is an expansion of integer order integration and differentiation. The most commonly used definition for the general fractional integro-differential operator is the GL (Grumwald-Letnikov) definition, which is given by:

$${}_a D_t^\alpha = \lim_{h \rightarrow 0} h^{-\alpha} \sum_{j=0}^{\lfloor (t-a)/h \rfloor} (-1)^j \binom{\alpha}{j} f(t-jh) \quad (2)$$

Where $\lfloor \cdot \rfloor$ implies the integer part, $\binom{\alpha}{j}$ implies a quadratic polynomial.

GL definition shows that fractional order differential operator provides a powerful description of the memory effect in every substance: result at time t depends on the results from time a to t . So we have to save the result at every time, it's the normal method named "global-Memory". Obviously, as the time t increases, memory load and computation time will grow, which makes the computational efficiency to be very low. Another point to notice is that as the time point t_1 further away from t , less influence from time point t_1 will have on the result of time point t . So when computing the result at time point t , results that of time far away from time point t can be ignored, and computation can be realized on a fixed time interval. This is so-called "short-Memory", which results the following expression from (2):

$$D_t^\alpha f(t) \approx \sum_{j=0}^{\lfloor \frac{t-L}{h} \rfloor} (-1)^j \binom{\alpha}{j} f(t-jh) / h^\alpha \quad (3)$$

Where $[t-L, t]$ is the fixed time interval, L is a time constant, h is the computation step. The "short-Memory" method can well solve the computation efficiency problem, which makes the fractional order calculus to be useful in real situation.

Fractional order $PI^\lambda D^\mu$ controller is an expansion of integer order PID controller, and both the rank of integrator and differentiator can be non-integer. The integer order controller is just an instance of fractional order controller. Because of the introducing of rank μ, λ , history data will be remembered and have some effect on the temporal answer to compute. So the disturbance from the dynamical model of the plant or the noise from the sensors can be conquered, and to behave robustly. That is

very important to some situations which the accurate model can not be got or the accurate model is too complicated to use. In frequency domain, the transfer function of fractional order $PI^\lambda D^\mu$ controller definition is given by:

$$C(s) = K_p + K_i * S^{-\lambda} + K_d * S^\mu \quad (4)$$

Where μ, λ are the rank of the integrator and differentiator, which to be arbitrary nonnegative real numbers, and K_p, K_i, K_d are the coefficients of the normal PID control.

III. DESIGN OF FRACTIONAL ORDER $PI^\lambda D^\mu$

In the autonomous parking process, the speed is very low and always to be a constant. Path following mainly depends on the directional control of the autonomous vehicle. So it is reasonable to primary define the autonomous parking process as a lateral control problem of the autonomous vehicle[11]. With the properties discussed above, we propose the use of fractional $PI^\lambda D^\mu$ controller for the lateral control of autonomous vehicle, and the block diagram is shown in Fig. 1. The fractional order $PI^\lambda D^\mu$ controller, model of the driver's delay and vehicle dynamical model is included in the diagram. To address the parameters tuning problem in the design process, an improved genetic algorithm is adopted and details of the algorithm will be discussed below.

A. Lateral dynamical model of vehicle

First we will discuss the dynamical model of the vehicle in the proposed control block diagram as shown in Fig. 1. Vehicle is a nonlinear system and the dynamical model is in fact very complicated. To describe it appropriately, many simple lateral dynamical model have been proposed, such as 2 degree of freedom model (DOF)[8], 3 DOF model, 6 DOF model, 17 DOF model[9] and so on. As 2 DOF model only has little parameters and mainly emphasizes the vehicle lateral control, it is widely used. This paper also adopts the 2 DOF model to describe the motion of the vehicle, and representation of it as following [8]:

$$\begin{cases} mv(\dot{\beta} + \gamma_\omega) + (k_1 + k_2)\beta - \frac{\gamma_\omega}{v}(ak_1 - bk_2) = k_1\delta \\ I_z \dot{\gamma}_\omega + (ak_1 - bk_2)\beta + \frac{\gamma_\omega}{v}(a^2k_1 + b^2k_2) = ak_1\delta \end{cases} \quad (5)$$

Thereinto, I_z represents the moment of inertia around the z axis, m is mass of the vehicle, δ is the steering angle, a is the distance between front axle and the vehicle center of gravity(CG) while b is the distance between rear axle and CG, k_1, k_2 are the front and rear tires cornering stiffness respectively, β is the sideslip angle of CG point, v implies the vehicle speed, γ_ω is the angular velocity of CG, and l implies the distance between front axle and rear axle. The transfer function between lateral acceleration and the steering angle δ is given by:

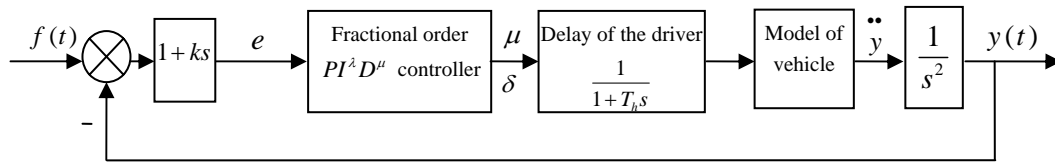


Figure 1. The proposed control block diagram of autonomous vehicle

$$\frac{y}{\delta}(s) = G_r \frac{\zeta_1 s + 1}{T_2 s^2 + T_1 s + 1}$$

$$G_r = \frac{v}{l(1 + Kv^2)}$$

$$K = \frac{1}{l} \left(\frac{1}{C_1} - \frac{1}{C_2} \right), C_1 = \frac{lk_1}{mb}, C_2 = \frac{lk_2}{ma}$$

$$T_1 = \frac{v}{l} \left(\frac{C_2}{1 + kv^2} + \frac{C_1}{1} \right), T_2 = \frac{\eta v^2}{C_1 C_2 (1 + Kv^2)}$$

$$\zeta_1 = \frac{v}{C_2}$$

In this paper, we use stepper motor to drive the actuator, and its dynamical equation is given by:

$$\delta = \mu \quad \mu \in [-\mu_{max}, \mu_{max}] \quad (7)$$

Where μ implies the turning angle of the actuator, which to be an arbitrary number between $[-\mu_{max}, \mu_{max}]$.

B. Preview-follower control

Now we will discuss the error signal used for the fractional $PI^\lambda D^\mu$ controller, which is shown in Fig. 1 as e . Obviously, the error signal should include the position error and the heading error between the vehicle and the planned trajectory. This paper will compute these errors by the preview-follower control. Preview-follower control[2] is widely used in path following of autonomous vehicle, and the sketch map of the method is shown in Fig. 2. As shown in the figure, $f(x)$ represents the planned trajectory and P represents the previewed single-point on the planned trajectory. Δy represents the position error between the predicted vehicle trajectory and the previewed single-point and $\Delta \theta$ represents the direction error. (x_0, y_0) represents the vehicle position and

θ_0 represents the heading of the vehicle at the computation interval. v_x represents the longitudinal velocity while v_y represents the lateral speed of the vehicle. T is the preview time, which comes from the normal human driving model. Δy and $\Delta \theta$ are given by:

$$\begin{cases} \Delta y = (f(x_0 + v_x T) - y_0 - v_y T) * \cos \theta_0 \\ \Delta \theta = \arctg(f'(x_0 + v_x T)) - \theta_0 \end{cases} \quad (8)$$

The error signal for the lateral control is given by the following expression:

$$e = \Delta y + k \Delta \theta \quad (9)$$

Where k represents the weight between the position error and the heading error.

C. Parameters tuning based on an improved genetic algorithm

With the proposed control block diagram as shown in Fig. 1, another important problem to resolve is the design of the controller parameters. Now we will discuss the parameters tuning method based on an improved genetic algorithm. Genetic algorithm has been proposed by J.Holland and his colleagues in Michigan University for about forty years ago. As a simple and robust algorithm, genetic algorithm has been widely used in optimization, machine learning, planning and so on. This paper uses an improved genetic algorithm[10] to tune the fractional order controller parameters as following:

Coding: In this paper, we start solving the problem from constructing the initial population. As the parameters to be tuned include the normal P, I, D coefficients and the rank μ, λ , size of each individual is 5, and each individual is represented as follow:

$$E^G = [K_p^G, K_i^G, K_d^G, \lambda^G, \mu^G] \quad (10)$$

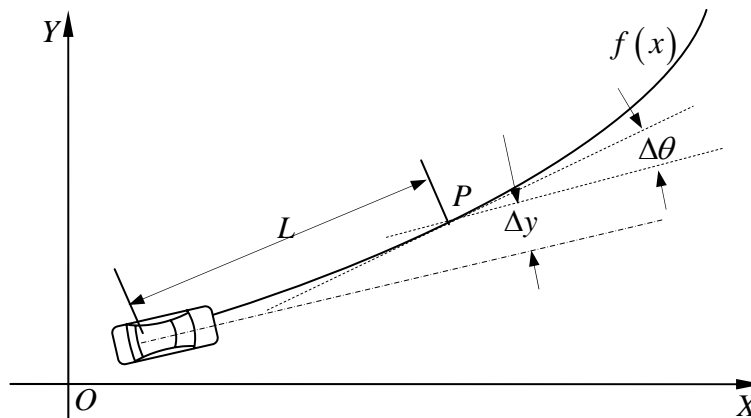


Figure 2. Sketch map of single point-preview control.

As the normal genetic algorithm, the initial population is generated by N individuals randomly:

$$K = \begin{bmatrix} E^1 \\ E^2 \\ \vdots \\ E^N \end{bmatrix} = \begin{bmatrix} k_p^1 & k_i^1 & k_d^1 & \lambda^1 & \mu^1 \\ k_p^2 & k_i^2 & k_d^2 & \lambda^2 & \mu^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ k_p^N & k_i^N & k_d^N & \lambda^N & \mu^N \end{bmatrix} \quad (11)$$

Computation of the fitness function: Fitness value of each individual represents the adapting capability to the environment. In this paper, the fitness function is defined as follow:

$$F = \begin{cases} \frac{1}{\int_0^\infty \omega_1 |e(t)| dt + \omega_2 t_u} & P.O. < 0.2 \\ \frac{1}{\int_0^\infty \omega_1 |e(t)| dt + \omega_2 t_u + 100 * (P.O. - 0.2)} & P.O. \geq 0.2 \end{cases} \quad (12)$$

Where ω_1, ω_2 implies the weights, t_u is the setting time, $P.O.$ is the overshoot of the system. In this paper, $\omega_1 = 1, \omega_2 = 2$.

Reproduction: Compare the fitness of this generation and next generation. If the best fitness value f_{next} in next generation is smaller than that in this generation, some copy process should be done. Assume that there are k sets in this generation, whose fitness value is bigger than f_{next} , we will copy the k sets from this generation to next generation, and randomly replace k sets or replace the k sets whose fitness values are smallest in the next generation.

Crossover: This paper uses the method of arithmetical crossover. With crossover, the i th and the $(i+1)$ th individuals are given by:

$$\begin{cases} TE(i,:) = \alpha * K(i+1,:) + (1-\alpha) * K(i,:) \\ TE(i+1,:) = \alpha * K(i,:) + (1-\alpha) * K(i+1,:) \end{cases} \quad (13)$$

Where α represents the weight of the i th and the $(i+1)$ th individuals to the generated individuals. The crossover probability p_c is defined as follow:

$$p_c = \begin{cases} p_{c1} & f' < f_{avg} \\ p_{c1} - \frac{(p_{c1} - p_{c2})(f' - f_{avg})}{f_{max} - f_{avg}} & f' \geq f_{avg} \end{cases} \quad (14)$$

Among above, $p_{c1} = 0.9, p_{c2} = 0.6$.

Mutation: The mutation rate p_m in this paper is given by:

$$p_m = \begin{cases} p_{m1} & f < f_{avg} \\ p_{m1} - \frac{(p_{m1} - p_{m2})(f_{max} - f)}{f_{max} - f_{avg}} & f \geq f_{avg} \end{cases} \quad (15)$$

Among above, $p_{m1} = 0.1, p_{m2} = 0.01$.

End: The algorithm will terminate when the best individual converge in some threshold. When the ending condition is reached, the set whose fitness value is the biggest is the best result.

D. Simulations

To verify the feasibility of the fractional order $PI^\lambda D^\mu$ controller with the improved genetic algorithm for the lateral control of the autonomous vehicle, simulation results will be given in this section. This paper uses the single-point preview control method and the improved genetic algorithm above to tune the fractional order $PI^\lambda D^\mu$ controller parameters. Table 1 shows the correlative parameters of the vehicle. To illustrate the predominance of the fractional order $PI^\lambda D^\mu$ controller, this paper uses fractional order $PI^\lambda D^\mu$ controller and integer order PID controller to simulate the block diagram in Fig. 1 respectively. Fig. 3 shows the convergence process of the genetic algorithm, represented by the best fitness value and Fig. 4 illustrates the simulation results of the unit-step response of these controllers, corresponding to the parameters computed from the genetic algorithm. From Fig.4 can see, overshoot of the system without controller is 33.5% and the setting time is 4.6s, overshoot of the system with integer order PID controller is 19% and the setting time is 0.32s. However, with fractional order $PI^\lambda D^\mu$ controller, overshoot of the system is 0.4%, and the setting time is 0.1s. Obviously, for the nonlinear plant with time delay, the performance of fractional order controller is better than that of the integer order controller. Another aspect, from Fig. 3 can see, even the best fitness value of the initial population is far from the final result, it can get close to the answer quickly and finally stably at the best solution. So the improved genetic algorithm can well adjusted to the need of the parameters tuning problem.

TABLE I.
PARAMETERS OF THE VEHICLE

Parameter	Value
$C_1, C_2 / (m/s^2)$	36.887, 55.0
$v / (km/h)$	2.5
T_h / s	0.1
$a, b / (m)$	1.84, 1.88
η	1.0
k	1.03

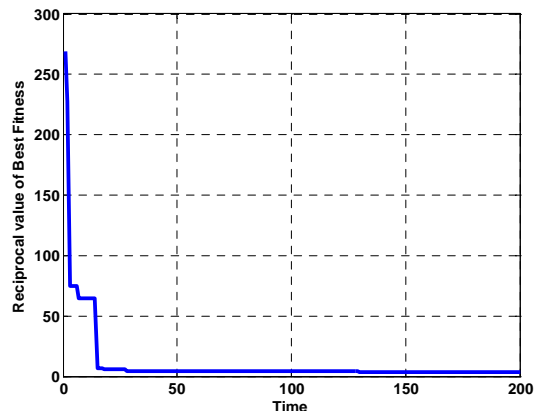


Figure 3. Convergence of the best fitness value

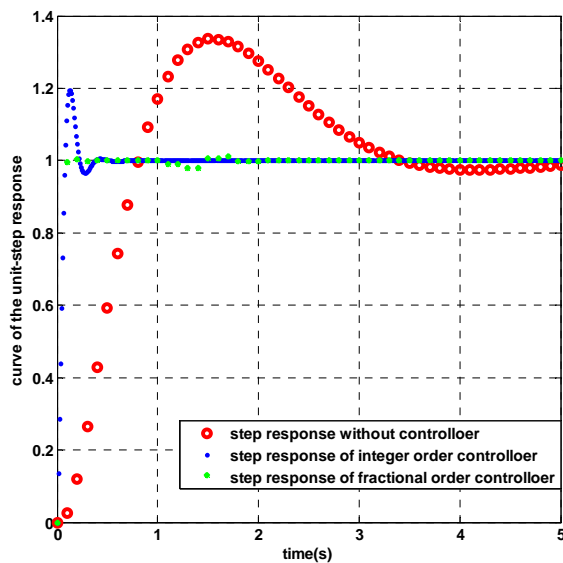


Figure 4. Comparison of results of unit-step response

IV. EXPERIMENT RESULTS

Autonomous parking experiments are carried out on an autonomous vehicle as shown in Fig. 5. The vehicle is modified for autonomous task, and the subsystem of brake, fuel and steering is controlled by computer. Detection of the parking space is fulfilled by some ultrasonic sensors equipped on the vehicle. Width of the vehicle is 1.85m, and length of it is 4.8m. Length of the parking space between two vehicles is 7.5m, which is normal for human driver. In the autonomous parking process, nominal speed of the vehicle keeps at 2.5km/h. Experiment results with fractional order $PI^\lambda D^\mu$ controller for the lateral control of the vehicle is shown in Fig. 6, and the sampling period is 0.01s. Because the process of autonomous parking lasts only several seconds, this paper uses the method of “short-Memory” to compute the fractional order calculus approximately and the mnemonic time lasts for 2s, so to meet the real time need of the control process.

Fig. 6(a)-(c) show the experiment results in different floor situations, such as on the flat ground or uphill and downhill. From these results we can see that, the vehicle can follow the planned trajectory well in these normal situations. Another difficult case to mention is shown in Fig. 6(d), in which heading of the vehicle is not parallel to the parking space, but with a small angle. However, the fractional order $PI^\lambda D^\mu$ controller can still manage the vehicle to follow the planned trajectory precisely and robustly. Other experiments such as in snow environment or rainy day are carried out and results are shown in Fig. 6(e) and Fig. 6(f). In these situations, the dynamical model of the vehicle can be inaccurate and changeable, especially the force from the ground to the vehicle, making the lateral control of the vehicle more challengeable. However, from these experiment results, we can see that the fractional order $PI^\lambda D^\mu$ controller for the lateral control of vehicle can still get good performance. Above all, the fractional order $PI^\lambda D^\mu$ controller can perform robustly and precisely for the

lateral control of the vehicle in kinds of situations at low speed, that to well satisfy the need of autonomous parking system.



Figure 5. The autonomous vehicle used in the parking experiments

V. CONCLUSION

We have presented a novel fractional order $PI^\lambda D^\mu$ controller for the lateral control of the vehicle in the process of autonomous parking. To address the parameters tuning problem in design of the controller, an improved genetic algorithm is incorporated. Simulation and real vehicle experiment results show that the controller performs precisely and robustly. The vehicle can follow the planned trajectory well at low speed, even when the vehicle is at different state, such as the vehicle parallel to the parking space or not, the vehicle is at uphill or downhill, or the flat ground, so can meet the need of autonomous parking system.

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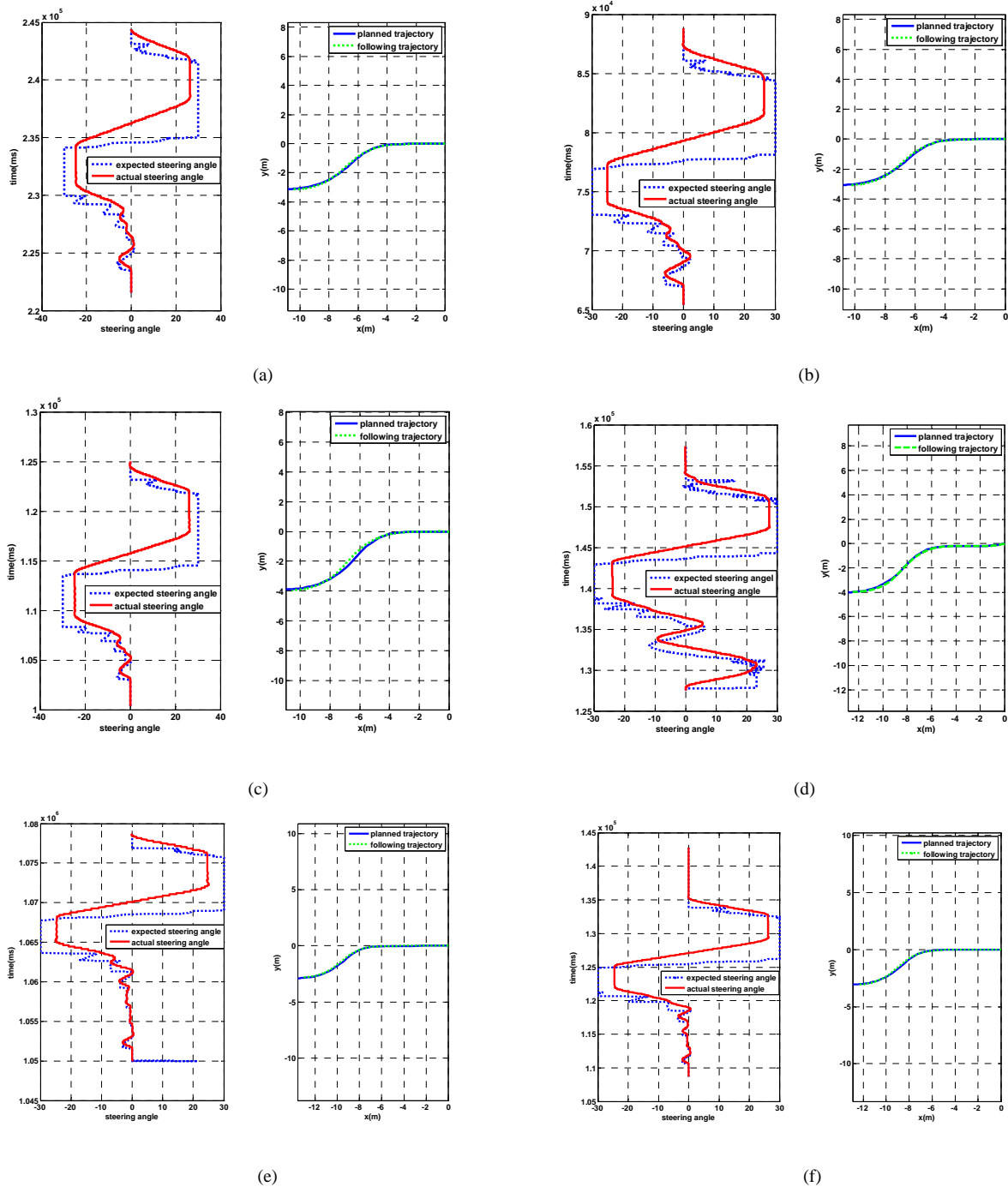


Figure 6. Real vehicle experiment results in different situations: (a) the vehicle moves on the flat ground and heading of the vehicle is parallel to the parking space. The left image shows the tracking performance of the steering angle and the right one shows the planned vehicle trajectory and the following trajectory. (b) The vehicle moves at a downhill situation. (c) The vehicle moves at an uphill situation. (d) The vehicle moves on the flat ground but heading of the vehicle is not parallel to the parking space. (e) The vehicle moves in the snow environment. (f) The vehicle moves on the wet ground in rainy day.



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