

# Sliding Mode Control of Surface-Mount Permanent-Magnet Synchronous Motor Based on Error Model with Unknown Load

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**Abstract**—Surface-mount permanent-magnet synchronous motor(SPMSM) has been widely applied in accurate motion control. In this paper, the SPMSM model is divided into electrical magnetic subsystem and electrical mechanical subsystem. Then a novel error model is given by which the controllers can be designed easily. In the general PMSM controllers design, the unknown load is often neglected. In this paper, a proportional speed error is used to take the place of the unknown load without using load estimation. Based on the error model of the SPMSM, sliding mode controllers are designed to enhance the robustness of the SPMSM drive system. At last, many simulation results are given to show the effectiveness of the proposed scheme.

**Index Terms**—Sliding mode control, Surface-mount permanent-magnet synchronous motor, Error model, robustness

## I. INTRODUCTION

In recent years, advancements in magnetic materials, semiconductor power devices, and control theory have made the SPMSM drive plays a vitally important role in accurate motion control in the low-to-medium power range. The desirable features of the SPMSM are its compact structure, high air-gap flux density, high power density, high torque-to-inertia ratio, and high torque capability [1-3]. At present, the research of SPMSM control design can be divided into three main aspects. The first aspect is to enhance the robustness of the drive system. The second aspect is to enhance the control accuracy. And the third aspect is to enhance the ratio of the performance and the price.

In the SPMSM control system, two types of SPMSM model are often used for the controllers design. One is

based on the  $d$ - $q$  reference frame model that rotates synchronously with an electrical angular velocity [4-6]. The other is based on the  $F$  or  $a$ - $b$  stationary reference frame model that keeps static in the state reference frame [7, 8]. In the controllers design of both types of SPMSM models, the unknown load torque is often neglected or estimated(observed) online [9, 10]. This makes the design of the controllers more complex to realize the high control performance of drive systems. So how to process the unknown load torque to enhance the robustness and accuracy of the SPMSM drive system is a very important problem.

Sliding mode control is one of the widely researched robust control approaches since it gives systems an invariance property to uncertainties once the system dynamics is in the sliding mode [11, 12]. It features good robustness, disturbance rejection, and fast response control. Sliding mode control has been widely used in the SPMSM control design [1, 8, 12].

The organization of this paper is as follows. Section 2 introduces the  $d$ - $q$  reference frame model of the SPMSM. The model is divided into electrical magnetic subsystem and electrical mechanical subsystem. Based on the departed model, the error model is given. In section 3, sliding mode controllers are designed based on the error model. In section 4, simulation results are shown in different conditions. Section 5 gives the conclusions of the paper.

## II. ERROR MODEL OF THE SPMSM

The  $d$ - $q$  model of the SPMSM is given as following equations.

$$\frac{di_d}{dt} = -\frac{R}{L}i_d + p\omega_r i_q + \frac{u_d}{L} \quad (1)$$

$$\frac{di_q}{dt} = -\frac{R}{L}i_q - p\omega_r i_d - \frac{p\phi_f}{L}\omega_r + \frac{u_q}{L} \quad (2)$$

$$\frac{d\omega_r}{dt} = \frac{3p\phi_f}{2J}i_q - \frac{T_L}{J} \quad (3)$$

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where  $R$  is the state resistance,  $L$  is the state inductance,  $p$  is the number of pole pairs of the SPMSM,  $\phi_f$  is the permanent magnet flux,  $J$  is the moment of inertia,  $T_L$  is the load torque which is assumed to be unknown in this paper,  $i_d$  is the  $d$ -axis stator current,  $\omega_r$  is the rotor speed,  $i_q$  is the  $q$ -axis stator current,  $u_q$  is the  $q$ -axis stator voltage,  $u_d$  is the  $d$ -axis stator voltage.

Based on the equations of the SPMSM, the model of the SPMSM can be divided into electrical magnetic subsystem and electrical mechanical subsystem. The electrical magnetic subsystem is given as equation(1), and the electrical mechanical subsystem is given as equation (2) and (3).

The steady states of the SPMSM are defined as  $i_d^*$ ,  $i_q^*$ ,  $u_{ds}$ ,  $u_{qs}$ ,  $\omega_r^*$  and  $T^*$ , where  $i_d^*$  is the steady  $d$ -axis stator current,  $i_q^*$  is the steady  $q$ -axis stator current,  $u_{ds}$  is the steady  $d$ -axis stator voltage,  $u_{qs}$  is the steady  $q$ -axis stator voltage,  $\omega_r^*$  is the reference rotor speed which is assumed to be two times differentiable, and  $T^*$  is the reference electrical magnetic torque. And we define  $e_d = i_d - i_d^*$ ,  $e_q = i_q - i_q^*$ , and  $e_\omega = \omega_r - \omega_r^*$ . The relation between the steady  $i_q^*$ ,  $u_{ds}$ ,  $u_{qs}$  and the reference

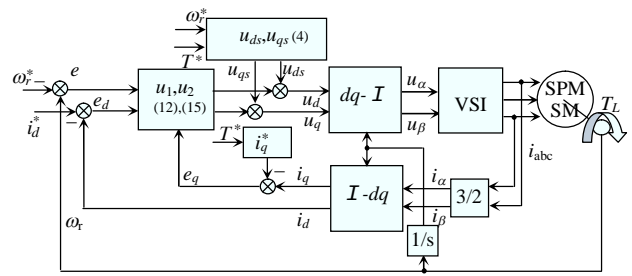


Figure 1 The control structure of the SPMSM

$\omega_r^*$ ,  $T^*$  can be given as following equation.

$$i_q^* = \frac{2T^*}{3p\phi_f}, u_{ds} = -\frac{2Lp\omega_r^*T^*}{3p\phi_f}, u_{qs} = \frac{2RT^*}{3p\phi_f} + p\phi_f\omega_r^* \quad (4)$$

We apply the errors  $e_{db}$ ,  $e_q$  and  $e_\omega$  as new state variables.

In the field-oriented control of the SPMSM,  $i_d^*$  is generally set to zero. Then the electrical magnetic subsystem can be written as

$$\frac{de_d}{dt} = -\frac{R}{L}e_d + \frac{u_1}{L} + \frac{2e_\omega T^*}{3p\phi_f} + pe_\omega e_q + p\omega_r^* e_q \quad (5)$$

where  $u_1 = u_q - u_{eq}$ . Then the electrical mechanical subsystem can be written as

$$\frac{de_q}{dt} = -\frac{R}{L}e_q - \frac{p\phi_f}{L}e_\omega + \frac{u_2}{L} - p\omega_r^* e_d - pe_\omega e_d$$

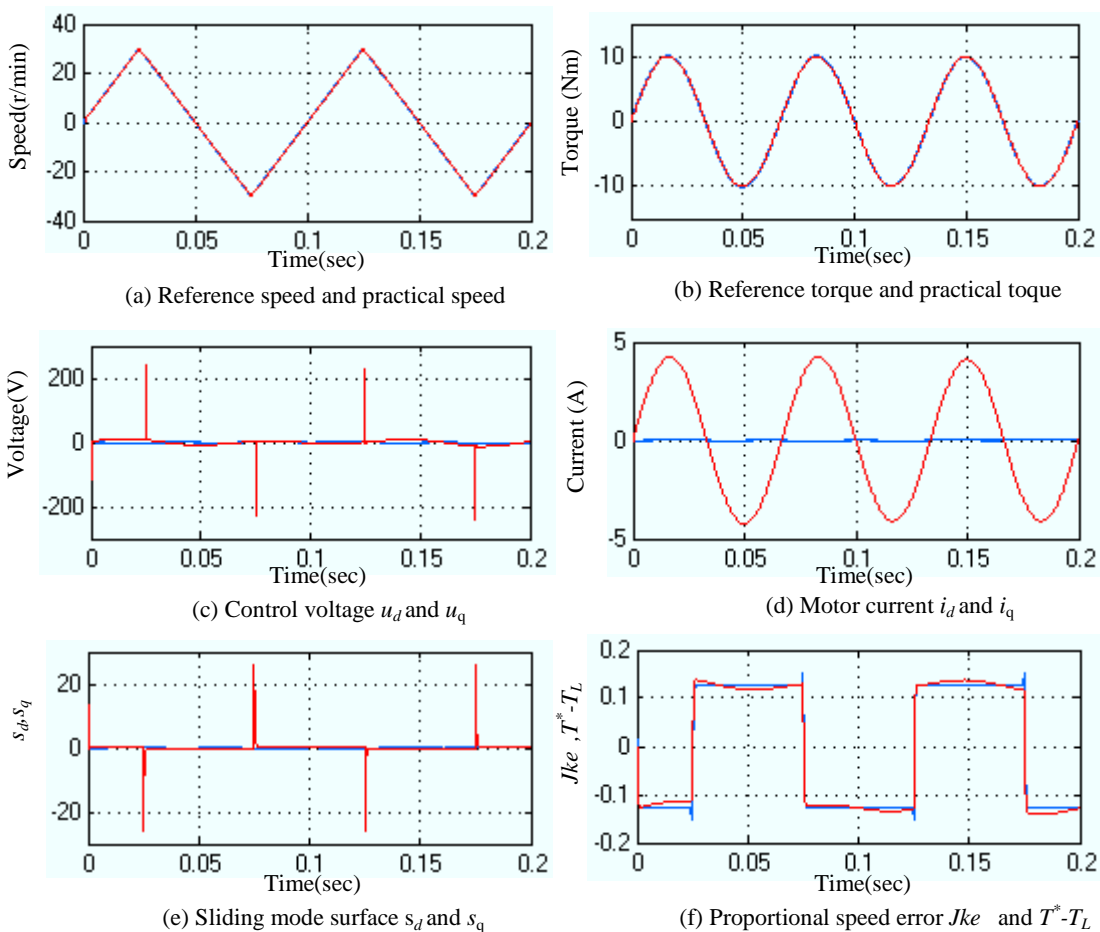


Figure 2 Simulation results of the first case

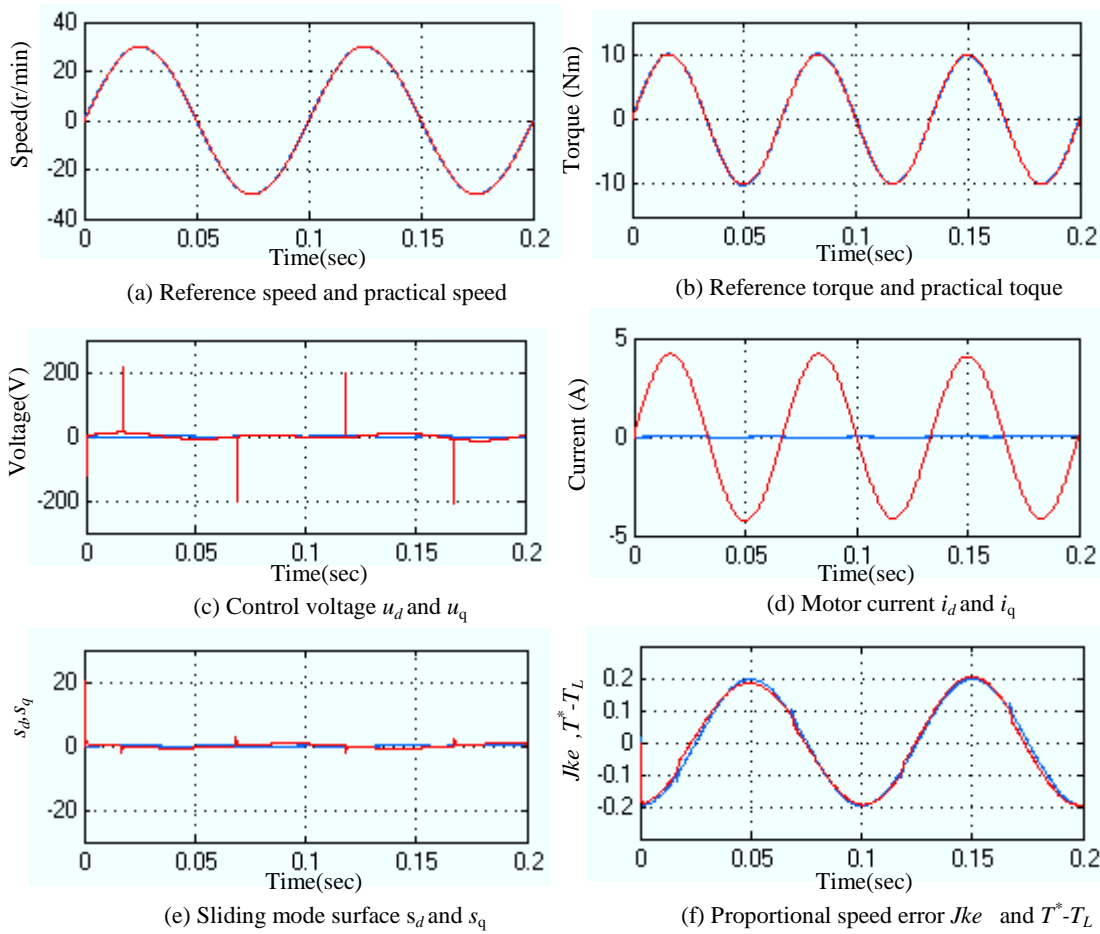


Figure 3 Simulation results of the second case

$$-\frac{2}{3p\phi_f} \frac{dT^*}{dt} \tag{6}$$

$$\frac{de_\omega}{dt} = \frac{3p\phi_f}{2J} e_q + \frac{T^* - T_L}{J} - \frac{d\omega_r^*}{dt} \tag{7}$$

where  $u_2 = u_q - u_{qs}$ . In equation (7), the unknown load  $T_L$  is generally neglected. From the design of the field-oriented control of the motor control, we can see that the output of the speed controller can be seen as the reference input of the current loop. Then we can use a proportional speed error  $ke_\omega$  to take the place of  $(T^* - T_L)/J$ . Now the equation (7) can be written as the following equation.

$$\frac{de_\omega}{dt} = \frac{3p\phi_f}{2J} e_q + ke_\omega - \frac{d\omega_r^*}{dt} \tag{8}$$

From the practice control, we can assume the following conditions.

$$|c_1| = \left| \frac{2e_\omega T^*}{3\phi_f} + pe_\omega e_q + p\omega_r^* e_q \right| \leq \rho_1 \tag{9}$$

$$|c_2| = \left| -p\omega_r^* e_d - pe_\omega e_d - \frac{2}{3p\phi_f} \frac{dT^*}{dt} \right| \leq \rho_2 \tag{10}$$

where  $\rho_1$  and  $\rho_2$  are positive number. If the coupling parameters  $c_1$  and  $c_2$  between the electrical magnetic

subsystem and the electrical mechanical subsystem are assumed to be disturbance of each system. Then each subsystem can be seen as linear system with out disturbance coming from other subsystem.

### III. SLIDING MODE CONTROL DESIGN OF SPMSM

The sliding mode surface of the electrical magnetic subsystem can be designed as following equation.

$$s_d = e_d + k_1 \int e_d dt, \quad k_1 > 0 \tag{11}$$

The control rule of the sliding mode surface is selected as following equation.

$$\dot{s}_d = -\mu_1 s_d - \rho_1 \text{sgn}(s_d), \quad \mu_1 > 0 \tag{12}$$

where  $\text{sgn}(\cdot)$  is sign function. Then from equation (5), (11) and (12), we can obtain  $u_1$ .

$$u_1 = -L\mu_1 s_d - L\rho_1 \text{sgn}(s_d) + Re_d - Lk_1 e_d \tag{13}$$

The sliding mode surface of the electrical mechanical subsystem can be designed as following equation.

$$s_q = e_\omega + k_2 \int e_q dt + k_3 \dot{e}_\omega, \quad k_2 > 0, \quad k_3 > 0 \tag{14}$$

The control rule of the sliding mode surface is selected as following equation.

$$\dot{s}_q = -\mu_2 s_q - \rho_2 \text{sgn}(s_q), \quad \mu_2 > 0 \tag{15}$$

Then from equation (6), (8), (14) and (15), we can obtain  $u_2$ .

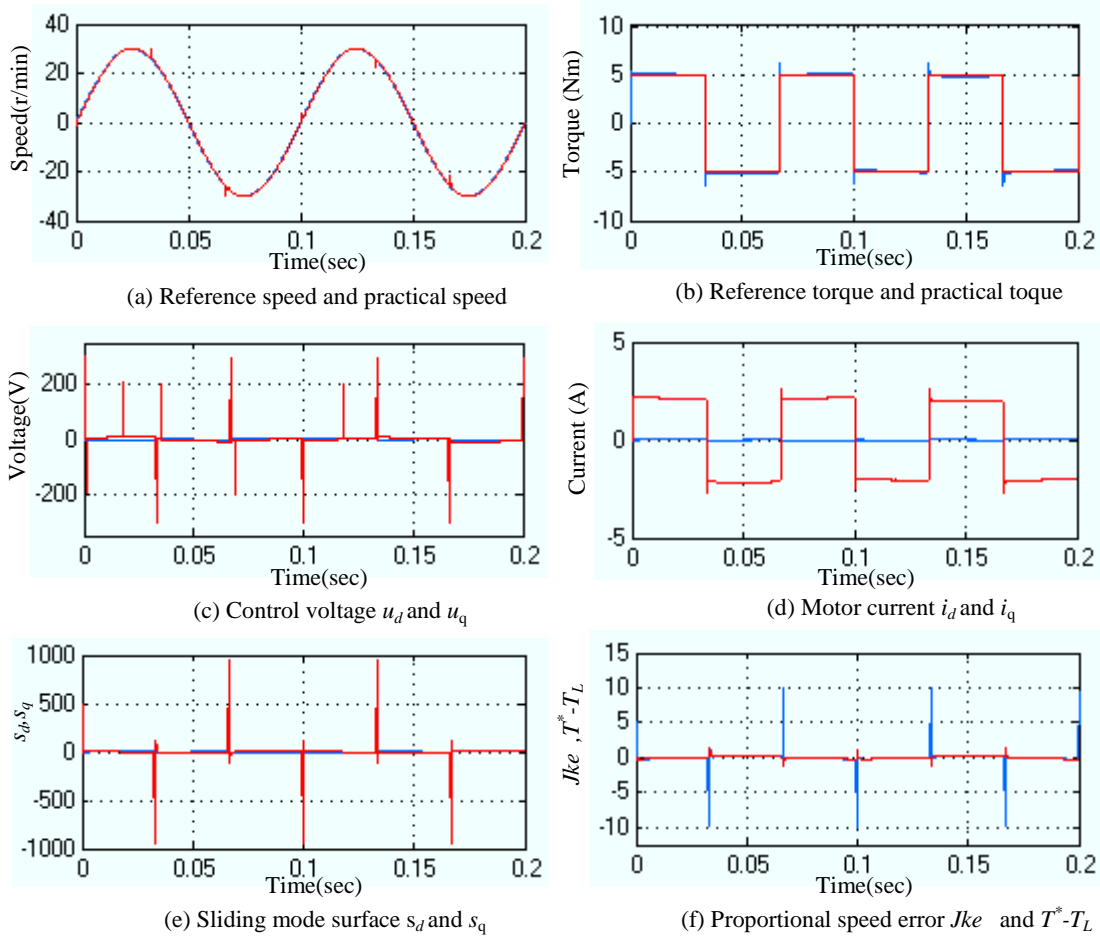


Figure 4 Simulation results of the third case

$$u_2 = \frac{2JL}{3p\phi_f k_3} [-\mu_2 s_q - \rho_2 \text{sgn}(s_q) - \frac{3p\phi_f}{2J} (1 - \frac{R}{L} k_3 + k k_3) e_q - (k + k_2 - \frac{3p^2\phi_f^2}{2JL} k_3 + k^2 k_3) e_\omega + \frac{d\omega_r^*}{dt} + k_3 \frac{d^2\omega_r^*}{dt^2}] \quad (16)$$

From the design, we can see that when the system states reach sliding mode surface the variable of the error model also reaches zero. The stability proof of the SPMSM controllers is a very easy task, so it is neglected in the paper.

IV. SIMULATION RESULTS AND ANALYSIS

The control structure of the SPMSM drive system is given in Figure 1, where VSI presents voltage source inverter, 3/2 presents stator *abc* current to *F* current. Table 1 gives the normal parameters of the SPMSM. And The control parameters are given in Table 2.

TABLE 1 PARAMETER OF THE SPMSM

$R(\Omega)$	$L(mH)$	$J(K_s m^2)$	$p$	$\phi_f(wb)$
2	6	0.001	2	0.8

TABLE 2 PARAMETERS OF THE SPMSM

$k$	$k_1$	$k_2$	$k_3$	$\mu_1$	$\mu_2$	$\rho_1$	$\rho_2$
300	5	5	0.2	20	15	30000	25000

In the first case simulation, the motor speed is set to be asymmetrical triangular wave whose magnitude is 30r/min and frequency is 10Hz. The reference electrical magnetic torque is set to be a sinusoidal wave whose magnitude is 10Nm and frequency is 15Hz. To test the speed and torque tracking performance, we deliberately set the reference speed and torque frequency as different frequency. The simulation results are given in Figure 2.

In the second case simulation, the motor speed is set to be a sinusoidal wave whose magnitude is 30r/min and frequency is 10Hz. The reference electrical magnetic torque is set to be a sinusoidal wave whose magnitude is 10Nm and frequency is 15Hz. The simulation results are given in Figure 3.

In the third case simulation, the motor speed is set to be a sinusoidal wave whose magnitude is 30r/min and frequency is 10Hz. The reference torque is set to be a symmetrical rectangle wave whose magnitude is 5Nm and frequency is 15Hz. The simulation results are given in Figure 4.

In the fourth case simulation, the motor speed is set to be a symmetrical rectangle wave whose magnitude is 30r/min and frequency is 10Hz. The reference electrical magnetic torque is set to be a symmetrical rectangle wave whose magnitude is 5Nm and frequency is 15Hz. The simulation results are given in Figure 5.

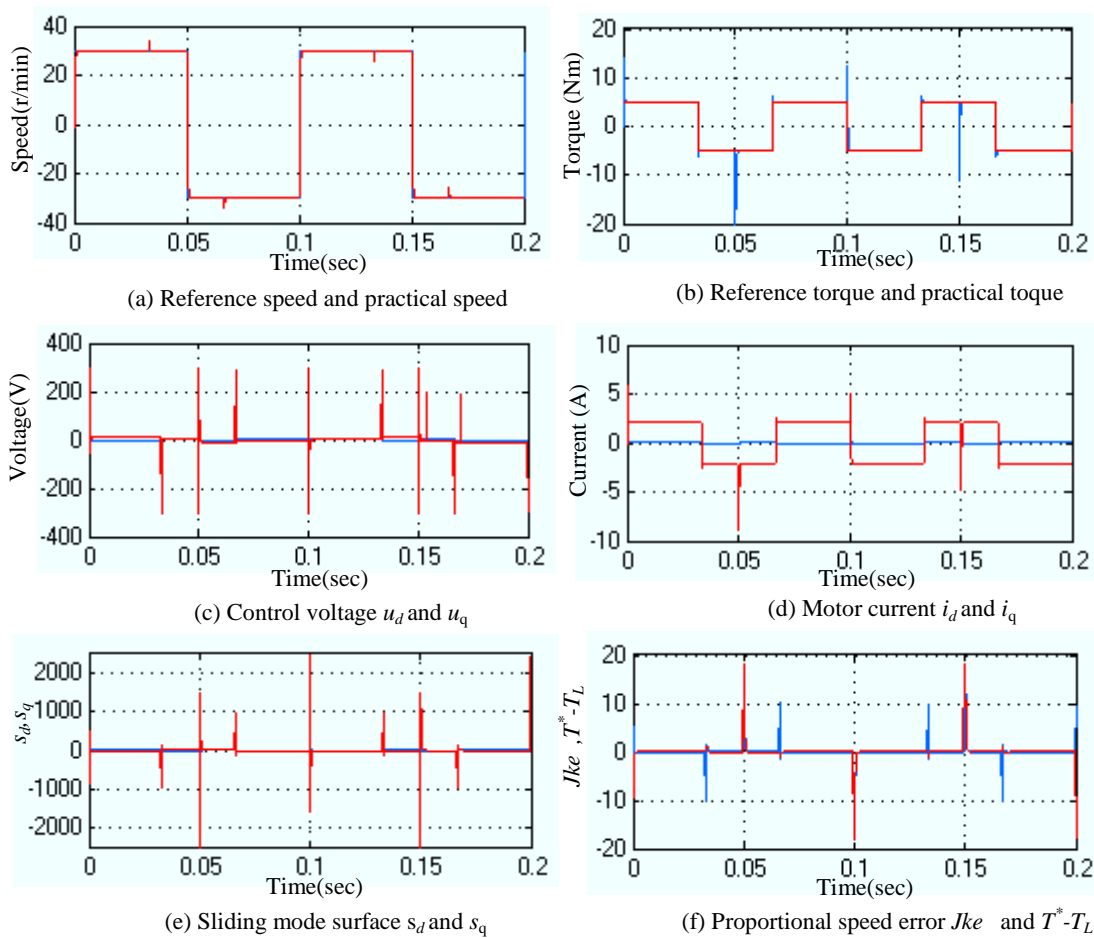


Figure 5 Simulation results of the fourth case

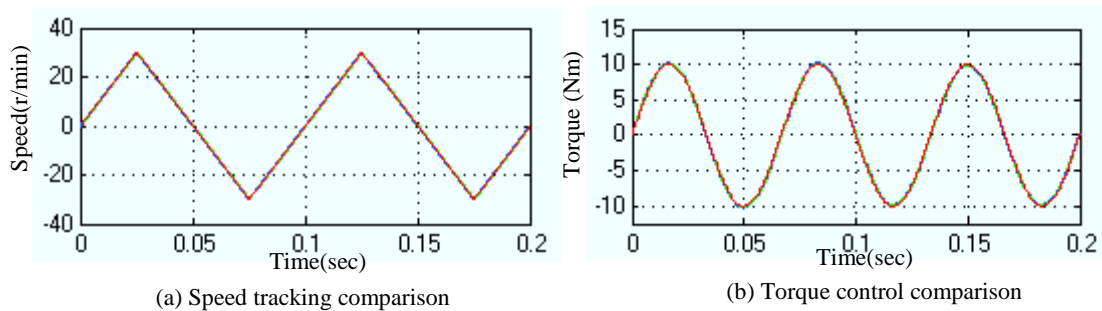


Figure 6 Simulation results of the fifth case

In the fifth case simulation, the stator  $R$  changes from  $2 \Omega$  to  $3 \Omega$ , and the moment of inertia changes from  $0.001 Kgm^2$  to  $0.002 Kgm^2$ . Other conditions are the same as the first case. The simulation results comparison of the speed and torque control with the first case are given in Figure 6.

In the sixth case simulation, the stator  $R$  changes from  $2 \Omega$  to  $3 \Omega$ , and the moment of inertia changes from  $0.001 Kgm^2$  to  $0.002 Kgm^2$ . Other conditions are the same as the second case. The simulation results comparison of the speed and torque control with the second case are given in Figure 7.

In the seventh case simulation, the stator  $R$  changes from  $2 \Omega$  to  $4 \Omega$ , and the moment of inertia changes from  $0.001 Kgm^2$  to  $0.003 Kgm^2$ . Other conditions are the same as the third case. The simulation results comparison

of the speed tracking control and torque control with the third case are given in Figure 8.

In the eighth case simulation, the stator  $R$  changes from  $2 \Omega$  to  $4 \Omega$ , and the moment of inertia changes from  $0.001 Kgm^2$  to  $0.003 Kgm^2$ . Other conditions are the same as the fourth case. The simulation results comparison of the speed and torque control with the fourth case are given in Figure 9.

From the system design and simulation results, we can conclude the followings:

- (1) The division of the SPMSM model is right and effective for the design of the controller. It can be seen as the decentralized system which is widely researched, and the decentralized control theory can be applied to the system design directly [13].

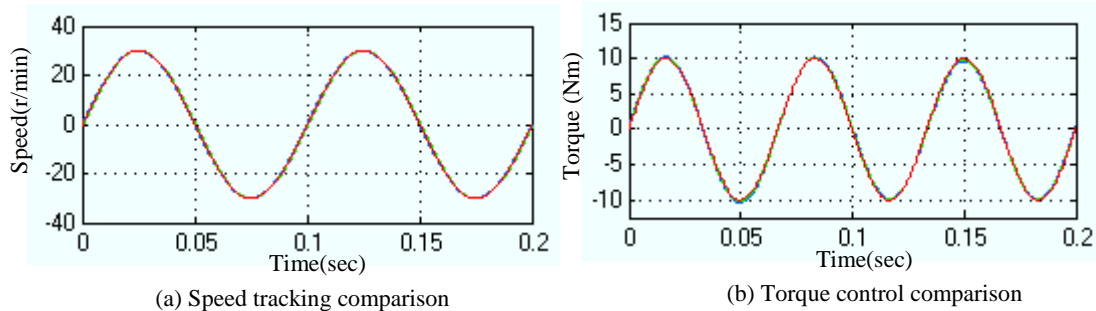


Figure 7 Simulation results of the sixth case

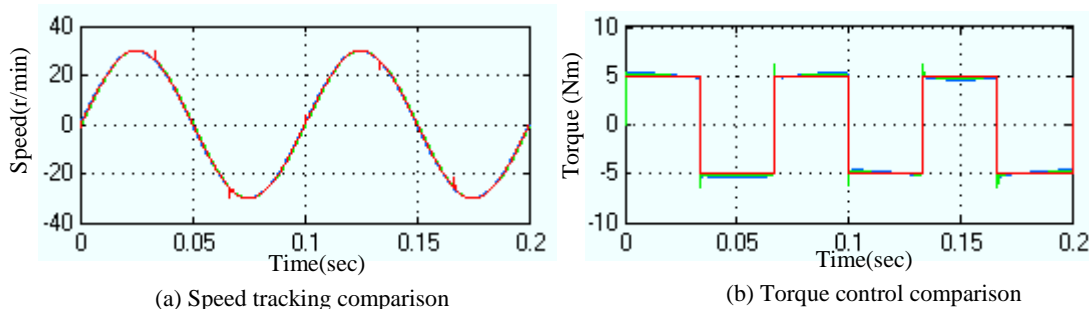


Figure 8 Simulation results of the seventh case

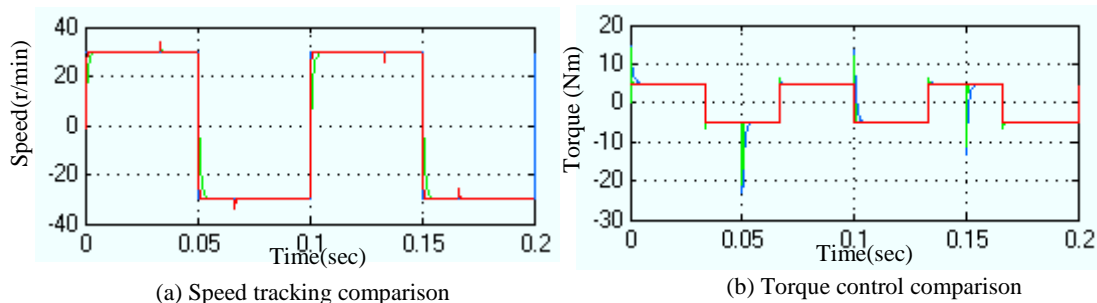


Figure 9 Simulation results of the eighth case

- (2) The error model of the SPMSM is a novel way by which a lot of linear control theory can be used. And the stability control theory can be easily used for the error model of the SPMSM drive system directly.
- (3) The proportional speed error can take the place of the unknown load though the adjustment of the proportional parameters. The simulation results show that the proportional speed error can tracking the variation of different types of the unknown loads with little error.
- (4) The sliding mode control of the SPMSM based on the error model not only has good fast speed tracking performance, but also has strong robustness to parameter variation and out disturbances.

## V. CONCLUSIONS

In this paper, we give a novel error model of the SPMSM which can be seen as the combination of two decentralized subsystems, the electrical magnetic subsystem and electrical mechanical subsystem. The proportional speed error is used to take the place of the unknown load. The sliding mode control is applied to the controllers design of the SPMSM drive system.

Simulation results show that the proposed scheme has good control performance.

It is well known that sliding mode control with high sliding mode gains can cause the chattering of the system. To enhance the control performance further, reduction of the chattering caused by high sliding mode gains is needed in the future.

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