Uncertain Linguistic Multiple Attribute Group Decision Making Approach and Its Application to Software Project Selection

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Abstract—This paper proposes a new approach to solve the multiple attribute group decision making (MAGDM) problem where attribute values are in the format of uncertain linguistic information and attribute weights are in the format of linguistic information. Firstly, concepts and comparison laws of uncertain 2-tuple are given. Then, an uncertain 2-tuple ordered weighted averaging (UTOWA) operator is used to calculate alternative appraisal values with respect to all attributes for each decision maker (DM) and to aggregate all DMs' preferences into a collective opinion. Furthermore, ranking alternatives or selecting the most desirable alternative(s) is conducted according to the comparison laws. Finally, a numerical example on software project selection is used to illustrate the applicability and effectiveness of the proposed approach.

Index Terms—Multiple attribute group decision making (MAGDM); Uncertain linguistic information; Uncertain 2tuple; Uncertain 2-tuple ordered weighted averaging (UTOWA) operator; Software project selection

I. INTRODUCTION

Multiple attribute group decision making (MAGDM) problems arise from many real-world situations [1]. In MAGDM analysis, the preference information on alternatives with regard to attributes provided by decision makers (DMs) is often aggregated to form a collective opinion. Based on the derived collective opinion, ranking alternatives or selecting the most desirable alternative(s) is obtained. A lot of research work for MAGDM problems has been conducted, one of the hot research topics is the use of linguistic approaches to solve MAGDM problems when DMs express their preferences in natural language because of the nature of the alternatives and their own vague knowledge over them [2, 3]. For some linguistic approaches, development of various operators is important to dispose or aggregate linguistic information directly.

Yager [4] developed an ordered weighted averaging (OWA) operator that has been used in a wide range of applications such as group decision making [5, 6]. Based on the OWA operator, many new operators have been developed to dispose linguistic information, such as the linguistic OWA (LOWA) operator [7, 8, 9, 10], the linguistic geometric averaging (LGA) operator [11], and so on. However, in practical MAGDM problems with linguistic information, DMs' preferences may be in the format of uncertain linguistic information because of time pressure, lack of knowledge, the DMs' limited attention and information processing capabilities [12, 13]. For instance, the DM can give uncertain linguistic information (e.g. "Slightly Good" or "Good") to express his/her preferences on an investment project because of the uncertainty of future market. To dispose the uncertain linguistic information, Xu developed several uncertain linguistic operators, such as the uncertain linguistic OWA (ULOWA) operator [12], the induced uncertain linguistic OWA (IULOWA) operator [14], and so on. These operators can be only used in the situation where attribute values are in the format of uncertain linguistic information and attribute weights are in the format of exact values in the MAGDM problem. Furthermore, Xu [15] also developed an uncertain linguistic weighted aggregation (ULWA) operator to solve the problem in which attribute values and attribute weights are both in the format of uncertain linguistic variables. It needs to be pointed out that the virtual linguistic term set in [15] has been extended to the infinite one and it is inconsistent with the original linguistic term set. Thus the aggregated result of uncertain linguistic information can not be found any position in the original linguistic term set and the result itself has less meaning. Besides, Herrera and Martinez [16] proposed a 2-tuple fuzzy linguistic representation model to deal with linguistic information. The 2-tuple is composed of a linguistic term and a numeric value. The main advantage of this representation model is that linguistic information can be disposed without loss of information. To dispose 2-tuples, an extended TOWA (ETOWA) operator [17] and uncertain

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2-tuple ordered weighted averaging (UTOWA) operator [13] have been developed, respectively.

The purpose of this paper is to propose an approach to solve the MAGDM problem, where attribute values are in the format of uncertain linguistic information and attribute weights are in the format of linguistic information. First, concepts and comparison laws of uncertain 2-tuple are given. Then, based on the UTOWA operator, alternative appraisal values with respect to all attributes for each DM are calculated, and all DMs' preferences are aggregated into a collective opinion. At last, ranking alternatives or selecting the most desirable alternative(s) is obtained according to the comparison laws.

The rest of this paper is organized as follows. In Section 2, basic concepts on linguistic information, 2tuple linguistic information, interval number and ordered weighted averaging (OWA) operator are introduced. In Section 3, concepts and comparison laws of two uncertain 2-tuples are given. In Section 4, an approach based on the UTOWA operator is presented to solve the MAGDM problem with uncertain linguistic information. Section 5 gives a numerical example on software project selection to illustrate the use of the proposed approach. Finally, Section 6 summarizes and highlights the main features of this paper.

II. PRELIMINARIES

For convenience of analysis, some basic concepts on linguistic information, 2-tuple linguistic information, interval number and ordered weighted averaging (OWA) operator are briefly reviewed.

A. Linguistic information

Many aspects of different activities in the real-world can not be assessed in a quantitative form, but rather in a qualitative one, i.e., with vague or imprecise knowledge. In that case, a better approach may be to use linguistic assessments instead of numerical values [16, 18-20]. In the following, the basic concept on linguistic information is given.

Suppose that $S = \{s_0, s_1, \dots, s_T\}$ is a finite and totally ordered discrete term set with odd cardinalities, where s_i denotes the *i*th linguistic term or label of S, $s_i \in S$, and T+1 is the cardinality of S. For example, a set of seven terms, S, could be $S = \{s_0 = \text{Very Poor} / \text{Very} \text{ Unimportant}, s_1 = \text{Poor } / \text{ Unimportant}, s_2 = \text{Slightly Poor} / \text{Slightly Unimportant}, s_3 = \text{Fair} / \text{Middle}, s_4 = \text{Slightly Good} / \text{Slightly Important}, s_5 = \text{Good} / \text{Important}, s_6 = \text{Very Good} / \text{Very} \text{Important}\}$. Ordinarily, it is required that set S has the following characteristics [16, 21]:

(1) The set is ordered: $s_i ">" s_j$, if i > j, where ">" denotes 'greater than'.

(2) There is the negation operator: $\text{Neg}(s_i)^{"}="s_j$, such that j = T - i, where "=" denotes 'equal to'.

(3) Maximization operator: $Max\{s_i, s_j\}^{"} = "s_i$, if $s_i^{"} \ge "s_j$, where " \ge " denotes 'greater than or equal to'.

(4) Minimization operator: $M \inf \{s_i, s_j\}^{"} = "s_i$, if $s_j^{"} \ge "s_j$.

B. 2-tuple linguistic representation model

The linguistic model has been applied successfully to solve many problems. However, there is a limitation which is the loss of information caused by the need to express the results in the initial linguistic term set that is discrete via an approximate process [16]. The loss of information implies a lack of precision in the final results from the computation of linguistic information. The 2-tuple linguistic representation model is effective to overcome this limitation [16]. That means that the linguistic information will be expressed by means of a 2-tuple, which is composed of a linguistic term and a numeric value assessed in [-0.5, 0.5].

Let $s_i \in S$ be a linguistic term. Then the function θ used to obtain the corresponding 2-tuple linguistic information of s_i is defined as

$$\theta: S \to S \times [-0.5, 0.5), \tag{1a}$$

$$\theta(s_i) = (s_i, 0), \quad s_i \in S.$$
(1b)

Let $S = \{s_0, s_1, \dots, s_T\}$ be a linguistic term set, $\beta \in [0,T]$ is a number value representing the aggregation result of linguistic symbol. Then the function Δ used to obtain the 2-tuple linguistic information equivalent to β is defined as

$$\Delta: [0,T] \to S \times [-0.5, 0.5), \qquad (2a)$$
$$\Delta(\beta) = (s_i, \alpha), \text{ with } \begin{cases} s_i, & i = \text{Round}(\beta), \\ \alpha = \beta - i, & \alpha \in [-0.5, 0.5), \end{cases}$$
(2b)

where 'Round' is the usual rounding operation. s_i has the closest index label to β and α is the value of the symbolic translation. If $S = \{s_0, s_1, \dots, s_T\}$ is a linguistic term set and (s_i, α) is 2-tuple linguistic information, then there exists a function Δ^{-1} , which is able to transform 2-tuple linguistic information into its equivalent numerical value $\beta \in [0, T]$. The function Δ^{-1} is defined as

$$\Delta^{-1}: S \times [-0.5, 0.5) \to [0, T], \quad (3a)$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta.$$
 (3b)

C. Interval number

For computation of uncertain linguistic operators hereinafter, we give preliminaries of interval numbers below.

Let $\overline{x} = [x^L, x^U]$ be an interval number, where $x^L, x^U \in \mathbf{R}, x^L \leq x^U$, and \mathbf{R} is a real number set. If $x^L = x^U$, then \overline{x} is a real number, i.e., $\overline{x} = x^L = x^U$. The rule for comparing interval numbers $\overline{x} = [x^L, x^U]$ and $\overline{y} = [y^L, y^U]$ is as follows:

- (1) $\overline{x} > \overline{y}$ if and only if $x^L > y^U$;
- (2) $\overline{x} = \overline{y}$ if and only if $x^L = x^U$ and $y^L = y^U$.

However, if there is an overlap between \overline{x} and \overline{y} , it is hard to tell whether interval number \overline{x} is greater than interval number \overline{y} or not. In order to rank interval numbers, we present a simple technique. In the technique, an interval can be transformed into a numerical point. A brief description of the technique is given below.

Let x^* be the center value of interval \overline{x} and ∇x^* be the error distribution of x^* , where $x^* = (x^U + x^L)/2$ and $\nabla x^* = (x^U - x^L)/2$. To rank interval numbers, interval number \overline{x} can be mapped into a crisp value by introducing the DM's risk attitude, i.e., the ranking value of each interval number can be defined as

$$\varphi_{\varepsilon}(\overline{x}) = x^* + \varepsilon \,\nabla x^*, \qquad (4)$$

where $\varepsilon \in [-1, 1]$. The parameter ε is an optimismpessimism degree, which denotes the DM's risk attitude. Usually, ε may be selected by the DM. Depending on the range of ε , the DM's risk attitude can be classified into three categories: 1) pessimistic, 2) neutral, and 3) optimistic. Since the possible ranges of ε may be $-1 \le \varepsilon < 0$, $\varepsilon = 0$, and $0 < \varepsilon \le 1$, which represents the DM's risk attitude being pessimistic, neutral, and optimistic, respectively. Therefore the corresponding ranges of ranking values $\varphi_{\varepsilon}(\bar{x})$ are $[x^* - \nabla x^*, x^*]$, x^* , and $[x^*, x^* + \nabla x^*]$, respectively. Obviously, given a risk attitude ε , interval number \bar{x} is greater than interval number \bar{y} , denoted $\bar{x} > \bar{y}$, if and only if $\varphi_{\varepsilon}(\bar{x}) > \varphi_{\varepsilon}(\bar{y})$, and \bar{x} is equal to \bar{y} , denoted $\bar{x} = \bar{y}$, if and only if $\varphi_{\varepsilon}(\bar{x}) = \varphi_{\varepsilon}(\bar{y})$.

Let $\overline{x} = [x^L, x^U]$ and $\overline{y} = [y^L, y^U]$ be positive interval numbers, we have the basic arithmetic of interval numbers as follows [22]:

(1) $\overline{x} + \overline{y} = [x^L + y^L, x^U + y^U];$ (2) $\overline{x} \times \overline{y} = [x^L y^L, x^U y^U];$ (3) $\overline{x} \div \overline{y} = [x^L / y^U, x^U / y^L];$ (4) $k\overline{x} = [kx^L, kx^U], k > 0.$

D. OWA operator

Yager [4] proposed an ordered weighted averaging (OWA) operator, which has been investigated in many documents [5-10, 12-14, 16, 17] and used in a wide range of applications.

An OWA operator of dimension *n* is a mapping, OWA: $R^n \rightarrow R$, that has an associated weight vector $w = (w_1, w_2, \dots, w_n)^T$ with the properties $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$, such that

OWA
$$(a_1, a_2, \dots, a_n) = \sum_{j=1}^n w_j b_j$$
, (5)

where b_j is the *j*th largest element in $(a_1, a_2, ..., a_n)$.

An important feature of the OWA operator is a recording step. During this step, an argument is ordered by its value. The OWA operator has some properties, such as monotonicity, idempotency, bounded, AA and commutativity [4].

III. UNCERTAIN 2-TUPLE AND ITS COMPARISON LAWS

A. Uncertain 2-tuples

Let $S = \{s_0, s_1, \dots, s_T\}$ be a linguistic term set, \overline{S} be the set of all uncertain linguistic variables and \widetilde{S} be the set of all uncertain 2-tuples, and then we give the following definitions.

Definition 1. Let $\overline{s}_i = [s_{k_i}, s_{l_i}]$ be an uncertain linguistic variable, $\overline{s}_i \in \overline{S}$, where s_{k_i} and s_{l_i} are the lower limit and upper limit, respectively, $s_{k_i}, s_{l_i} \in S$, $k_i, l_i \in \{0, 1, ..., T\}$, $k_i \leq l_i$.

Definition 2. Let $\tilde{s}_i = [(s_{k_i}, \alpha_k^i), (s_{l_i}, \alpha_l^i)]$ be an uncertain 2-tuple, $\tilde{s}_i \in \tilde{S}$, where (s_{k_i}, α_k^i) and (s_{l_i}, α_l^i) are the lower limit and upper limit, respectively; α_k^i and α_l^i are the difference values between respective calculated linguistic term and most approximate initial linguistic term, $s_{k_i}, s_{l_i} \in S$, $\alpha_k^i, \alpha_l^i \in [-0.5, 0.5)$, $k_i, l_i \in \{0, 1, \dots, T\}$, $k_i < l_i$ (or $k_i = l_i$ and $\alpha_k^i \le \alpha_l^i$).

Definition 3. Let $\tilde{s}_i = [(s_{k_i}, \alpha_k^i), (s_{l_i}, \alpha_l^i)]$ be an uncertain 2-tuple, $\tilde{s}_i \in \tilde{S}$, and $\bar{\beta}_i = [\beta^{L_i}, \beta^{U_i}]$ be an interval number representing the aggregation result of linguistic symbol, $\beta^{L_i}, \beta^{U_i} \in [0, T], \beta^{U_i} \ge \beta^{L_i}$. Then the function Δ used to obtain the uncertain 2-tuple equivalent to $\bar{\beta}_i$ is defined as

$$\Delta: [0, T] \to S \times [-0.5, 0.5),$$
 (6a)

$$\Delta(\beta^{L_i}) = \begin{cases} s_{k_i}, & k_i = \text{Round}(\beta^{L_i}), \\ \alpha_k^i = \beta^{L_i} - k_i, & \alpha_k^i \in [-0.5, \ 0.5), \end{cases}$$
(6b)

and

$$\Delta: [0, T] \to S \times [-0.5, 0.5),$$
 (7a)

$$\Delta(\beta^{U_i}) = \begin{cases} s_{l_i}, & l_i = \text{Round}(\beta^{U_i}), \\ \alpha_l^i = \beta^{U_i} - l_i, & \alpha_l^i \in [-0.5, \ 0.5), \end{cases}$$
(7b)

where 'Round' is the usual rounding operation, s_{k_i} and s_{l_i} are the closest labels to ' β^{L_i} ', and ' β^{U_i} ', respectively; ' α_k^i ', and ' α_l^i ', are values of the symbolic translation.

Definition 4. Let $\tilde{s}_i = [(s_{k_i}, \alpha_k^i), (s_{l_i}, \alpha_l^i)]$ be an uncertain 2-tuple, $\tilde{s}_i \in \tilde{S}$. Then there exists a function Δ^{-1} , which can be used to transform uncertain 2-tuple \tilde{s}_i into its equivalent interval numerical value $\bar{\beta}_i = [\beta^{L_i}, \beta^{U_i}]$. The function Δ^{-1} is given by

$$\Delta^{-1}: S \times [-0.5, 0.5) \to [0, T], \qquad (8a)$$

$$\Delta^{-1}(s_k, \alpha_k^i) = k_i + \alpha_k^i = \beta^{L_i}, \qquad (8b)$$

and

$$\Delta^{-1}: S \times [-0.5, 0.5) \to [0, T], \qquad (9a)$$

$$\Delta^{-1}(s_{l_i}, \alpha_l^i) = l_i + \alpha_l^i = \beta^{U_i}.$$
 (9b)

Definition 5. Let $\tilde{S} = {\tilde{s}_1, \tilde{s}_2, ..., \tilde{s}_n}$ be a collection of uncertain 2-tuples, then an UTOWA operator is defined as

UTOWA $(\tilde{s}_1, \tilde{s}_2, \dots, \tilde{s}_n) = \Delta(h_1 \overline{c}_1 + h_2 \overline{c}_2 + \dots + h_n \overline{c}_n)$, (10)

where $\overline{\boldsymbol{c}} = (\overline{c}_1, \overline{c}_2, \dots, \overline{c}_n)^T$ is an associated ordered interval number vector, and $\boldsymbol{h} = (h_1, h_2, \dots, h_n)^T$ is an associated weight vector, $h_i \in [0, 1]$, $\sum_{i=1}^n h_i = 1$.

Using the above rule for comparing interval numbers, each element \overline{c}_j ($\overline{c}_j \in \overline{c}$) is the *j*th largest label in the collection $\widetilde{s}_1, \widetilde{s}_2, \dots, \widetilde{s}_n$. Furthermore, the element of associated weight vector $\boldsymbol{h} = (h_1, h_2, \dots, h_n)^T$ is given by

$$h_i = \frac{\Delta^{-1}(w_i)}{\sum_{i=1}^n \Delta^{-1}(w_i)}, \quad i = 1, 2, \dots, n, \qquad (11)$$

where w_i is a weight concerning \tilde{s}_i , $w_i \in S =$, i = 1, 2, ..., n.

B. Comparison laws of uncertain 2-tuples

In order to set up comparison laws of uncertain 2tuples, we need some results from Probability Theory. Probability Theory has always allowed people to deal quantitatively with the lack of precision [9]. As for uncertain 2-tuple $\tilde{s}_i = [(s_k, \alpha_k^i), (s_l, \alpha_l^i)]$, from Eqs. (8) and (9), function Δ^{-1} can be used to make \tilde{s}_i return to its equivalent interval number $\beta_i = [\Delta^{-1}(s_k, \alpha_k^i)]$, $\Delta^{-1}(s_{l}, \alpha_{l}^{i})$]. Furthermore, the set of all possible values in $[\varDelta^{-1}(s_k, \alpha_k^i), \varDelta^{-1}(s_l, \alpha_l^i)]$ is regarded as a space $\Omega_{\tilde{s}}$ and each possible value in $[\varDelta^{-1}(s_k, \alpha_k^i)]$, $\Delta^{-1}(s_{l_i}, \alpha_l^i)$] is a point $\omega_{\tilde{s}}$ in $\Omega_{\tilde{s}}$. Each point $\omega_{\tilde{s}}$ only represents a real number $\eta_{\Omega_{\tilde{s}}}(\omega_{\tilde{s}})$ (or $\eta_{\tilde{s}}$ thereafter) which can be regarded as a continuous random variable $\eta_{\tilde{s}}$ taking on an infinite number of values. Two basic indexes are used to describe $\eta_{\widetilde{s}}$, i.e., expected value $E(\eta_{\tilde{s}})$ and variance $D(\eta_{\tilde{s}})$. The expected value of $\eta_{\tilde{s}}$, denoted $E(\eta_{\tilde{s}})$, is a weighted average of all possible values of $\eta_{\widetilde{s}}$. There are various ways to measure how far $\eta_{\tilde{s}}$ is from its expected value $E(\eta_{\tilde{s}})$, but the simplest one to work with algebraically is the use of the squared difference, i.e., variance $D(\eta_{\tilde{s}})$ [23]. It denotes a degree of uncertainty, i.e., the bigger $D(\eta_{\tilde{s}})$ is, the greater the degree of uncertainty will be.

Here, the expected value and variance on random variable $\eta_{\tilde{s}}$ are respectively given by

$$E(\eta_{\tilde{s}}) = \int_{\Delta^{-1}(s_{l_{i}},\alpha_{k}^{i})}^{\Delta^{-1}(s_{l_{i}},\alpha_{k}^{i})} x_{\eta_{\tilde{s}}} p(x_{\eta_{\tilde{s}}}) dx_{\eta_{\tilde{s}}} , \qquad (12)$$

$$D(\eta_{\tilde{s}}) = \int_{A^{-1}(s_{k_{i}},a_{k}^{i})}^{A^{-1}(s_{l_{i}},a_{k}^{i})} [x_{\eta_{\tilde{s}}} - E(\eta_{\tilde{s}})]^{2} p(x_{\eta_{\tilde{s}}}) dx_{\eta_{\tilde{s}}}.$$
 (13)

Since the outcome of a random experiment is assigned unique numeric value, random variable $\eta_{\tilde{s}}$ takes on each value with the same probability, i.e. $\eta_{\tilde{s}}$ has a uniform probability distribution. The probability distribution function of $\eta_{\tilde{s}}$ is given by

$$p(x_{\eta_i}) = \begin{cases} \frac{1}{\Delta^{-1}(s_{l_i}, \alpha_l^i) - \Delta^{-1}(s_{k_i}, \alpha_k^i)}, \Delta^{-1}(s_{l_i}, \alpha_l^i) \le x_{\eta_i} \le \Delta^{-1}(s_{k_i}, \alpha_k^i), \\ 0, & \text{others.} \end{cases}$$

Let $\tilde{s}_1 = [(s_{k_1}, \alpha_k^1), (s_{l_1}, \alpha_l^1)]$ and $\tilde{s}_2 = [(s_{k_2}, \alpha_k^2), (s_{l_2}, \alpha_l^2)]$ be two uncertain 2-tuples, $E(\eta_{\tilde{s}_1})$ and $D(\eta_{\tilde{s}_1})$ denote the expected value and variance of random variable $\eta_{\tilde{s}_1}$ obtained using Eqs. (12) and (13),

respectively; $E(\eta_{\tilde{s}_2})$ and $D(\eta_{\tilde{s}_2})$ denote the expected value and variance of random variable $\eta_{\tilde{s}_2}$ obtained using Eqs. (12) and (13), respectively. Then, comparison laws of uncertain 2-tuples are given as follows:

(1) if
$$E(\eta_{\tilde{s}_1}) > E(\eta_{\tilde{s}_2})$$
, then $\tilde{s}_1 ">"\tilde{s}_2$;
(2) if $E(\eta_{\tilde{s}_1}) = E(\eta_{\tilde{s}_2})$, then
a. if $D(\eta_{\tilde{s}_1}) = D(\eta_{\tilde{s}_2})$, then $\tilde{s}_1 "="\tilde{s}_2$;
b. if $D(\eta_{\tilde{s}_1}) < D(\eta_{\tilde{s}_2})$, then $\tilde{s}_1 ">"\tilde{s}_2$.

If $\alpha_k^i = \alpha_l^i = 0$ in \tilde{s}_i , $i = 1, 2, \dots, n$, it is obvious that the comparison laws of uncertain 2-tuples can be also used to comparison of uncertain linguistic variables.

IV. A MAGDM APPROACH BASED ON THE UTOWA OPERATOR

A brief description of the MAGDM problem with uncertain linguistic variables is given below.

Let $S = \{s_0, s_1, ..., s_T\}$ be a set of T+1 linguistic terms and \overline{S} be a set of uncertain linguistic variables. Let $X = \{X_1, X_2, ..., X_m\}$ $(m \ge 2)$ be a set of m discrete alternatives and $P = \{P_1, P_2, ..., P_n\}$ $(n \ge 2)$ be a set of n attributes. Let $D = \{d_1, d_2, ..., d_l\}$ $(l \ge 2)$ be a set of l DMs and $\mathbf{v} = (v_1, v_2, ..., v_l)^T$ be a weight vector of DMs, $v_k \in S$, k = 1, 2, ..., l. Let $\mathbf{w}^k = (w_1^k, w_2^k, ..., w_n^k)^T$ be a weight vector of attributes given by DM d_k $(d_k \in D)$, $w_j^k \in S$, k = 1, 2, ..., l, j = 1, 2, ..., n. Suppose that $\overline{A}^k = [\overline{a}_{ij}^k]_{m \times n}$ is an uncertain linguistic decision matrix, where \overline{a}_{ij}^k is the attribute value for alternative X_i with respect to attribute P_j provided by DM d_k , and it is in the format of uncertain linguistic variables, $\overline{a}_{ij}^k \in \overline{S}$.

The MAGDM problem concerned in this paper is to rank alternatives or to select the most desirable alternative(s) among a finite set X based on uncertain linguistic decision matrix $\overline{A}^{k} = [\overline{a}_{ij}^{k}]_{m \times n}$, weight vectors \boldsymbol{v} and \boldsymbol{w}^{k} . Here, we propose an approach to solve the MAGDM problem with uncertain linguistic variables using the UTOWA operator. Calculating steps of the approach are presented below.

Step 1. Normalize $\overline{A}^{k} = [\overline{a}_{ij}^{k}]_{m \times n}$ into $\overline{B}^{k} = [\overline{b}_{ij}^{k}]_{m \times n}$, the element in matrix \overline{B}^{k} , \overline{b}_{ij}^{k} , is given by

$$\overline{b}_{ij}^{k} = \begin{cases} \overline{a}_{ij}^{k}, & \text{for benefit attribute } P_{j}, \\ \text{Neg}(\overline{a}_{ij}^{k}), & \text{for cost attribute } P_{j}, \end{cases}$$
(15)

where $\overline{a}_{ij}^k \in \overline{S}$. Let $\overline{s} = [s_a, s_b]$, then $\operatorname{Neg}(\overline{s}) = [s_{a'}, s_{b'}], a' = T - b, b' = T - a$, and it is obvious that $\overline{b}_{ij}^k \in \overline{S}$.

Step 2. Build alternative appraisal value matrix $\tilde{Z} = [\tilde{z}_{ik}]_{m \times l}$ using the UTOWA operator. The element in matrix \tilde{Z} , \tilde{z}_{ik} , is given by

$$\begin{aligned} \tilde{z}_{ik} &= \text{UTOWA}(\overline{b}_{i1}^k, \overline{b}_{i2}^k, \dots, \overline{b}_{in}^k) \\ &= \varDelta(u_1^k \overline{c}_{i1}^k + u_2^k \overline{c}_{i2}^k + \dots + u_n^k \overline{c}_{in}^k), \quad i = 1, 2, \dots, m; \\ &\quad k = 1, 2, \dots, l, \end{aligned}$$

where $\overline{\boldsymbol{c}}_{ij}^{k} = (\overline{c}_{i1}^{k}, \overline{c}_{i2}^{k}, \dots, \overline{c}_{in}^{k})^{T}$ is an associated ordered interval number vector, and $\boldsymbol{u}^{k} = (\boldsymbol{u}_{1}^{k}, \boldsymbol{u}_{2}^{k}, \dots, \boldsymbol{u}_{n}^{k})^{T}$ is a weight vector of attributes associated with $\boldsymbol{w}^{k} = (w_{1}^{k}, w_{2}^{k}, \dots, w_{n}^{k})^{T}$. The element in vector \boldsymbol{u}^{k} , \boldsymbol{u}_{j}^{k} , is given by

$$u_{j}^{k} = \frac{\varDelta^{-1}(w_{j}^{k})}{\sum_{j=1}^{n} \varDelta^{-1}(w_{j}^{k})}, \quad j = 1, 2, \dots, n; \ k = 1, 2, \dots, l,$$
(17)

where
$$u_j^k \in [0,1]$$
, $\sum_{j=1}^n u_j^k = 1$, $k = 1, 2, ..., l$.

Step 3. Aggregate all the DMs' opinion \tilde{z}_{ik} into a collective alternative appraisal value \tilde{z}_i using the UTOWA operator. \tilde{z}_i is given by

$$\tilde{z}_{i} = \text{UTOWA}(\tilde{z}_{i1}, \tilde{z}_{i2}, \dots, \tilde{z}_{il})$$
$$= \varDelta(r_{1}\overline{y}_{i1} + r_{2}\overline{y}_{i2} + \dots + r_{l}\overline{y}_{il}), i = 1, 2, \dots, m, (18)$$

where $\overline{\mathbf{y}}_{ik} = (\overline{y}_{i1}, \overline{y}_{i2}, \dots, \overline{y}_{il})^T$ is an associated ordered interval number vector and $\mathbf{r} = (r_1, r_2, \dots, r_l)^T$ is a weight vector of DMs associated with $\mathbf{v} = (v_1, v_2, \dots, v_l)^T$. The element in vector \mathbf{r} , r_k , is given by

$$r_{k} = \frac{\Delta^{-1}(v_{k})}{\sum_{k=1}^{l} \Delta^{-1}(v_{k})}, \ k = 1, 2, \dots, l , \qquad (19)$$

where $r_k \in [0, 1]$, $\sum_{k=1}^{l} r_k = 1$.

Step 4. Rank all the alternatives or select the best one(s) using the comparison laws of uncertain 2-tuples.

The greater the value \tilde{z}_i is, the better the corresponding alternative X_i will be.

V. ILLUSTRATIVE EXAMPLE

The activities in software industry center on decision making in very complex situations [24]. When considering investing software projects, the issue of optimizing investments when value is measured in multiple attributes that are not easily commensurate is the topic of MAGDM. In this section, an example on software project selection is used to illustrate the use of the proposed approach. An investment company wants to invest a sum of money in the best opinion. There are four possible software projects (alternatives) to be considered: X_1 , X_2 , X_3 , X_4 . When making a decision, the attributes considered by the investment company include: (1) P_1 denotes the risk analysis; (2) P_2 denotes the technology analysis; (3) P_3 denotes the project team analysis; (4) P_4 denotes the market analysis. Among the four attributes, P_1 is of cost type, P_2 , P_3 and P_4 are of benefit type.

To select the desirable alternative(s), three experts (d_1 , d_2 and d_3) are invited to participate in the decision analysis. In the decision process, a linguistic term set S is used to express preference information, i.e., $S = \{ s_0 = \text{Very Poor} / \text{Very Unimportant}, s_1 = \text{Poor} / \text{Unimportant}, s_2 = \text{Slightly Poor} / \text{Slightly Unimportant}, s_3 = \text{Fair} / \text{Middle}, s_4 = \text{Slightly Good} / \text{Slightly Important}, s_5 = \text{Good} / \text{Important}, s_6 = \text{Very Good} / \text{Very Important} \}$. Suppose that the weight vector of experts provided by the investment company is $\mathbf{v} = (s_6, s_5, s_4)^{\text{T}}$. Furthermore, the preference information on alternatives with respect to attributes and the weight vectors of attributes provided by the three experts are presented as follows:

$$\overline{\mathbf{A}}^{1} = \begin{bmatrix} [s_{1}, s_{2}] & [s_{5}, s_{6}] & [s_{2}, s_{3}] & [s_{5}, s_{6}] \\ [s_{0}, s_{2}] & [s_{3}, s_{4}] & [s_{1}, s_{3}] & [s_{3}, s_{4}] \\ [s_{0}, s_{1}] & [s_{3}, s_{5}] & [s_{4}, s_{6}] & [s_{2}, s_{3}] \\ [s_{0}, s_{2}] & [s_{3}, s_{5}] & [s_{4}, s_{5}] & [s_{5}, s_{6}] \end{bmatrix}$$

$$\overline{\mathbf{A}}^{2} = \begin{bmatrix} [s_{2}, s_{3}] & [s_{6}, s_{6}] & [s_{3}, s_{4}] & [s_{4}, s_{5}] \\ [s_{1}, s_{2}] & [s_{5}, s_{6}] & [s_{3}, s_{4}] & [s_{2}, s_{4}] \\ [s_{2}, s_{3}] & [s_{4}, s_{5}] & [s_{5}, s_{6}] & [s_{3}, s_{5}] \\ [s_{1}, s_{3}] & [s_{4}, s_{5}] & [s_{5}, s_{6}] & [s_{5}, s_{6}] \end{bmatrix}$$

$$\overline{A}^{3} = \begin{bmatrix} [s_{3}, s_{4}] & [s_{6}, s_{6}] & [s_{4}, s_{5}] & [s_{6}, s_{6}] \\ [s_{1}, s_{2}] & [s_{5}, s_{6}] & [s_{3}, s_{4}] & [s_{4}, s_{6}] \\ [s_{2}, s_{3}] & [s_{4}, s_{5}] & [s_{5}, s_{6}] & [s_{4}, s_{5}] \\ [s_{1}, s_{2}] & [s_{4}, s_{6}] & [s_{6}, s_{6}] & [s_{3}, s_{5}] \end{bmatrix}$$
$$w^{1} = (s_{2}, s_{5}, s_{4}, s_{4})^{T},$$
$$w^{2} = (s_{2}, s_{5}, s_{4}, s_{5})^{T},$$
$$w^{3} = (s_{3}, s_{5}, s_{6}, s_{4})^{T}.$$

To obtain the ranking result of alternatives, a brief description of the resolution process using the proposed approach is given below.

Firstly, matrices \overline{A}^1 , \overline{A}^2 and \overline{A}^3 can be normalized into matrices \overline{B}^1 , \overline{B}^2 and \overline{B}^3 using Eq.(15), respectively, i.e.,

$$\overline{B}^{1} = \begin{bmatrix} [s_{4}, s_{5}] & [s_{5}, s_{6}] & [s_{2}, s_{3}] & [s_{5}, s_{6}] \\ [s_{4}, s_{6}] & [s_{3}, s_{4}] & [s_{1}, s_{3}] & [s_{3}, s_{4}] \\ [s_{5}, s_{6}] & [s_{3}, s_{5}] & [s_{4}, s_{6}] & [s_{2}, s_{3}] \\ [s_{4}, s_{6}] & [s_{3}, s_{5}] & [s_{4}, s_{5}] & [s_{5}, s_{6}] \end{bmatrix},$$

$$\overline{B}^{2} = \begin{bmatrix} [s_{3}, s_{4}] & [s_{6}, s_{6}] & [s_{3}, s_{4}] & [s_{4}, s_{5}] \\ [s_{4}, s_{5}] & [s_{5}, s_{6}] & [s_{3}, s_{4}] & [s_{2}, s_{4}] \\ [s_{3}, s_{4}] & [s_{4}, s_{5}] & [s_{5}, s_{6}] & [s_{3}, s_{5}] \\ [s_{3}, s_{5}] & [s_{4}, s_{5}] & [s_{5}, s_{6}] & [s_{5}, s_{6}] \\ [s_{4}, s_{5}] & [s_{5}, s_{6}] & [s_{3}, s_{4}] & [s_{4}, s_{5}] \\ [s_{4}, s_{5}] & [s_{5}, s_{6}] & [s_{3}, s_{4}] & [s_{4}, s_{6}] \\ [s_{3}, s_{4}] & [s_{4}, s_{5}] & [s_{5}, s_{6}] & [s_{4}, s_{5}] \\ [s_{4}, s_{5}] & [s_{4}, s_{5}] & [s_{5}, s_{6}] & [s_{4}, s_{5}] \\ [s_{4}, s_{5}] & [s_{4}, s_{6}] & [s_{6}, s_{6}] & [s_{3}, s_{4}] \\ [s_{4}, s_{5}] & [s_{4}, s_{6}] & [s_{6}, s_{6}] & [s_{3}, s_{5}] \end{bmatrix}.$$

Then, according to weight vectors \boldsymbol{w}^1 , \boldsymbol{w}^2 and \boldsymbol{w}^3 , three associated weight vectors \boldsymbol{u}^1 , \boldsymbol{u}^2 and \boldsymbol{u}^3 can be obtained using Eq. (17), i.e.,

- $\boldsymbol{u}^{1} = (0.133, 0.333, 0.267, 0.267)^{T},$
- $\boldsymbol{u}^2 = (0.125, 0.3125, 0.25, 0.3125)^T$,
- $\boldsymbol{u}^{3} = (0.167, 0.278, 0.333, 0.222)^{T}$.

Next, based on the obtained matrices (\overline{B}^1 , \overline{B}^2 and \overline{B}^3) and vectors (u^1 , u^2 and u^3), the alternative appraisal value matrix \tilde{Z} can be obtained using Eq. (16), i.e.,

 $\tilde{z} = \begin{bmatrix} [(s_4, -0.068), (s_5, -0.068)] & [(s_4, -0.313), (s_5, -0.438)] & [(s_4, 0.446), (s_5, 0.001)] \\ [(s_3, -0.401), (s_4, -0.001)] & [(s_3, 0.25), (s_5, -0.438)] & [(s_4, -0.055), (s_5, 0.223)] \\ [(s_3, 0.332), (s_5, -0.068)] & [(s_4, -0.438), (s_5, -0.188)] & [(s_4, -0.055), (s_5, -0.055)] \\ [(s_4, -0.134), (s_5, 0.466)] & [(s_4, 0.125), (s_5, 0.438)] & [(s_4, 0.112), (s_6, -0.555)] \end{bmatrix}$ According to weight vectors \boldsymbol{v} , associated weight vector \boldsymbol{r} can be obtained using Eq. (19), i.e., $\boldsymbol{r} = (0.4, 0.333, 0.267)^T$. Furthermore, based on obtained matrix \tilde{Z} and vector r, the collective alternative values of the four alternatives can be obtained using Eq. (18), i.e.,

$$\begin{split} \tilde{z}_1 &= [(s_4, -0.072), (s_5, -0.139)], \\ \tilde{z}_2 &= [(s_3, 0.354), (s_5, -0.324)], \\ \tilde{z}_3 &= [(s_4, -0.346), (s_5, -0.103)], \\ \tilde{z}_4 &= [(s_4, 0.051), (s_5, 0.492)]. \end{split}$$

Finally, according to the comparison laws of uncertain 2-tuples, the comparison result of the four alternatives is presented in Fig. 1. Thus a ranking order among the four alternatives is $X_4 \succ X_1 \succ X_3 \succ X_2$, and the best alternative is X_4 .

$$\begin{split} \widetilde{z}_1 &\longrightarrow E(\eta_{\widetilde{z}_1}) = 4.395 \\ \widetilde{z}_2 &\longrightarrow E(\eta_{\widetilde{z}_2}) = 4.015 \\ \widetilde{z}_3 &\longrightarrow E(\eta_{\widetilde{z}_3}) = 4.276 \\ \widetilde{z}_4 &\longrightarrow E(\eta_{\widetilde{z}_4}) = 4.772 \end{split}$$

Fig. 1. The comparison result of the four alternatives

VI. CONCLUSIONS

This paper presents a new approach to solve the MAGDM problem with uncertain linguistic information. In the approach, the comparison laws of uncertain 2tuples and UTOWA operator are given. Using the UTOWA operator, alternative appraisal values with respect to each DM can be obtained and all DMs' opinions can be aggregated into a collective opinion. According to the comparison laws, the ranking order of alternatives can be obtained. The proposed approach is theoretically sound and computationally simple for solving the MAGDM problem under uncertain linguistic environment. It is also a supplement or an extension of the existing approaches. Besides, the approach proposed in this paper is helpful to decision makers in software industry from economic perspective, that is, help people to make decisions in resource-limited situations.

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