

A Novel PSO Algorithm Based on Local Chaos & Simplex Search Strategy and its Application

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Abstract—To improve particle swarm optimization (PSO) computing performance, the centroid of particle swarm is firstly introduced in standard PSO model to enhance inter-particle cooperation and information sharing capabilities, then combining randomness and ergodicity of the strong chaotic motion and fast convergence of the simplex method, a novel particle swarm optimization algorithm with adaptive space mutation (CSM-CPSO) is proposed to improve local optimum efficiency and global convergence performance of PSO algorithm. Results of Benchmark function simulation and the material balance computation (MBC) in alumina production show the new algorithm has not only steady convergence and better stability, but also higher precision and faster convergence speed, and also can avoid the premature convergence problem effectively.

Index Terms—Particle Swarm Optimization, Centroid, Chaos, Simplex, Information Sharing

I. INTRODUCTION

In practical engineering applications, we encounter many computing problems in which a problem can be formulated as a global optimization problem of the objective function having nonlinear or multi-peaked characteristics. Since it is felt necessary in recent years to derive a global solution for nonlinear and multi-peaked optimization problems, global optimization is one of the most important topics in optimization. Particle Swarm Optimization (PSO) is one of the most powerful methods for solving unconstrained and constrained global optimization problems in recent years. PSO is a biologically inspired computational search and optimization method inspired by the social behavior of a swarm such as bird flocking or fish schooling and proposed by Eberhart and Kennedy in 1995[1-2]. It is computationally effective and easier to implement when compared with other mathematical algorithms and evolutionary algorithms. It has also fast converging characteristics and more global searching ability at the beginning of the run and a local searching near the end of the run. In recent years there have been a lot of reported works focused on the PSO. The PSO has been applied widely in the function optimization[3], artificial

neural networks' training[4-5], pattern recognition[6], fuzzy control[7] and some other fields[8-10].

However, while solving problems with more local optima, PSO has the premature convergence shortcomings of easily leading to fall into local solution, as well as slow local convergence speed and low convergence accuracy. Since its first publication, more and more research has been carried out so far to study the characteristics of PSO and to improve its convergence performance through parameter selection and optimization[11], mutating of particle [12], merging with other optimal algorithms[13-14] and other improved mechanisms[15-20], all these are on the basis of the standard PSO model. In this paper, based on the main framework of particle swarm optimization algorithm, from start to upgrade its computing performance, the centroid of particle swarm is firstly introduced in standard PSO model to enhance individual and population cooperation and information sharing capabilities, then combined randomness and ergodicity of the strong chaotic motion and fast convergence of the simplex algorithm, an effective particle swarm cooperative optimization (CSM-CPSO) algorithm is proposed. Results of benchmark functions experiment and engineering application show that new method abstains effectually the disadvantages of original methods, and has faster convergence speed and higher globally convergence ability than PSO and some improved PSO methods with both a better stability and a steady convergence.

II. PARTICLE SWARM OPTIMIZATION ALGORITHM

PSO is a stochastic optimization algorithm which maintains a swarm of candidate solutions, referred to as particles, they are members in the population, have their own positions and velocities, and they fly around the problem space in the swarms searching for the position of optima. PSO is initialized with a group of random particles and then searches for optima by updating generations. In every iteration, each particle is updated by following two "best" values. The first one is the best solution it has achieved so far. This value is called

pbest. Another "best" value tracked by the particle swarm optimizer is the best value, obtained so far by any particle in the population. This best value is a global best and called *gbest*. After finding the two best values, each particle of PSO updates its velocity and position according to its own and its companion's flying experience by the following equations

$$v_{id}^{k+1} = w \times v_{id}^k + c_1 \times rand() \times (p_{id} - x_{id}^k) + c_2 \times rand() \times (p_{gd} - x_{id}^k) \quad (1)$$

$$x_{id}^{k+1} = x_{id}^k + v_{id}^{k+1} \quad (2)$$

where $d = 1, 2, \dots, N$; $i = 1, 2, \dots, M$, M is the swarm size, and k indicates the iteration number; w is called inertia weight; c_1 and c_2 are two constant numbers called social or cognitive confidence respectively; $rand()$ is a function which can generate a random number between 0 and 1; p_u is the position at which the particle has achieved its best fitness so far, and p_{gd} is the position at which the best global fitness has been achieved so far; x_{id}^{k+1} is the next position of particle i according to its previous position and new velocity at time k ; v_{id}^{k+1} is new velocity of particle i at the k^{th} iteration. Every particle finds the optimal solution through cooperation and competition among the particles.

III. IMPROVED PSO ALGORITHM MODEL BASED ON LOCAL CHAOS & SIMPLEX SEARCH STRATEGY

The optimization performance of PSO depends on the abilities of exploration and exploitation of particles. In traditional PSO, every particle updates its next velocity and position only according to the velocity and position at the previous time, as well as individual best position and the best position of population, obviously, it lacks of collaboration and information sharing with other particles, most particles often contact quickly a local optimum position. Especially, as the iteration goes on, particles become very similar and almost have no ability to explore new area, and not easy for the particles to escape from it. In allusion to such circumstances, an improved PSO algorithm in combination with certain excellent characteristics and mechanisms of other optimization algorithms is proposed.

A. PSO Model Embeded Centroid (CPSO)

In PSO, because particles fly in the search space guided only by their individual experience and the best experience of the swarm. Here, the centroid of the swarm is introduced in traditional PSO model to enhance inter-particle cooperation and information sharing capabilities.

Let x_c^k and p_c^k be the centroid and the individual best centroid of all particles at time k , they can be defined respectively as follows

$$x_c^k = \sum_{i=1}^M x_i^k / M \quad (3)$$

$$p_c^k = \sum_{i=1}^M p_i^k / M \quad (4)$$

Then, x_c^k and p_c^k are introduced into formula (1), so the formula (1) can be rewritten into

$$v_{id}^{k+1} = w \times v_{id}^k + c_1 \times rand() \times (\lambda \times p_{id} + (1-\lambda) \times (\eta \times x_{cd}^k + (1-\eta) \times p_{cd}^k) - x_{id}^k) + c_2 \times rand() \times (p_{gd} - x_{id}^k) \quad (5)$$

where λ and η are two constants between 0 and 1, which are called weight adjustment factors.

The formula (5) and (2) are called as improved particle swarm optimization model with centroid (CPSO). So the running track of every particle is also related with position and individual best position of other particles, the cooperation and information sharing capabilities are enhanced effectively, the premature convergence of PSO can be decreased greatly.

B. PSO Algorithm Model Based on Local Chaos & Simplex Search Strategy

At the later of the searching process, particles often have closed to the region containing the global optimum solution, at this time, we only need to search carefully the optimum solution in a smaller area nearby the best position, but not other areas adequately. However, as each particle's velocity closes to zero, all the particles tend to equilibrium, they haven't enough abilities of exploration and exploitation, and not easy to escape from local optimum for the particles. To this question, based on the analyses of the advantages and disadvantages of each algorithm, then combining with ergodicity of the chaotic motion and fast convergence of the simplex algorithm, CSM-CPSO algorithm is proposed to improve global optimum efficiency and accuracy of particle swarm optimization algorithm. so they can achieve the purpose to exploit fully one's favorable conditions and avoid the unfavorable ones.

Let $x_g = (x_{g1}, x_{g2}, \dots, x_{gN})$ be the best position of all the particles at time K , where $x_{gd} \in region(d) = [sleft_d, sright_d]$, $d = 1, 2, \dots, N$, $gbest(K)$ is its best fitness value, during iteration process, for given integer $L > 0$, if

$$gbest(K) - gbest(K-L) < \varepsilon \quad (6)$$

where $K-L > 0$, ε is the variation accuracy of the best fitness value, then we can obtain S particles through following chaos motion.

According to Logistic map, the chaotic variables and the decision variables are mutually transformed in the following manner[21]:

(1) Mapping the decision variables x_k to chaotic variables cx_k located in the interval (0, 1) by using the following equation.

$$cx_{k,i} = (x_{k,i} - a_i) / (b_i - a_i) \quad (7)$$

(2) According to logistic map, then generate a chaotic sequence $\{cx_k\}$, $k = 1, 2, \dots, S$.

$$cx_{k+1,i} = u \times cx_{k,i} \times (1 - cx_{k,i}), \quad (8)$$

$$k = 1, 2, \dots, S, \quad i = 1, 2, \dots, N$$

(3) On the contrary, converting the chaotic variables cx_k to decision variables $x_k = (cx_{k1}, cx_{k2}, \dots, cx_{kN})$ using the following equation.

$$x_{k,i} = a_i + cx_{k,i} \times (b_i - a_i), x_{k,i} \in [a_i, b_i] \quad (9)$$

Then, we use these particles as S vertex of a convex polyhedron, and the value of each vertex is obtained by calculation, the maximum, hypo-maximum and minimum value are produced, and then a better solution is sought through the strategies of reflection, expansion and shrinking edges, using them to replace best or worst, which constitutes a new polyhedron. It can approached to a very small point with better performance through a lot of iterations. Finally, x_g' is obtained by simplex method, $gbest(K)$ ' is its best fitness value, if

$$gbest(K) - gbest(K') < \varepsilon', \text{ for } \forall \varepsilon' > 0 \quad (10)$$

we update x_g with x_g' , the new algorithm continues to run, otherwise we can adjust particle's search space with following formulas

$$region(d) = \begin{cases} [x_{gd} - \alpha * (1 - \alpha) * (x_{gd} - sleft_d), \\ x_{gd} + \alpha * (1 - \alpha) * (sright_d - x_{gd})] \\ \text{if } \max\{sright_d - sleft_d\} > \delta \\ [sleft_d, sright_d] \\ \text{others} \end{cases}$$

$$d = 1, 2, \dots, N \quad (11)$$

where α is space adjustment factor in (0, 1), δ is the given maximum of the interval length of x .

As the iteration goes on, the local exploration ability of each particle is greatly improved by shrinking particle's search space, when the maximum of interval length of x is less than the known δ , we can expand particle's search space and make particles adequately explore other areas to improve the ability of searching a global solution.

Then, the CSM-CPSO algorithm can be summarized as follows:

Step1: Initialize position and associated velocity of all the particles randomly in the N dimension space.

Step2: Evaluate the fitness value of each particle, and update the individual and global optimum positions.

Step3: For positive integer L , According to formulas (6) and (10), determine whether running local chaos & simplex searching or mutating search space.

Step4: Reassign $pbest$ and $gbest$ according to the current fitness values of particles: compare the p_i of every individual with its current fitness value. If the current fitness value is better, assign the current fitness value to p_i ; determine the current best fitness value in the entire population. If the current best fitness value is better than the p_g , then assign the current best fitness value to p_g .

Step5: For each particle, Update particle velocity according formula (5), Update particle position according formula (2).

Step6: Repeat Step2 - 5 until a stop criterion is satisfied or a predefined number of iterations is completed.

IV. COMPUTATION RESULTS AND ANALYSIS

To test the performance of the new algorithm, firstly, four benchmark functions are introduced to test the new model, then, it is applied to MBC in alumina production, the final results of the new model are compared with standard PSO and other improved methods.

A. Benchmark Function Simulation

(1) Ackley function

$$f(x) = -20 \exp \left[-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2} - \exp \left(\frac{1}{n} \sum_{i=1}^n \cos(2\pi x_i) \right) \right] + 20 + e, \quad -32 \leq x_i \leq 32 \quad (12)$$

(2) Griewank function

$$f(x) = \frac{1}{4000} \sum_{i=1}^n x_i^2 + \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1, \quad -600 \leq x_i \leq 600 \quad (13)$$

(3) Rastrigin function

$$f(x) = \sum_{i=1}^n (x_i^2 - 10 \cos(2\pi x_i) + 10), \quad -5.12 \leq x_i \leq 5.12 \quad (14)$$

(4) Rosenbrock function

$$f(x) = \sum_{i=1}^n [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2], \quad -30 \leq x_i \leq 30 \quad (15)$$

PSO, AM-PSO[17], AF-PSO[18], SM-PSO[19], SASM-PSO[20] and CSM-CPSO are respectively run for 50 times. The swarm sizes are set as 60 for PSO, AM-PSO, AF-PSO, SM-PSO, SASM-PSO, 40 for CSM-CPSO; $\alpha = \lambda = \eta = 0.5$, $\delta = 1e-2$, $\varepsilon = 1e-3$, $L = 100$, $c_1 = c_2 = 2.0$, $S = 20$, w is declined linearly from 0.9 to 0.4. Other parameters are set in Table 1. Comparisons of computation results among PSO, AM-PSO, AF-PSO, SM-PSO, SASM-PSO and CSM-CPSO are shown in Table 2, Fig. 1-4 show comparisons of convergence curve for two functions (only give a comparison of the convergence curve of PSO, AF-PSO, SM-PSO and CSM-CPSO).

TABLE 1. CONFIGURATION OF SOME PARAMETERS

| Function | Dimension | Generation | Precision |
|------------|-----------|------------|-----------|
| Ackley | 30 | 2000 | 5 |
| Griewank | 30 | 2000 | 0.1 |
| Rastrigrin | 30 | 2000 | 50 |
| Rosenbrock | 30 | 2000 | 100 |

TABLE 2. COMPARISONS OF THE COMPUTATIONAL RESULTS

| Function | Algorithm | Fitness value | | | | Success Rate (%) |
|------------|-----------|---------------|---------|---------|-----------|------------------|
| | | Best | Worst | Mean | Deviation | |
| Ackley | PSO | 8.e-16 | 17.357 | 8.5822 | 55.477 | 42 |
| | AF-PSO | 8.e-16 | 4.1437 | 0.3469 | 0.4952 | 100 |
| | AM-PSO | 8.e-16 | 2.8377 | 0.1898 | 0.3138 | 100 |
| | SM-PSO | 8.e-16 | 8.e-16 | 8.e-16 | 0 | 100 |
| | SASM-PSO | 3.9e-3 | 2.8519 | 0.3280 | 0.3572 | 100 |
| Griewank | CSM-CPSO | 8.e-16 | 9.78e-6 | 3.96e-7 | 3.51e-12 | 100 |
| | PSO | 0 | 1.0914 | 0.2998 | 0.1641 | 58 |
| | AF-PSO | 0 | 0.5225 | 4.95e-2 | 8.62e-3 | 90 |
| | AM-PSO | 0 | 0 | 0 | 0 | 100 |
| | SM-PSO | 0 | 0.4356 | 3.57e-2 | 8.16e-3 | 100 |
| Rastrigrin | SASM-PSO | 4.46e-11 | 0.0599 | 4.29e-3 | 1.18e-4 | 100 |
| | CSM-CPSO | 0 | 3.64e-6 | 1.38e-7 | 3.61e-13 | 100 |
| | PSO | 0 | 115.69 | 33.753 | 922.36 | 76 |
| | AF-PSO | 0 | 82.770 | 17.849 | 414.31 | 94 |
| | AM-PSO | 0 | 54.464 | 16.462 | 170.98 | 98 |
| Rosenbrock | SM-PSO | 0 | 88.769 | 29.76 | 701.64 | 72 |
| | SASM-PSO | 8.971 | 25.954 | 16.053 | 17.218 | 100 |
| | CSM-CPSO | 0 | 35.818 | 13.199 | 1.4e+2 | 100 |
| | PSO | 232.3 | 3696.8 | 872.96 | 423616 | 0 |
| | AF-PSO | 35.59 | 142.37 | 79.089 | 485.07 | 86 |
| | AM-PSO | 40.07 | 153.44 | 75.762 | 368.62 | 90 |
| | SM-PSO | 0.362 | 9 | 7.626 | 7.605 | 100 |
| | SASM-PSO | 2.2e-4 | 74.974 | 7.7548 | 142.83 | 100 |
| | CSM-CPSO | 6.9e-7 | 2.9e-6 | 2.16e-6 | 5.9e-13 | 100 |

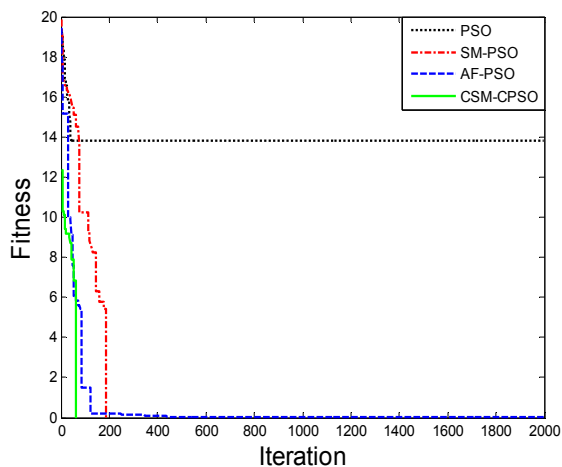


Figure 1. Comparisons of convergence curve for Ackley function

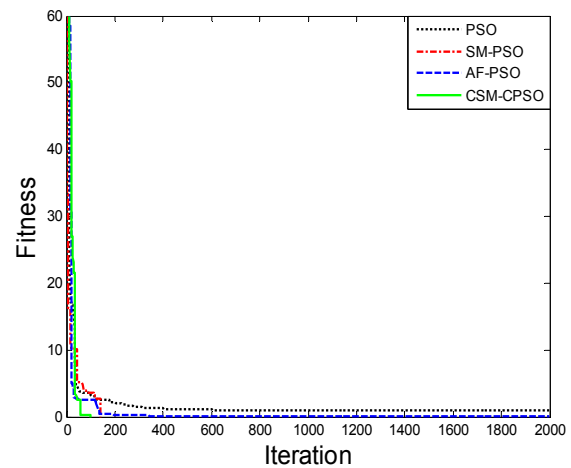


Figure 2. Comparisons of convergence curve for Griewank function

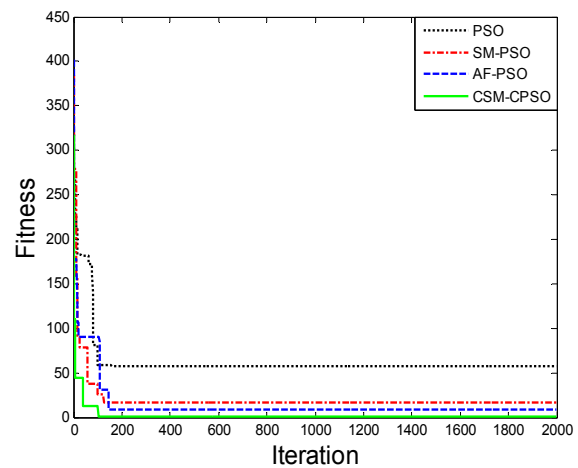


Figure 3. Comparisons of convergence curve for Rastrigrin function

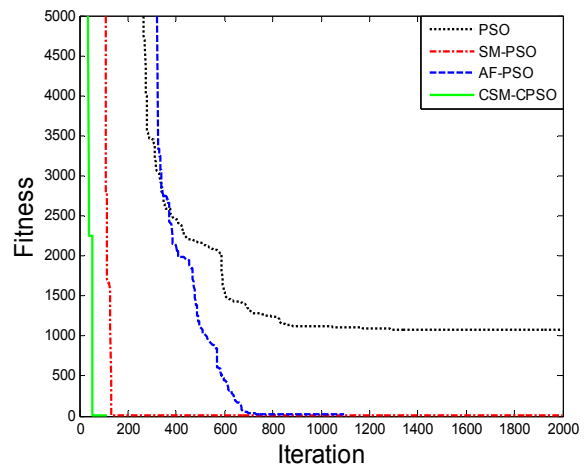


Figure 4. Comparisons of convergence curve for Rosenbrock function

From Table 2, It is easy to see that there are higher convergence accuracy and rate for CSM-CPSO than that for PSO, AM-PSO, AF-PSO, SM-PSO, SASM-PSO, from the mean and deviation in Table 2, CSM-CPSO has a better stability than PSO, AM-PSO, AF-PSO, SM-PSO except SASM-PSO for function Rastrigrin, The average success rate of CSM-CPSO reaches 100% for each function, and obviously better than PSO, AM-PSO and

AF-PSO. From Fig.1-4, we can see that CSM-CPSO has higher convergence performance than PSO, AF-PSO and SM-PSO, CSM-CPSO can effectively avoid falling into local optimum solution through inter-particle cooperation and information sharing and local chaos & simplex searching, and attain global better solution while other algorithms cannot. These show that CSM-CPSO has better optimization solving capability and faster convergence performance than PSO, AM-PSO, AF-PSO, SM-PSO, SASM-PSO.

B. MBC in Alumina Production Process

MBC is the core of alumina production, which is an important method for guiding the production and the technical design, although there are many technical projects to construct a new alumina plant, only through it, can we select the best technical process and production method, and achieve the purpose of the lowest cost and the lowest investment. The production process of alumina can be seen as a complex control system[22-23], and a lot of processes of which come down to the revert computation, each of these processes has a direct impact on the results of material balance calculation of the entire process, as a result, it results in calculating complexity of its material balance with tediousness.

Through the analysis and the actual deduction of material balance calculation of the entire process without the storage and transportation of limestone, lime burning process, and the composition of lime is known, the model is obtained, which satisfies seven equation and two balance relation formula: ① the conservation of additive soda quantity, ② the conservation of alumina, ③ the conservation of alumina of cycle mother liquor, ④ the conservation of caustic alkali of cycle mother liquor, ⑤ the conservation of alumina of red mud washing, ⑥ the conservation of caustic alkali of red mud washing, ⑦ the conservation of carbon alkali of red mud washing, ⑧ the balance relation between finished alumina hydroxide and aluminum in the roasting process, ⑨ the control relation of water quality in the entire flow.

The equations $\square - \square$ are objective functions bound to meet two constraint conditions \square and \square . Then the material balance calculation on the whole can be turned into solving a nonlinear multi-objective constrained optimization problem:

$$\begin{aligned} \min F(X) &= \min(f_1(X), f_2(X), \dots, f_7(X))^T \\ X &\in R^8 \\ R &= \{X \mid f_c(X) \leq 0\}, f_c(X) = (f_8(X), f_9(X))^T \\ X &= (x_1, x_2, \dots, x_8)^T, X \in R \subset E^8 \end{aligned} \quad (16)$$

where $F(X)$ is objective function vector, $f_c(X)$ is constraint vector, X is variable vector, $x_i (i = 1, \dots, 8)$ is respectively the quality of alumina in finished aluminum products, the quality of alumina in finished aluminum hydroxide products, the quality of additive soda in recombined process of mother liquid, the quality of alumina in red mud lotion, the quality of caustic alkali in

finished red mud lotion, the quality of carbon alkali in red mud lotion, the total quality of red mud lotion and the quality of water in the evaporation process.

Here, PSO, AM-PSO, AF-PSO, SASM-PSO and CSM-CPSO are respectively run for 50 times. The population sizes are set as 200 for PSO, AM-PSO and AF-PSO, 100 for SASM-PSO, and 40 for CSM-CPSO; The maximum evolution generation is set as 2000 for PSO, AM-PSO and AF-PSO, 1000 for SASM-PSO and CSM-CPSO; Other parameters of algorithms are set as above. The ranges of X 's value in multi-objective optimization problem is set in Table 3. Comparisons of computation results among PSO, AM-PSO, AF-PSO, SASM-PSO and CSM-CPSO are shown in Table 4, Fig. 5-12(only for PSO, AM-PSO, AF-PSO and CSM-CPSO).

TABLE 3. THE RANGE OF X 'S INITIAL VALUE

| Variable | x_1, x_2 | x_3, x_4, x_5 | x_6 | x_7 | x_8 |
|---------------|------------|-----------------|----------|-----------|------------|
| Initial range | [200, 600] | [5, 300] | [5, 200] | [5, 5000] | [100, 500] |

TABLE 4. COMPARISON OF COMPUTATION RESULTS OF PSO, AM-PSO, AF-PSO, SASM-PSO AND CSM-CPSO

| Function | Algor-ithm | Fitness value | | | | Succ - Rate (%) |
|---|------------|---------------|------------|------------|------------|-----------------|
| | | best | worst | mean | devia tion | |
| Multi-objective optimization problem (16) | PSO | 0.17956576 | 6.8599641 | 0.59293555 | 1.11078 | 72 |
| | AF-PSO | 0.17956574 | 1.6981267 | 0.37532116 | 0.13555 | 80 |
| | AM-PSO | 0.17956576 | 1.0322167 | 0.22759169 | 1.987e-3 | 96 |
| | SASM-PSO | 0.17957035 | 0.19593058 | 0.17990193 | 5.3502e-6 | 100 |
| | CSM-CPSO | 0.179565 | 0.179575 | 0.179572 | 8.5556e-12 | 100 |

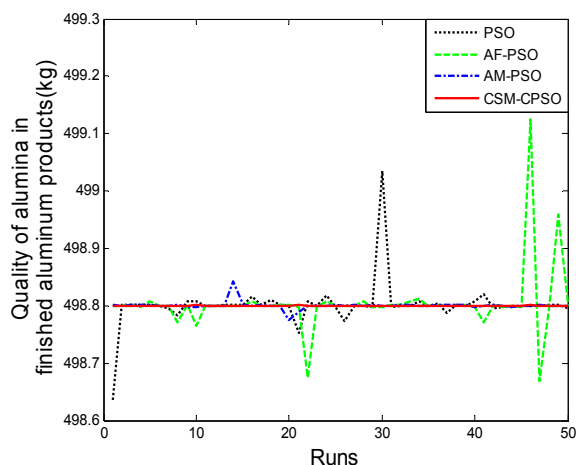


Figure 5. The quality of alumina in the finished aluminum products for 50 times iteration

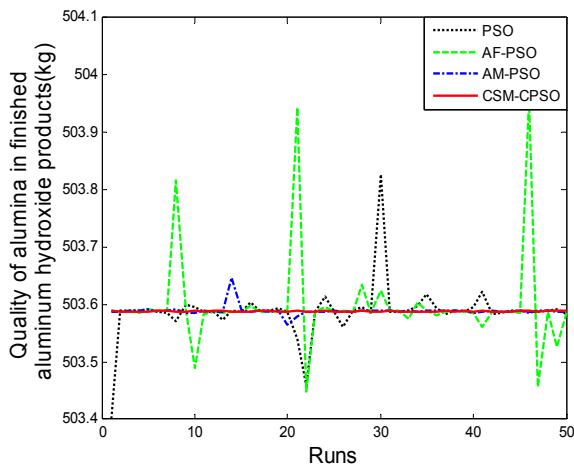


Figure 6. The quality of alumina in the finished aluminum hydroxide products for 50 times iteration

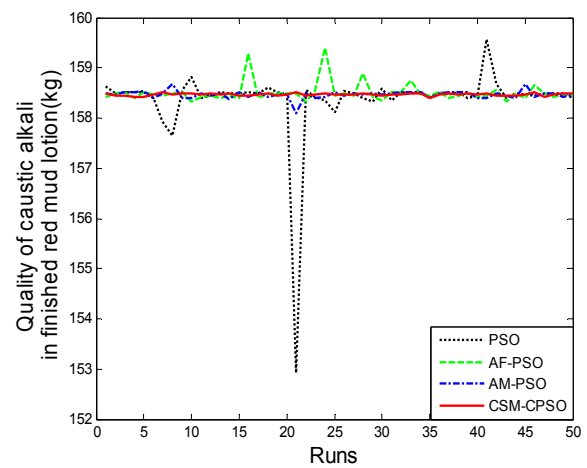


Figure 9. The quality of caustic alkali in the finished red mud lotion for 50 times iteration

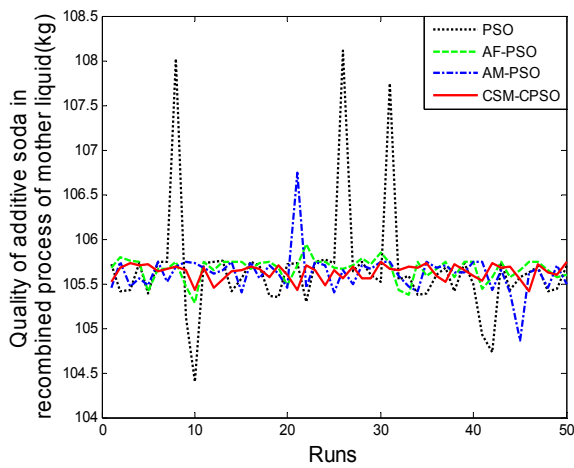


Figure 7. The quality of additive soda in the recombined process of mother liquid for 50 times iteration

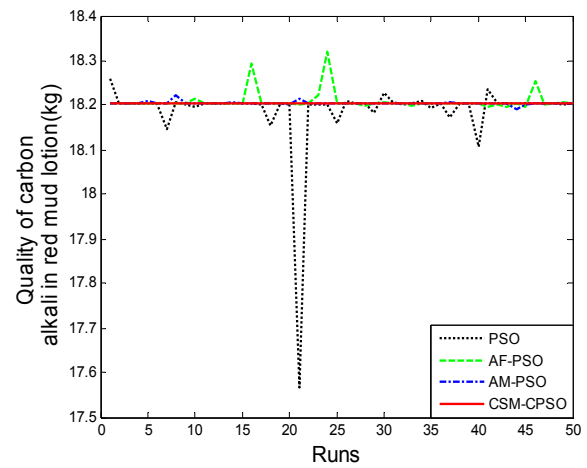


Figure 10. The quality of carbon alkali in the red mud lotion for 50 times iteration

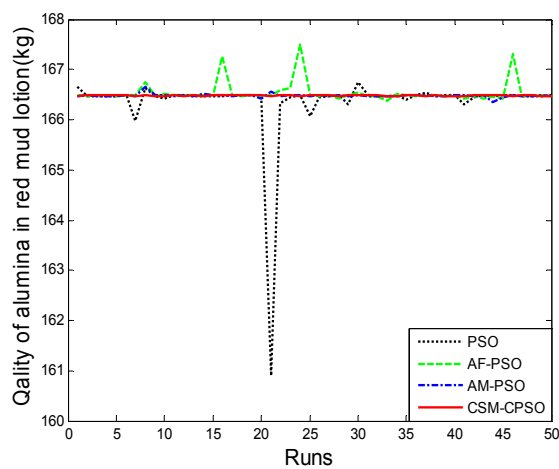


Figure 8. The quality of alumina in the red mud lotion for 50 times iteration

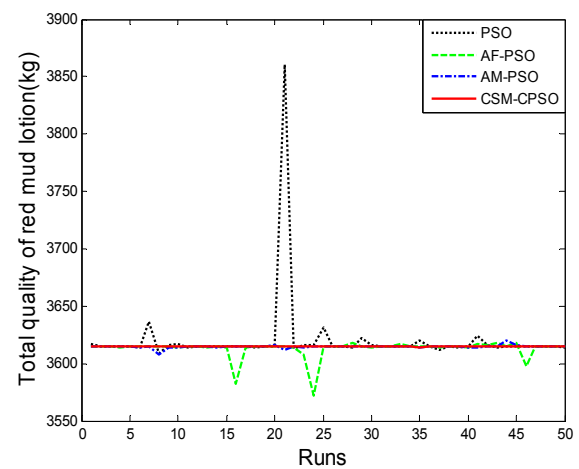


Figure 11. The total quality of red mud lotion for 50 times iteration

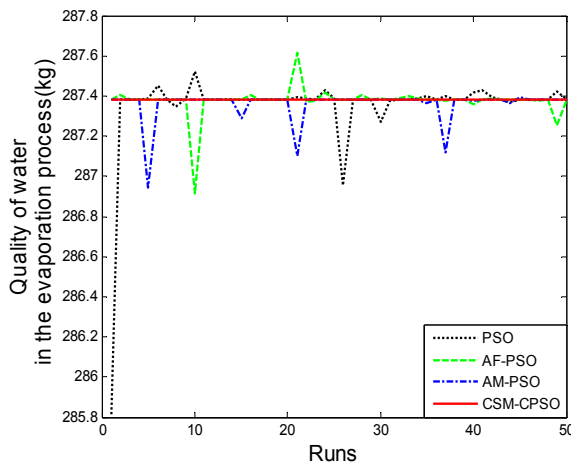


Figure 12. The quality of water in the evaporation process for 50 times iteration

From Table 4, we can see that there are higher convergence success rate and accuracy for CSM-CPSO and SASM-PSO than for the PSO, AF-PSO and AM-PSO, CSM-CPSO can especially avoid falling into local optimum solution. From mean and deviation in Table 4 and Fig. 5~12, CSM-CPSO has a better stability than for PSO, AF-PSO, AM-PSO and SASM-PSO, the average success rate of new algorithm is better than those of PSO, AF-PSO, AM-PSO. All these results demonstrate CSM-CPSO is more feasible and efficient than PSO, AF-PSO, AM-PSO and SASM-PSO.

V. CONCLUSION

Based on the analyses of the advantages and disadvantages of the PSO, chaos and simplex method, in order to avoid getting stuck in a local optimum to a certain degree and improve the optimal performance of PSO, a novel PSO algorithm model with adaptive space mutation embedded chaos & simplex method is given. CSM-CPSO retains the original advantages of PSO, the disadvantages were offset by the merits of chaos & simplex method and new model, the new algorithm can enhance individual and group collaboration and information sharing capabilities effectively through introducing the centroid, the exploration ability of CSM-CPSO is greatly improved, and the probability of falling into local optimum is efficiently decreased. Experiment and application results have proved that CSM-CPSO has not only the powerful ability to search the global optimal solutions, but also the ability to effectively avoid falling into local optimum solution. In the future, the application of the CSM-CPSO in widespread areas and theoretical analysis can be discussed further, and the convergence pattern, dynamic and steady-state performances of the algorithm can be improved more to specific complex optimization functions through combining with other optimal mechanisms.

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