Research on Algorithm and Model for Indefinite Multi-objective Decision Making

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***Abstract*—Decision Support System is playing an important role in computer science, technology and engineering. Intelligent decision-making is one of the current hotspots in the decision support system research. Intelligent decision-making methods and algorithms are one of the most important basics and key cores in intelligent information processing, intelligent pervasive computing and so on. In this paper, conduct the research to two kinds of indefinite multi-objective decision making question: the indefinite sector and the indefinite language. (1) In view of multi-attribute decision-making under linguistic setting, propose one new decision method. Firstly construct a range pole plan and introduce the policy-maker risk-preference weight. Then with three tuples (Limit low similarity, Risk degree, Risk-preference value) reflect the risk-degree existing in the decision-making process. At last, construct the risk-weighted similarity measure operator (RWSMO) to measure the risk balance similarity's size between each of decision schemes and the range pole plan. (2) In view of multi-attribute decision-making under the indefinite sector, propose one new decision method based on the multiple-valued intuitive fuzzy sets.**

***Index Terms*—Intelligent Method, Range pole plan, Risk balance similarity, Multiple-valued intuitive fuzzy sets, Iisomorphism, Indefinite multi-objective decision-making**

I. Constructing Policy-making Algorithm for the Linguistic Setting Value Multi-attributes Decision-making Based on Risk-weighted

The multi-objective decision making question is the current hotspots in decision-making science, systems engineering, management science and so on, also has a very extensive and important using in the practical application. The multi-objective decision making under that the policy-making attribute take the single real value has been already studied quite thoroughly. But in the practical application, because of incomplete information about attributes or attributes characteristics, we often cannot evaluate one plan on some attributes with precise values, for example: Automobile's performance, personnel's quality, equipment performance and so on. When carry on evaluating to plans on those attributes, we often use the linguistic value for example: worst, worse, bad, good, better, best and so on. Literature [1] ~ [4] has conducted the research to this kind of language decision-making. Often we can not use a precise linguistic value to evaluate plans on some attributes, but can only use a Linguistic Setting value to estimate approximately. This kind of multi-objective decision making is the indefinite multi-objective decision making. Recent years this kind of policy-making question receives some scholar's attention gradually, and has obtained a series of research results [5]~[11]. As a result of attribute evaluation value is indefinite, therefore has uncertainty in the decision-making process. Making decision in the indefinite condition, people will face with the risk. Because of the different degree of risk preferences, the different policy-maker may have the different evaluation value to the identical plan. The existing indefinite multi-objective decision making methods have not considered the policy-maker's risk-preference. In this part, the innovation is that the policy-maker's risk-preference will be considered in the decision-making process by constructing three tuples (Limit similarity, Risk degree, Risk preference value) to reflect the risk-degree existing in the decision-making process. Then construct risk-weighting similar measure operator (RWSMO) to measure similarity between the decision plan and the range pole plan. The risk-weighted similarity is closest to 1, the decision scheme is the optimalizing plan.

A. Preparation Knowledge

Suppose $A = \{A_1, A_2, \cdots, A_n\}$ as the decision plan set and $U = \{u_1, u_2, \cdots, u_m\}$ as the attribute set. When carry on the qualitative measurement to attributes, generally need suitable language evaluation scale.

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Therefore, we should establish language evaluation scale S = { $s_a \mid a=-t,\cdots,t$ } where s_a express language varity. Specially, with s_{-t} , s_t separately express the scale's low limit, the up limit. The commonly used language evaluation scale may be: Third-level evaluation scale S1= {bad, general, good}; Seven-level evaluation scale S2= {worst, worse, bad, general, good, better, best} or {smallest, smaller, small, general, big, bigger, biggest} and so on. Introduce symbol “$\prec$”to express relation in various linguistic values. Definite “bad $\prec$ general $\prec$ good”, in this formula, rank-number of a linguistic value setting the left of “$\prec$” is smaller “1” to that of a linguistic value setting the right of “$\prec$”, and the rank-number can be accumulated. For example: In this third-level evaluation scale “bad $\prec$ general $\prec$ good” may promote the rank-number of the linguistic value “good” is bigger 2 to that of the linguistic value “bad”. But cannot promote “bad $\prec$ good”, because “bad” is smaller two ranks to “good”, but is not one rank. May similarly definite in seven- level of evaluation scales S2“worst $\prec$ worse $\prec$ bad $\prec$ general $\prec$ good $\prec$ better $\prec$ best” or “smallest $\prec$ smaller $\prec$ small $\prec$ general $\prec$ big $\prec$ bigger $\prec$ bi ggest”. When carry on the evaluation on each attribute, we may select appropriate evaluation scale according to the characteristics of attributes as well as the policy-maker's knowledge, we may use the different evaluation scale to carry on the evaluation to the different attributes.

Symbol“$\tilde{v}_{ki}$” represent linguistic sector value. Evaluate the plan A_k ($k=1,2,\cdots,n$) on the attribute u_i $(i=1,2,\cdots,m)$, then get a linguistic sector value, recording $\tilde{v}_{ki}$ = $[v_1^{ki},v_2^{ki},\cdots,v_{p[k,i]}^{ki}]$, where v_j^{ki} $(j=1,2,\cdots,p[k,i])$ represent the linguistic value whose rank-number is “j”in the linguistic sector value “$\tilde{v}_{ki}$” . Linguistic values' ordering rule is: When for cost attribute, linguistic values carry on sorting according to the rank-number's descending sequence. When for efficient attribute, carry on sorting according to the rank-number's rising sequence. The symbol “ $p[k,i]$ ” expresses the number of linguistic values in the linguistic sector value“$\tilde{v}_{ki}$”. Thus may obtain the linguistic sector value decision-making matrix. The symbol “ v_L^{ki} ” represents the left limit value in the language sector value“ $\tilde{v}_{ki}$ ”,“ v_R^{ki} ” represents the right limit value in language sector value “$\tilde{v}_{ki}$”. “v_i^L” represents the left limit value in language evaluation scale which policy-maker use to evaluate each plan on the attribute “u_i”, “ v_i^R ” represents the right limit value in language evaluation scale that we use to evaluate each plan on the attribute “u_i”. Suppos the expert weight value of the attribute u_i is ω_i $(i=1,2,\cdots,m)$ .where $\omega_i \ge 0\,(i=1,2,\cdots,m)$ $\omega_1+\omega_2+\cdots+\omega_m=1$.

Definition 1. Suppose v_{0i} to be the best value of attribute u_i $(i=1,2,\cdots,m)$ (when only considering the attribute u_i, linguistic value v_{0i} is the optimalizing value in view of policy-making goal), then say that the plan $(v_{01},v_{02},\cdots,v_{0m})$ is the range pole plan, recording A_0.

TABLE I.
POLICY-MAKER RISK-PREFERENCE DEGREE TABLE

Risk evaluation scale	Risk-preference degree
$R_2=\{r_1^2,r_2^2\}$	$W=\{\lambda_1^2,\lambda_2^2\}$,where $\lambda_1^2+\lambda_2^2=1$, $0\le\lambda_1^2,\lambda_2^2\le1$
$R_3=\{r_1^3,r_2^3,r_3^3\}$	$W=\{\lambda_1^3,\lambda_2^3,\lambda_3^3\}$,where $\lambda_1^3+\lambda_2^3+\lambda_3^3=1$ $0\le\lambda_1^3,\lambda_2^3,\lambda_3^3\le1$
⋮	⋮
$R_p=\{r_1^p,\cdots,r_p^p\}$	$W=\{\lambda_1^p,\cdots,\lambda_p^p\}$,where $\lambda_1^p+\lambda_2^p+\cdots+\lambda_p^p=1$ $0\le\lambda_1^p,\lambda_2^p,\cdots,\lambda_p^p\le1$

Often one specific policy-maker displays certain risk-preference (risk-preference degree possibly base on policy-maker's individuality, objective environment or both). Therefore before carrying on the decision-making, the policy-makers may make the evaluation to their risk-preference degree and construct the policy-maker risk-preference degree table shown as Figure 1. Notice: On this table, in every risk evaluation scale, the risk degree along with the subscript increases. The value λ expresses risk-preference degree of policy-maker. The value of λ is more big, the policy-maker is more like to the corresponding risk degree. $p=\max(p[k,i],$ $(i=1,2,\cdots,m)$, $(k=1,2,\cdots,n)$) .

Define the following four operators:

(1) $\|u_i \dot{-} v_{0i}\|_L$ is equal to grading rank-number between the linguistic value v_{0i} and the linguistic value v_i^L $(i=1,2,\cdots,m)$.

(2) $\|u_i \dot{-} v_{0i}\|_R$ is equal to grading rank-number between the linguistic value v_{0i} and the linguistic value v_i^R $(i=1,2,\cdots,m)$.

(3) $\|u_i\| = \max(\|u_i \dot{-} v_{0i}\|_L, \|u_i \dot{-} v_{0i}\|_R)$, call $\|u_i\|$ as the biggest deviation of the attribute $u_i (i = 1,2,\cdots,m)$.

(4) $\|v_j^{ki} \dot{-} v_{0i}\|$ is equal to grading rank-number between the linguistic value v_{0i} and the linguistic value v_j^{ki}, $(i = 1,2,\cdots,m)$ $(j = 1,2,\cdots,p[k,i])$.

Definition2. Call $1 - \frac{\|v_L^{ki} \dot{-} v_{0i}\|_L}{\|u_i\|}$ as low limit similarity between the plan A_k and the range pole plan A_0 about the attribute“ u_i ”, recording ℓ_{ki}^L $(i = 1,2,\cdots,m)$, $(k = 1,2,\cdots,n)$.

Definition3. Call $1 - \frac{\|v_R^{ki} \dot{-} v_{0i}\|_R}{\|u_i\|}$ as up limit similarity between the plan A_k and the range pole plan A_0 about the attribute“ u_i ”, recording ℓ_{ki}^R $(i = 1,2,\cdots,m)$, $(k = 1,2,\cdots,n)$.

Obviously similarity between the plan A_k $(k = 1,2,\cdots,n)$ and the range pole plan A_0 about the attribute u_i $(i = 1,2,\cdots,m)$ situates between ℓ_{ki}^L and ℓ_{ki}^R. The policy-maker is unable to determine the position precisely with the existing information. This means that when carrying on the decision-making, the policy-maker must undertake the corresponding risk. People already constructed some methods to solve this kind of policy-making problem, for example: With the probability method, the fuzzy set method and so on. In this paper, the author will study this kind of policy-making issue from new angle: the policy-maker risk preference. Introduce three tuples (Limit low similarity, Risk degree, Risk preference value) to precisely quantify the risk-degree existing in the decision-making process and policy-maker's risk-preference degree to corresponding risk degree. These three tuples express the following meaning: When thinking that the similarity is not smaller than a value (Limit low similarity), the policy-maker needs to undertake the corresponding risk degree (Risk degree) and the policy-maker's risk-preference degree (Risk-preference degree) to the corresponding risk degree. Recording “Limit low similarity” in the three tuples as “ ℓ_j^{ki} ”, $\ell_j^{ki} = 1 - \frac{\|v_j^{ki} \dot{-} v_{0i}\|}{\|u_i\|}$ $(j = 1,2,\cdots,p[k,i])$. For example: suppose the attribute “ u_i ” is an efficient attribute and the evaluation scale is “worst $\prec$ worse $\prec$ bad $\prec$ general $\prec$ good $\prec$ better $\prec$ best”. Use this evaluation scale to measure attribute “ u_i ”. When $\tilde{v}_{ki}$ = [good, better, best], therefore in view of this risk (as evaluation value's of plan A_k on attribute u_i is uncertainty), the policy-maker can use the following risk evaluation scale $R_3 = \{r_1^3, r_2^3, r_3^3\}$ in the policy-maker risk-preference degree table to measure this risk degree. Then obtain the following three tuples: (4/6, r_1^3, λ_1^3)、(5/6, r_2^3, λ_2^3) and (1, r_3^3, λ_3^3).

Definition4. Call $\sum_{j=1}^{p[k,i]} \lambda_j^{p[k,i]} \ell_j^{ki}$ as risk-weighted similarity between the plan A_k and the range pole plan A_0 about the attribute“ u_i ”, recording“ ℓ_{ki} ”, obviously $0 \le \ell_{ki} \le 1$ $(i = 1,2,\ldots,m)$, $(k = 1,2,\cdots,n)$.

Definition5. Call $\sum_{i=1}^{m} \omega_i \ell_{ki}$ as risk-weighted similarity between the plan A_k and the range pole plan A_0. recording“ ℓ_k ”, obviously $0 \le \ell_k \le 1$, $(k = 1,2,\cdots,n)$. When $\ell_k = 1$, the risk-weighted similarity attrchs the biggest value. When $\ell_k = 1$, the risk-weighted similarity attrchs the smallest value.

B. Constructing Policy-making Algorithm

Risk-weighted similar measure operator (RWSMO)

RWSMO: $\tilde{S}^m \to R$, $\tilde{S}^m$ is a set which constructed by m-dimension vectors.

$$\begin{aligned}\ell_k &= RWSMO_{W,B}(\tilde{v}_{k1}, \tilde{v}_{k2}, \cdots, \tilde{v}_{km}) \\ &= \sum_{i=1}^{m} \omega_i \ell_{ki} \\ &= \sum_{i=1}^{m} \omega_i \sum_{j=1}^{p[k,i]} \lambda_j^{p[k,i]} \ell_j^{ki} \\ &= \sum_{i=1}^{m} \omega_i \sum_{j=1}^{p[k,i]} \lambda_j^{p[k,i]} \left(1 - \frac{\|v_j^{ki} \dot{-} v_{0i}\|}{\|u_i\|}\right)\end{aligned}$$

($k = 1,2,\cdots,n$)

Where $W = (\lambda_1^{p[k,i]}, \lambda_2^{p[k,i]}, \cdots, \lambda_{p[k,i]}^{p[k,i]})$ is $\ell_1^{ki}, \ell_2^{ki}, \cdots, \ell_{p[k,i]}^{ki}$ policy-maker risk- preference weight vector. $B = (\omega_1, \omega_2, \cdots, \omega_m)$ is the expert weight vector to the attributes $u_1, u_2, \cdots, u_m$. ω_i is the weight value of attribute u_i $(i = 1,2,\cdots,m)$. The symbol $\tilde{v}_{ki}$ represents a linguistic sector value. Measure the plan A_k ($k=1,2,\cdots,n$) on the attribute u_i $(i=1,2,\cdots,m)$ and get a linguistic sector value, recording $\tilde{v}_{ki}=[v_1^{ki},v_2^{ki},\cdots,v_{p[k,i]}^{ki}]$. v_{0i} expresses the attribute u_i $(i=1,2,\cdots,m)$ value to the range pole plan A_0. The symbol $p[k,i]$ expresses the number of linguistic value in the linguistic sector value"$\tilde{v}_{ki}$".

Policy-making algorithm

Before constructing the algorithm make the following work: determine each attribute is the cost attribute or the efficient attribute, as the following method: If the value of attribute u_i $(i=1,2,\cdots,n)$ and policy-making goal are revasly changing, the attribute u_i is an cost attribute and u_i =0; Otherwise attribute u_i is an efficient attribute, and u_i =1.

Multi-attribute Decision-making Algorithm

Input:

(1) attributes' value $\tilde{v}_k=(\tilde{v}_{k1},\tilde{v}_{k2},\cdots,\tilde{v}_{km})$, $k=(1,2,\cdots,n)$, where m is the number of attributes, n is the number of plans. Total $m\times n$ the linguistic sector values, The symbol "v_L^{ki}" represents the left limit value in the linguistic sector value"$\tilde{v}_{ki}$", "v_R^{ki}" represents the right limit value in linguistic sector value "$\tilde{v}_{ki}$".

(2) $(u_1,u_2,\cdots,u_n)$

(3) $\{\lambda_1^2,\lambda_2^2\}$, $\{\lambda_1^3,\lambda_2^3,\lambda_3^3\}$, …, $\{\lambda_1^p,\cdots,\lambda_p^p\}$, m, n.

(4) Expert weight vector($\omega_1,\omega_2,\cdots,\omega_m$)

Output: The optimalizing plan.

Begin

Step1: Separately extract the right limit value and left limit value of the linguistic value set $\tilde{v}_{1i}\cup\tilde{v}_{2i}\cup\cdots\cup\tilde{v}_{ni}$, respectively recording v_i^R and v_i^L $(i=1,2,\cdots,n)$.

Step2: $v_{0i}=v_i^R$ $(i=1,2,\cdots,n)$, then may extract the range pole plan $A_0=(v_{01},v_{02},\cdots,v_{0m})$.

Step3: According to the formula $\|u_i\|=\max(\ \|u_i\dot{-}v_{0i}\|_L,\ \|u_i\dot{-}v_{0i}\|_R\ )$ $(i=1,2,\cdots,m)$, extract the biggest deviation of each attribute value.

Step4: Extract the value $p[k,i]$, according to the following method: $p[k,i]$ equals the number of linguistic value in the linguistic sector value"$\tilde{v}_{ki}$". According to the formula $p=\max(p[k,i],\ (i=1,2,\cdots,m),\ (k=1,2,\cdots,n))$, get the value "$p$". Then policy-makers construct their risk-preference degree table according to their risk-preference.

Step5: According to the formula $\ell_j^{ki}=1-\dfrac{\|v_j^{ki}\dot{-}v_{0i}\|}{\|u_i\|}$, ($j=1,2,\cdots,p[k,i]$), get all possible limit similarity between the plan A_k and the range pole plan A_0 about the attribute u_i. According to the number of linguistic value in the linguistic sector value"$\tilde{v}_{ki}$", determine the corresponding risk evaluation scale (method: select the risk evaluation scale in which the number of risk-degree value is equal to $p[k,i]$). Then can construct the corresponding three tuples (ℓ_j^{ki}, Risk-degree, Risk preference value), $(i=1,2,\cdots,m)$ ($k=1,2,\cdots,n$) ($j=1,2,\cdots,p[k,i]$).

Step6: According to the formula $\ell_{ki}=\sum_{j=1}^{p[k,i]}\lambda_j^{p[k,i]}\ell_j^{ki}$, get risk-weighted similarity between the plan A_k and the range pole plan A_0 about the attribute u_i, $(i=1,2,\cdots,m)$ ($k=1,2,\cdots,n$).

Step7: According to the formula $\ell_k=\sum_{i=1}^{m}\omega_i\ell_{ki}$ $(k=1,2,\cdots,n)$, get risk-weighted similarity between the plan A_k and the range pole plan A_0.

Step8: A_k $(k=1,2,\cdots,n)$ carry on sorting according to the corresponding the descending sequence of value ℓ_k. The first plan is the best plan.

End

II. CONSTRUCTING POLICY-MAKING ALGORITHM FOR THE SECTOR VALUE MULTI-ATTRIBUTE DECISION-MAKING BASED ON THE MULTIPLE-VALUED INTUITIVE FUZZY SETS

In decision-making process, as the decision information is unprecise, incomplete and so on, in addition the policy-maker's information-handling capacity is limit. So sometimes gain the precise attribute's evaluation value is very difficult, even was impossible. Conducting the research to this kind of multi-objective decision making containing the incomplete information is further expansion to the research of the traditional multi-objective decision making question. For the fundamental research and solving actual problems, the sector multi-objective decision making question gains more and more people's attention. In this part, for the two major difficulties (As the information about attributes is indefinite, how to express the indefinite information; as the information has multi-channels, how to fuse the information into synthetic information.) in the multi-objective decision making process, introduce the multiple-valued intuitive fuzzy sets into the multi-objective decision making question, and construct one new algorithm for interval value multi-objective decision making based on the multiple-valued intuitive fuzzy sets.

A. Multiple-valued intuitive fuzzy sets information fusion

Definition6. (multiple-valued intuitive fuzzy sets [12]) suppose X as the given domain, then a multiple-valued intuitive fuzzy sets in X is:

$$A=\{<x,[\mu_1^A(x),\mu_2^A(x),\cdots,\mu_n^A(x)],[\gamma_1^A(x),\gamma_2^A(x),\cdots,\gamma_n^A(x)]>|x\in X\}$$

Where, $\mu_i^A(x):X\to[0,1]$ $\gamma_i^A(x):X\to[0,1]$

Represent the first "i" membership function $\mu_i^A(x)$ and the non-membership function $\gamma_i^A(x)$, and $\forall x\in X$, $0\le\mu_i^A(x)+\gamma_i^A(x)\le1,\ (i=1,\ 2,\ \cdots,n)$ is establishment.

Represent the multiple-valued intuitive fuzzy sets A as:

When given domain X is the continual space:

$$A=\int_A<[\mu_1^A(x),\mu_2^A(x),\cdots,\mu_n^A(x)],[\gamma_1^A(x),\gamma_2^A(x),\cdots,\mu_n^A(x)]>/x,x\in X\ ;$$

When given domain X is the discrete space, suppose $X=\{x_1,x_2,\cdots,x_m\}$:

$$A=\sum_{j=1}^{m}<[\mu_1^A(x_j),\mu_2^A(x_j),\cdots,\mu_n^A(x_j)],[\gamma_1^A(x_j),\gamma_2^A(x_j),\cdots,\gamma_n^A(x_j)]>/x_j,\ x_j\in X,$$

$j=1,2,\cdots,m$.

B. Degree of membership or non-degree of membership of multiple-valued intuitive fuzzy set information fusion

The degree of membership or non-degree of membership of multiple-valued intuitive fuzzy sets information fusion refers to fusing the degree of membership or non-degree of membership into one degree of membership or non-degree of membership, thus multiple-valued intuitive fuzzy sets will be transformed into a general intuitive fuzzy sets. Suppose A as a multiple-valued intuitive fuzzy set:

$$A=\{<x,[\mu_1^A(x),\mu_2^A(x),\cdots,\mu_n^A(x)],[\gamma_1^A(x),\gamma_2^A(x),\cdots,\gamma_n^A(x)]>|x\in X\}.$$

Following, construct several methods to fuse this multiple-valued intuitive fuzzy sets into a general intuitive fuzzy sets

$$B=\{<x,\mu_B(x),\gamma_B(x)>|x\in X\}.$$

(1) Median method

(1.1) The median of the material not grouped

Firstly group various degrees of membership or the non-degree of membership's value by ascending. Then, compute median:

When n is an odd number:

$$\mu_B(x)=\mu_{(n+1)/2}^A(x) \qquad \gamma_B(x)=\gamma_{(n+1)/2}^A(x)$$

When n is an even number:

$$\mu_B(x)=\frac{\mu_{n/2}^A(x)+\mu_{(n+1)/2}^A(x)}{2}$$

$$\gamma_B(x)=\frac{\gamma_{n/2}^A(x)+\gamma_{(n+1)/2}^A(x)}{2}$$

(1.2) The median of the material grouped

If the material has grouped, and establishes distribution list, then calculate the median using the distribution list, its formula is:

$$\mu_B(x)=L_\mu+\frac{i_\mu}{f_\mu}(\frac{n}{2}-c_\mu)$$

$$\gamma_B(x)=L_\gamma+\frac{i_\gamma}{f_\gamma}(\frac{n}{2}-c_\gamma)$$

In the formula: L_μ,L_γ — lower limit; i_μ,i_γ — interval; f_μ,f_γ —number of times; n—total degree; c_μ,c_γ —number of times smaller than the median.

(2) Simple weighted arithmetic average method

$$\mu_B(x)=\sum_{i=1}^{n}\lambda_i\mu_i^A(x)$$

$$\gamma_B(x)=\sum_{i=1}^{n}\lambda_i\gamma_i^A(x)$$

(3) Harmonic mean method

(3.1) Simple harmonic mean method

$$\mu_B(x)=\frac{1}{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{\mu_i^A(x)}}=\frac{n}{\sum_{i=1}^{n}\frac{1}{\mu_i^A(x)}}$$

$$\gamma_B(x)=\frac{1}{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{\gamma_i^A(x)}}=\frac{n}{\sum_{i=1}^{n}\frac{1}{\gamma_i^A(x)}}$$

(3.2) Weighting harmonic mean method

$$\mu_B(x)=\frac{1}{\sum_{i=1}^{n}\frac{\lambda_i}{\mu_i^A(x)}} \qquad \gamma_B(x)=\frac{1}{\sum_{i=1}^{n}\frac{\lambda_i}{\lambda_i^A(x)}}$$

Where $\lambda_1,\lambda_2,\cdots,\lambda_n$ satisfy the following conditions:

$$\sum_{i=1}^{n}\lambda_i=1,\ 1\ge\lambda_i\ge0\ (i=1,2,\cdots,n)$$

(4) Combination of mean values [13]

The combination mean value defers that many kinds of traditional mean values carry on the weighted average. Therefore, its formula is:

$$p_0=\sum_{i=1}^{n}\omega_i p_i$$

In the formula: P_0—combination mean value; p_i—different type mean value, where $i=1,2,\cdots,n$ (similarly hereinafter, omitted); ω_i —weight of various mean values, they satisfy $\sum_{i=1}^{n} \omega_i = 1$. Combination mean value may collect each kind of mean value the superiority, reflectS more accurately the information in the general level of data.

(5) Mathematics optimization method

Regarding each pair $< \mu_i^A, \gamma_i^A > \ (i = 1,2, \cdots, n)$ in the multiple-valued intuition fuzzy set. Each of them expresses information which obtains from the different attributes. When carry on the information fusion, a very natural idea is: In the information fusion process, as far as possible to make the modifiation to the existing information to a minimum. We may establish the following mathematical programming model according to this principle:

When is X a continual space:

$$\min(\int_X [\sum_{i=1}^{n} \left(\mu_i^A(x) - \mu_B(x)\right)^2 + \sum_{i=1}^{n} \left(\gamma_i^A(x) - \gamma_B(x)\right)^2]dx$$
$$s.t. 0 \le \mu_B(x) + \gamma_B(x) \le 1$$
$$0 \le \mu_B(x) \le 1$$
$$0 \le \gamma_B(x) \le 1$$
$$x \in X$$
$$(A1)$$

Through solving the optimize question $(A1)$, may obtain the various parameters' estimated value in the function $\mu_B(x)$ and $\gamma_B(x)$, $(x \in X)$.

When is X the discrete space:

$$\min\left(\sum_{i=1}^{n}\sum_{j=1}^{m} \left(\mu_i^A(x_j) - \mu_B(x_j)\right)^2 + \sum_{i=1}^{n}\sum_{j=1}^{m} \left(\gamma_i^A(x_j) - \gamma_B(x_j)\right)^2 \right)$$
$$s.t. 0 \le \mu_B(x_j) + \gamma_B(x_j) \le 1$$
$$0 \le \mu_B(x_j) \le 1$$
$$0 \le \gamma_B(x_j) \le 1$$
$$(j = 1,2, \cdots, m)$$
$$(A2)$$

Through solving the optimize question $(A2)$, may obtain the following values: $\mu_i^A(x_j), \gamma_i^A(x_j)$, $x_j \in X \ (j = 1,2, \cdots, m) \ (i = 1,2, \cdots, n)$.

C. Constructing decision method

Suppose $A = \{A_1, A_2, \cdots, A_n\}$ to the decision plan set and $U = \{u_1, u_2, \cdots, u_m\}$ to the attribute set. When carry on the qualitative measure to attributes, generally need suitable language evaluation scale. Therefore, we should establish language evaluation scale S = $\{ s_a \mid a = -t, \cdots, t \}$ where s_a express language Variables. Specially s_{-t} and s_t separately expresse the scale's low limit and the up limit. The commonly used language evaluation scale may be: Third-level evaluation scale S_1 ={bad, general, good}, seven-level evaluation scale S_2= {worst, worse, bad, general, good, better, best} or {smallest, smaller, small, general, big, bigger, biggest} and so on. Introduce mark"$\prec$" to express relation in various linguistic values. Definite"bad $\prec$ general $\prec$ good", in this formul rank-number of a linguistic value seting the left of "$\prec$" is smaller "1" to that of a linguistic value seting the right of "$\prec$", and the rank-number can be accumulated. For example: In this third-level evaluation scale"bad $\prec$ general $\prec$ good" may promote the rank-number of the linguistic value "good" is biger 2 to that of the linguistic value "bad". But can not promote "bad $\prec$ good", because "bad" is smaller two ranks to "good", but is not one rank. May similarly definite in seven- level of evaluation scales S_2"worst $\prec$ worse $\prec$ bad $\prec$ general $\prec$ good $\prec$ better $\prec$ best" or "smallest $\prec$ smaller $\prec$ small $\prec$ general $\prec$ big $\prec$ bigger $\prec$ biggest". Symbol $\tilde{v}_{ki}$ represent a value by measuring the attribute u_i $(i = 1,2, \cdots, m)$ of plan A_k ($k = 1,2, \cdots, n$) . In $\tilde{v}_{ki}$ various values' arrangement rule is: When u_i is cost-attribut, various linguistic values carry on sorting according to the descending sequence of rank number (or real number size), otherwise according to riseing sequence to carry on sorting. Recording the right margin value of $\tilde{v}_{ki}$ as $\tilde{v}_{ki}^R$.

Supposes $\tilde{v}_i = \bigcup_{k=1}^{n} \tilde{v}_{ki}$, the right margin value which records is, Records the right margin value of $\tilde{v}_i$ as $\tilde{v}_i^R$, the left margin value of $\tilde{v}_i$ as $\tilde{v}_i^L$.

Definition7. Policy-maker takes a value in an indefinite value, and supposes that attribute's value is not smaller than this value. Then calls this spot as the vacillation decision point.

Obviously as the vacillation decision point toward the right migration, the plan's performance is better in this attribute. When carries on the decision-making at this kind of suppose, policy-maker must withstand the bigger risk. Therefore the vacillation decision point's integer and policy-maker's risk manner has the relation.

Policy-makers risk preferences

People carry on the decision-making at the definite condition, because policy-maker risk preferences is different. With a programme, to a certain decision-makers policy makers it is a certain optimal programme, but in terms of other policy-makers it is't necessarily optimal programme. Therefore in the indefinite multi-objective decision making, considers policy-maker's risk preferences is very essential.

TABLE II.
POLICY-MAKER RISK - INCOME BALANCE TABLE

Risk scale	Risk-income balance
$R_2 = \{r_1^2, r_2^2\}$	$W_2 = \{\lambda_1^2, \lambda_2^2\}, where, \lambda_1^2 + \lambda_2^2 = 1$ $0 \le \lambda_1^2, \lambda_2^2 \le 1$
$R_3 = \{r_1^3, r_2^3, r_3^3\}$	$W_3 = \{\lambda_1^3, \lambda_2^3, \lambda_3^3\}, \lambda_1^3 + \lambda_2^3 + \lambda_3^3 = 1$ $0 \le \lambda_1^3, \lambda_2^3, \lambda_3^3 \le 1$
$\vdots$	$\vdots$
$R_p = \{r_1^p, r_2^p, \cdots, r_p^p\}$	$W_P = \{\lambda_1^p, \lambda_2^p, \cdots, \lambda_P^p\}, \lambda_1^p + \lambda_2^p + \cdots + \lambda_p^p = 1$ $0 \le \lambda_1^p, \lambda_2^p, \cdots, \lambda_p^p \le 1$

Note: On this table, in every risk evaluation scale, the risk degree along with the subscript increases. The value λ expresses risk-preference degree of policy-maker. The more the value of λ is big, the more policy-maker is like to the corresponding risk degree. $p = \max(p[k,i], (i = 1,2,\cdots,m), \ (k = 1,2,\cdots,n))$

Sector value discretization: Suppose M as the most district of span in all sector value (Before asks district of span, carry on standardized processing. Approach is that the right margin value and the left margin value respectively divide maximum value of this attribute). Supposes $g = \dfrac{M}{p-2}$, divide the various standardized sectors with g, then obtain a series of break points (including the right margin value and the left margin value), separately record as $v_1^{ki}, v_2^{ki}, \cdots, v_{p[k,i]}^{ki}$, and take these break points as vacillation decision point.

Using the intuitive fuzzy sets to express the indefinite information

In 3.1.We can express the uncertainty information of the value of $\tilde{v}_{ki}$ as a series of intuitive fuzzy values owing different degrees of risk, respectively records as $(<\mu_j^{ki}, \gamma_j^{ki}>, \lambda_j^{p[k,i]}) \ j = 1,2,\cdots,p[k,i]$.

Transformation method:

$$\mu_j^{ki} = \frac{\left\|\tilde{v}_j^{ki} \dot{-} \tilde{v}_i^L\right\|}{\left\|\tilde{v}_i^R \dot{-} \tilde{v}_i^L\right\|}$$

$$\gamma_j^{ki} = \frac{\left\|\tilde{v}_i^R \dot{-} \tilde{v}_{ki}^R\right\|}{\left\|\tilde{v}_i^R \dot{-} \tilde{v}_i^L\right\|} \qquad (A)$$

Where symbolic $\|x \dot{-} y\|$ represent distance or grading number between value x and value y.

With the information fusion methods which are constructed in the previous section, can transform these multiple-valued intuitive fuzzy sets into an ordinary intuitive fuzzy sets, record as:

$$B = \{<\mu_1, \gamma_1> / A_1, <\mu_2, \gamma_2> / A_2, \cdots, <\mu_n, \gamma_n> / A_n\}$$

According to the indefinite multi-objective decision making's characteristic, the weighted average method is a good fusion method:

$$\mu_k(A_k) = \sum_{j=1}^{p[k,i]} \lambda_j^{p[k,i]} \mu_j^{ki}(A_k)$$

$$\gamma_k(A_k) = \sum_{j=1}^{p[k,i]} \lambda_j^{p[k,i]} \gamma_j^{ki}(A_k) \qquad (B)$$

Speaking of each decision scheme, the most ideal result is $<1,0>$.

Intuitive fuzzy value similar measure method:

The massive literature has conducted the research to the intuitive fuzzy value similarity measure method (Vague similarity measure method). The intuitive fuzzy value's similar measure method may profit from the fuzzy similar measure method. In the literature [14] construct a method, as follows:

Suppose $x = [\mu_x, \gamma_x]$ and $y = [\mu_y, \gamma_y]$ as two intuitive fuzzy value in the given domain, the similar measure formula as follows:

$$S(x,y) = 1 - \sqrt{\frac{(\mu_x - \mu_y)^2 + (\gamma_x - \gamma_y)^2}{2}} \qquad (C)$$

Policy-making algorithm:

Step 1: Respectively appraisal each decision scheme according to each attributes and carries on standardized processing to each appraisal result.

Step 2: The policy-maker determines the policy-maker risk-income balance table according to the subjective and objective condition. Based on this carries on discretization processing to each sector value, and determines vacillation decision point of each indefinite value.

Step 3: With formula (A), can express the uncertainty information of the value of $\tilde{v}_{ki}$ as a series of intuitive fuzzy values owing different degrees of risk.

Step 4: With the information fusion methods which are constructed in the Part 2, can transform these multiple-valued intuitive fuzzy sets into an ordinary intuitive fuzzy sets.

Step 5: With formula (C), extract similarity between $<A_i, \mu_i, \gamma_i> (i = 1,2, \cdots, n)$ and $<1,0>$, record as ℓ_i.

Step 6: A_k $(k=1,2,\ \cdots\ ,n)$ carry on sorting according to the corresponding the descending sequence of value ℓ_k. The first plan is the best plan.

III. EXAMPLE

Consider one venture capital company which carries on the high tech project investment. Five alternative enterprises (plan) A_k ($k=1,2,\ \cdots\ ,5$) can be chosen. Carry on the appraisal from the angle of those enterprises ability's, firstly formulate seven appraisal targets (attribute) [15]: The marketing capacity (u_1), the managed capacity (u_2), productivity (u_3), technical ability (u_4), fund ability (u_5), risk exposure ability (u_6), the uniformity of enterprise strategy (u_7). Obviously, these seven attributes are the efficient attribute. Use the seven-level evaluation scale to measure these seven attributes, which is "worst $\prec$ worse $\prec$ bad $\prec$ general $\prec$ good $\prec$ better $\prec$ best" or "smallest $\prec$ smaller $\prec$ small $\prec$ general $\prec$ big $\prec$ bigger $\prec$ biggest". Might as well use the mark " $s_1 \prec s_2 \prec s_3 \prec s_4 \prec s_5 \prec s_6 \prec s_6 \prec s_7$ " to express the corresponding linguistic value. Then obtain the policy-making matrix (shown as Table 2). Try to determine the best enterprise.

TABLE III.
EXAMPLE'S POLICY-MAKING MATRIX

	u_1	u_2	u_3	u_4	u_5	u_6	u_7
A_1	$[s_2,s_3,s_4]$	$[s_3,s_4]$	$[s_3,s_4,s_5]$	$[s_5,s_6]$	$[s_5,s_6]$	$[s_6,s_7]$	$[s_3,s_4]$
A_2	$[s_5,s_6]$	$[s_4,s_5,s_6]$	$[s_6,s_7]$	$[s_6,s_7]$	$[s_3,s_4]$	$[s_3,s_4]$	$[s_5,s_6]$
A_3	$[s_2,s_3,s_4]$	$[s_3,s_4]$	$[s_6,s_7]$	$[s_5,s_6]$	$[s_3,s_4]$	$[s_6,s_7]$	$[s_3,s_4]$
A_4	$[s_6,s_7]$	$[s_4,s_5,s_6]$	$[s_3,s_4]$	$[s_5,s_6]$	$[s_3,s_4]$	$[s_5,s_6]$	$[s_3,s_4]$
A_5	$[s_4,s_5,s_6,s_7]$	$[s_5,s_6]$	$[s_3,s_4]$	$[s_3,s_4]$	$[s_6,s_7]$	$[s_5,s_6]$	$[s_4,s_5,s_6]$

Obviously the above 7 attributes are the efficient attribute, then $(u_1,u_2,\ \cdots\ ,u_7) = (1,1,\ \cdots\ ,1)$. Policy-makers carry on measuring their risk preference, Obtain the following policy-maker risk-preference degree table (shown as Table 3).

Carry on the above algorithm to this multi-objective decision making, Obtain the following result (shown as Table 4):

Obviously, $\ell_2 > \ell_4 > \ell_3 > \ell_5 > \ell_1$. Therefore, the enterprise is the best enterprise.

TABLE IV.
POLICY-MAKER RISK-PREFERENCE DEGREE TABLE

Risk evaluation scale	Risk-preference degree
$R_2=\{r_1^2,r_2^2\}$	$W=(\lambda_1^2,\lambda_2^2\}$ =（0.3，0.7）
$R_3=\{r_1^3,r_2^3,r_3^3\}$	$W=(\lambda_1^3,\lambda_2^3,\lambda_3^3)$ =（0.1，0.3，0.6）
$R_4=\{r_1^4,r_2^4,r_3^4,r_4^4\}$	$W=(\lambda_1^4,\lambda_2^4,\lambda_3^4,\lambda_4^4)=$ (0.05, 0.15, 0.25, 0.55)

TABLE V.
MULTI-OBJECTIVE DECISION MAKING RESULT

	A_1	A_2	A_3	A_4	A_5
ℓ_k	*0.6133*	*0.7800*	*0.6367*	*0.6383*	*0.6175*

IV. CONCLUSION

As society developing, the questions which the people face in the actual decision-making are getting more and more complex. Thus in many situations, as the existing information is insufficient, people often can't carry on precisely quantitative assessment to the preelection plans. So the very major part of policy-making questions are the linguistic setting or sector multi-objective decision making. In view of practical needs, these multi-objective decision-making gradually receive the numerous researcher's attention.

In the first partion, conduct the research on the linguistic setting multi-objective decision making question, constructe risk-weighted similar measure operator (RWSMO) to measure similar degree size between the preelection plans and the range pole plan, and introduce the risk preferences of the policy-maker to the decision marking. At last, construct one new decision method based on these. This decision method's merit: On the one hand, calculate easily; the final result has the explicit significance—similar degree, It's advantageous for the policy-maker to understand the model result. On the other hand, consider risk-preference of policy-maker in the decision-making process. So that decision-makers can independently decide some parameters in the decision-making model according to their own characteristics and decision-making environmental changing. The dialogue between decision-making model and decision-makers can make the result of decision-making meeting with specific decision-making environment.Thus let the policy-maker be satisfied to the policy-making result.

Innovation in the second partion is that: for the two major difficulties in the multi-objective decision making process, introduce the multiple-valued intuitive fuzzy sets into the multi-objective decision making question, then study the multiple-valued intuitive fuzzy set's information fusion and construct some methods to fuse the information included in the degree of memberships or non-degree of memberships of multiple-valued intuitive fuzzy set. At last, use the isomorphism mind to research the interval multi-objective decision making and construct one new algorithm for interval value multi-objective decision making based on isomorphism information fusion.

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