Methodology of Fuzzy Linear Symmetrical Bi-level Programming and its Application in Supply Chain Management

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***Abstract*—Fuzzy linear symmetrical bi-level programming is the most extensive problem in multi-level programming. A new method based on tolerance degree has been introduced in this paper. The method mainly concerns the modeling of complicated Supply Chain with bi-level Stackelberg structure. We analyze the reason lead to uncertainties in supply chain, summarize methods of dealing with uncertainties, and present a fuzzy bi-level programming modeling method which could not only describe the layered structure but also construct the uncertainties. An actual mathematical model based on fuzzy bi-level programming is applied in supply chain management. At last, a numerical example is given to prove the validity of the new method.**

***Index Terms*—supply chain management, bi-level program ming, fuzzy sets, Stackelberg decision making, algorithm**

I. INTRODUCTION

With the development of science and technology, consumption level of customers enhances constantly, competitive degree of different corporation divisions increases drastically. It leads to great transformation of market environment, as shown in paper [1]. In recent years, researches on supply chain have become the hotspot field in management [2].

In paper [3-7], decentralized supply chain with multi-level has been researched, where Stackelberg decision making method is applied to model layered structures, such as the structure between vendee and bargainor, structure between production and distribution, structure between storage and vendition. However, there are still some problems, such as Bullwhip Effect, uncertain factors[8] in requirement and manufacturing process, harmony of customers' satisfactory degree, link of different departments, etc. In this paper, we introduce fuzzy sets and numbers to describe the uncertainty, and integrate the layered structure in supply chain, present fuzzy multi-level programming modeling method to solve the problem.

The paper is organized as follows: In section II, uncertainties in supply chain is analyzed and describing method is summarized; In section III, fuzzy bi-level programming and its algorithm is given to solve a model; In section IV, hierarchy of supply chain is analyzed; In section V, we apply fuzzy bi-level programming modeling method to construct a supply chain model which comprises both layered structure and uncertainties; In section VI, an actual case of supply chain management is given to prove the validity of the method.

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II. UNCERTAINTIES IN SUPPLY CHAIN MANAGEMENT

Uncertainty is one of main causations lead to the difficulty in the supply chain management[9]. Abundant researches have done to analyze the mechanism. Uncertain factors could be concluded by three types:

- ***Stochastic: price fluctuation,timetable delay;***
- ***Subjective:rational countermeasure,disharmony;***
- ***Fuzzy: requirement forecast,batch order.***

In actual modeling in supply chain, dealing with uncertainties recurs to experiences and judgment of managers; whereas, to describe more complicated information, we should seek for more effective method. Fuzzy set is a mathematic method can be applied to reflect the essential attribute of uncertainty. Fuzzy number and fuzzy arithmetic is the important theory of fuzzy sets.

A. Fuzzy Set and Fuzzy Arithmetic

Definition 1[8] X is a set, if $\forall x \in X$,exist mapping $\mu_{\tilde{A}}$:

$$X \to [0,1] \quad (1\alpha)$$

$$x \mapsto \mu_{\tilde{A}}(x) \quad (1\beta)$$

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then , set $\tilde{A}=\{(x,\mu_{\tilde{A}}(x))\,|\,x\in X\}$ is called a fuzzy set of X , $\mu_{\tilde{A}}$ is called membership function of set $\tilde{A}$; and $\mu_{\tilde{A}}(x)$ is called grade of membership of $\tilde{A}$ at point x , $\mu_{\tilde{A}}(x)$ is also noted by $\tilde{A}(x)$.

Definition 2[9] X is a set, $\tilde{A},\tilde{B}$ are the fuzzy sets of X , the algorithm is:

(1) $\tilde{A}\cup\tilde{B}=\{(x,\mu_{\tilde{A}\cup\tilde{B}}(x))\,|\,\forall x\in X\}$,where

$$\mu_{\tilde{A}\cup\tilde{B}}(x)=(\tilde{A}\cup\tilde{B})(x)=\tilde{A}(x)\vee\tilde{B}(x),\forall x\in X \quad (2)$$

arithmetic operators " $\vee$ " means maximum of two membership degree.

(2) $\tilde{A}\cap\tilde{B}=\{(x,\mu_{\tilde{A}\cap\tilde{B}}(x))\,|\,\forall x\in X\}$, where

$$\mu_{\tilde{A}\cap\tilde{B}}(x)=(\tilde{A}\cap\tilde{B})(x)=\tilde{A}(x)\wedge\tilde{B}(x),\forall x\in X \quad (3)$$

arithmetic operators " $\wedge$ " means minimum of two membership degree.

(3) $\tilde{A}^c=\{(x,\mu_{\tilde{A}^c}(x))\,|\,\forall x\in X\}$,where

$$\mu_{\tilde{A}^c}(x)=\tilde{A}^c(x)=1-\tilde{A}(x),\forall x\in X \quad (4)$$

Definition 3[10] $\tilde{A}$ is a fuzzy set of X , $\forall\alpha\in[0,1]$, note

$$A_\alpha=\{x\,|\,\tilde{A}(x)\ge\alpha,x\in X\} \quad (5)$$

and call A_α is α -cut set or α -level set of $\tilde{A}$.

Definition 4[11] $\alpha\in[0,1]$, $\tilde{A}$ is a fuzzy set of X , note $\alpha\tilde{A}$ is the numerical multiply of α and $\tilde{A}$, and the membership function is defined by

$$\alpha\tilde{A}(x)=\alpha\wedge\tilde{A}(x). \quad (6)$$

Theorem 1[11] For all fuzzy set $\tilde{A}$ of X , $\alpha\in[0,1]$,

$$\tilde{A}=\bigcup_{\alpha\in[0,1]}[\alpha\wedge A_\alpha] \quad (7)$$

Theorem 1 is also called decomposed principle of fuzzy sets. Translation processing of common set into fuzzy set is described by the theorem.

Definition 5[11] $\tilde{A}$ is a fuzzy set of real number R ,

(1) $\tilde{A}$ is called normal set, if $\exists x\in R$,satisfies

$$\tilde{A}(x)=1; \quad (8)$$

(2) I is the total set of limitary closed intervals, for all $\alpha\in[0,1]$, $A_\alpha\in I$, that is, $A_\alpha=[A_\alpha^-,A_\alpha^+]$;

Then $\tilde{A}$ is called a **fuzzy number**.

Definition 6[11] Suppose l,r are arbitrary real numbers, if the membership function of fuzzy number $\tilde{A}$ satisfy:

$$\mu_{\tilde{A}}(x)=\begin{cases}1, & l\le x\le r\\ 0, & otherwise\end{cases} \quad (9)$$

Then $\tilde{A}$ is called a interval number, and noted by $\tilde{A}=[l,r]$.

Definition 7[11] $\bar{x},\bar{y}$ are interval numbers, the arithmetic is:

(1) $\bar{x}+\bar{y}=[x^-+y^-,x^++y^+]$; (10)

(2) $\bar{x}-\bar{y}=[x^--y^-,x^+-y^+]$ (11)

(3) $\bar{x}\times\bar{y}=[x^-y^-\wedge x^+y^+\wedge x^-y^+\wedge x^+y^-,$ $x^-y^-\vee x^+y^+\vee x^-y^+\vee x^+y^-]$ (12)

(4) $\bar{x}/\bar{y}=[x^-,x^+]\times[1/y^+,1/y^-],0\notin\bar{y}$ (13)

B. Uncertainties in Supply Chain Management

For the Supply Chain comprised with multi node corporations, there are usually complicated characters:

- Many participators form a complicated value network;
- Participators decentralize in geography, not locating in a country and always in all over the world. There are great differences in commerce civilization, technology level and management style;
- The logistic, fund and information in different participators are diversiform and reciprocal, having great otherness.

Suppliers in SC deliver goods by different ways, there exists uncertainties in implementing and producing process of production. The vehicle in transportation may have unpredictable instance, so there are all kinds of uncertain factors in supply chain management [12,13].

In the supply chain, almost all manufacturers have a certain amounts of inventory; from another perspective, uncertain factors, such as daily delivery delays, and its equipment failures, cancellation of orders will lead to stock increasing. The existence of uncertainty of supply chain inventory coordination is one of the obstacles, while the steady increasing in stocks become confrontational and control the uncertainty of the most primitive method. In fact, stock leads to production and demand fluctuations in the stability of the principal means.

How to identify sources of uncertainty, analyzing the impact of uncertainty on the supply chain mechanism, well-designed supply chain structure to reduce the impact of uncertainty, how to manage and control the uncertainty on the effective supply chain management is of great significance. For the supply chain uncertainty causes, is generally believed that cause and affect the dynamic characteristics of the supply chain needs of seven main factors: the demand forecasting, batch ordering, price fluctuations, lead time, rational responses, time delays and lack of coordination and so on.

In order to describe the uncertainties in modeling, abundant fruit has been got in literature. Pyke D (1993) [14] studied such an issue, he considered a manufacturer, finished product warehouse, retailers, consisting of three single-product production-distribution problems. Multi-stage inventory management inventory system appears in a class of important issues. Kouvelis P (1997)[15] researched a decentralized decision-making program, his seasonal differences in market pricing and in case of transfer pricing issues. Subrahmanyan S (1996)[16] studied the case of uncertainty in the demand for retail pricing - inventory policy. Using dynamic programming approach to the problem analyzed. Sudip B (2000)[17] studied the distribution system in multi-period profit maximization problem. On the assumption that a single product, which has a shorter shelf-life conditions, allowing prices to change overtime conditions, considering the product cycle inventory/pricing decision-making, the establishment of a multi-period model of distribution problems. Viswanathan S (2001)[18] considered a single-vendor, multi-product supply chain, a single buyer, the seller determine the price discount, the buyer at a specific time replenishment of inventory coordination problems. According to the second floor planning methods, the response problem is solved - the optimal replenishment period of the buyer and seller the best price discounts. Abdul JB (2003)[19] considered a warehouse, multiple retailers distribution system, also studied the strategy of decentralized decision-making to the parties.

Paper [21] give a modeling method to describe supply chain, especially for the relation between requirement and supply. The method focus on the construction of membership function and transformation of uncertain information into fuzzy sets, becomes the foundation of fuzzy supply chain modeling.

Fig 1 describes respectively the situation which member ship functions of requirement may be appropriately d_m , enormously exceed d_m , great possibility in $[d_l', d_n']$.

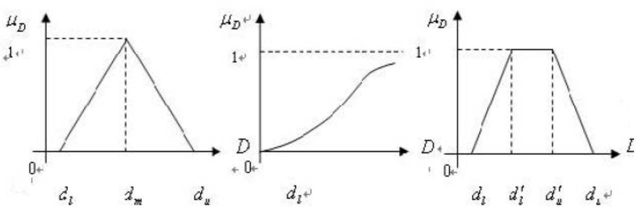

Figure 1. Membership function of requirement

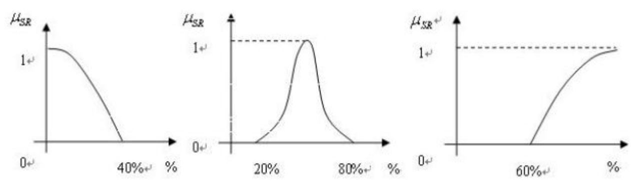

Figure 2. Membership function of supply

Fig 2 describes respectively situations which dealers supply may be trustless, moderate reliable, dependable.In bi-level supply chain, basal relationship is two nodes' producing-dealing, storage-dealing and buying-selling; ultimately, is the relationship between requirement and supply. So, once confirming membership functions of requirement and supply, we can describe uncertainties of supply chain in substance.

III. Fuzzy Bi-level Programming and Algorithm

A. Fuzzy Bi-level Programming

Mathematics Programming is called a Fuzzy Bi-level Programming. When it has two layered structure, decision making principle determined by Stackelberg game, and the objective and restriction have fuzzy information.

According to the difference of fuzzy information in objective and restriction, fuzzy bi-level programming has different model. In this paper, suppose the model is described by programming (14).

$$\begin{cases} \underset{x_1}{\tilde{\max}} \quad f_1(x_1, x_2) \\ s.t. \ \ g_0(x_1) \prec 0 \\ \underset{x_2}{\tilde{\max}} \quad f_2(x_1, x_2) \\ s.t. \ \ g_1(x_1, x_2) \prec 0 \\ \quad x_1, x_2 \geq 0 \end{cases} \tag{14}$$

Where, x_1, x_2 respectively represents the upper and lower's decision vector, and decision making principle is determined by Stackelberg game. $\tilde{\max}$ means fuzzy maximum, $\prec$ means fuzzy restriction $\leq$.

In order to solve the programming, based on Zimmermann's method, we transform the fuzzy into certain programming.

First transform fuzzy restriction into certain restriction. Suppose $D = (D_1, D_2, \cdots, D_p)$, $d = (d_1, d_2, \cdots d_p)$ respect-tively represent fuzzy restriction's allowance error, construct membership functions of fuzzy restriction:

$$\mu_{g_{0i}}(x_1) = \begin{cases} 1, & g_0(x_1)_i \leq 0 \\ 1 - \dfrac{g_0(x_1)_i}{D_i}, & 0 \leq g_0(x_1)_i \leq D_i \\ 0, & g_0(x_1)_i \geq D_i \end{cases} \tag{15}$$

$$\mu_{g_{1j}}(x_1, x_2) = \begin{cases} 1, & g_1(x_1, x_2)_j \leq 0 \\ 1 - \dfrac{g_1(x_1, x_2)_j}{d_j}, & 0 \leq g_1(x_1, x_2)_j \leq d_j \\ 0, & g_1(x_1, x_2)_j \geq d_j \end{cases} \tag{16}$$

where $i = 1, 2, \cdots, p$; $j = 1, 2, \cdots q$.

$g_0(x_1)_i \prec 0$ represents the *i*th upper fuzzy restriction, $g_1(x_1, x_2) \prec 0$ represents the *j*th lower fuzzy restriction. Equation (15) and (16) represent upper and lower level's fuzzy restriction by a membership function based on tolerance and membership degree. We can transform the fuzzy into accurate restriction. The transforming method is following:

$p_{1j}, p_{2j} \in [0,1]$, and i, j is decided by the number of fuzzy restriction g_0, g_1 , then the equivalent programming is equation (17).

$$\begin{cases} \tilde{\max_{x_1}} \quad f_1(x_1, x_2) \\ s.t. \;\; p_{1i} \le \mu_{g_{0i}}(x_1) \\ \tilde{\max_{x_2}} \quad f_2(x_1, x_2) \\ s.t. \;\; p_{2j} \le \mu_{g_{1j}}(x_2) \\ x_1, x_2 \ge 0 \end{cases} \tag{17}$$

Second transform fuzzy objective into certain objective. Construct the membership function of fuzzy objective:

$$\mu_{f_t}(x) = \begin{cases} 1, & f_t(x) \ge f_t^U \\ \dfrac{f_t(x) - f_t^L}{f_t^U - f_t^L}, & f_t^L \le f_t(x) \le f_t^U \\ 0, & f_t(x) \le f_t^L \end{cases} \tag{18}$$

where $f_t^U (t = 1, 2)$ are the optimal solution of following mathematics programming (19) and (20), suppose the optimal solution are $(x_{1,t}^U, x_{2,t}^U)$ and $(x_{1,t}^L, x_{2,t}^L)$, $t = 1, 2$.

$$\begin{cases} \max_x \quad f_t(x_1, x_2) \\ s.t. \quad g_0(x_1) \le D \\ g_1(x_1, x_2) \le d \\ g_2(x_1, x_2) \le 0 \\ x_1, x_2 \ge 0 \end{cases} \tag{19}$$

$$\begin{cases} \max_x \quad f_t(x_1, x_2) \\ s.t. \quad g_0(x_1) \le 0 \\ g_1(x_1, x_2) \le 0 \\ g_2(x_1, x_2) \le 0 \\ x_1, x_2 \ge 0 \end{cases} \tag{20}$$

The two solutions separately represent upper and lower 's decision-making goals when the tolerance is maximum and minimum. They can be considered as the satisfactory solution and reflect the degree of risk preference when the restriction diversification is best or worst.

Let $f_t^U = f_t(x_{1,t}^U, x_{2,t}^U)$, take f_t^U as a reference to equation (18), f_t^U represents the t-th ($t = 1, 2$) level's decision maker's Most likely objective. Let

$$f_1^L = \min\{ f_1(x_{1,1}^U, x_{2,1}^U),\ f_1(x_{1,1}^L, x_{2,1}^L),\ f_1(x_{1,2}^U, x_{2,2}^U),\ f_1(x_{1,2}^L, x_{2,2}^L)\} \tag{21}$$

$$f_2^L = \min\{ f_2(x_{1,1}^U, x_{2,1}^U),\ f_2(x_{1,1}^L, x_{2,1}^L),\ f_2(x_{1,2}^U, x_{2,2}^U),\ f_2(x_{1,2}^L, x_{2,2}^L)\} \tag{22}$$

Equation (21) and (22) describe the maximum and minimum value of upper and lower objective function based on tolerance degree. Equation (18) constructs the member ship function of fuzzy objective which is a Monotonic linear function related to the upper and lower fuzzy objective. For practical problems, the selection of f_t^L and f_t^U may consult by the decision-makers, according to directly define the light of experience reflects the decision-makers to grasp the actual problems

Last transform the fuzzy bi-level programming into certain mathematics programming (23).

$$\begin{cases} \max \min\ (\mu_{g_{0i}}, \mu_{f_1}) \\ s.t. \quad \mu_{g_{1j}}(x) \ge \sigma \\ \mu_{f_2}(x) \ge \sigma \\ g_2(x_1, x_2) \le 0 \\ x_1, x_2 \ge 0 \end{cases} \tag{23}$$

It also could be described by programming (24)

$$\begin{cases} \max \min \omega \\ s.t. \quad \mu_{g_{0i}}(x_1) \ge \omega \\ \mu_{f_1}(x_1) \ge \omega \\ \mu_{g_{1j}}(x_1) \ge \sigma \\ \mu_{f_2}(x) \ge \sigma \\ g_2(x_1, x_2) \le 0 \\ x_1, x_2 \ge 0 \end{cases} \tag{24}$$

To facilitate the calculation, the programming has the coequal form (25):

$$\begin{cases} \min \beta \\ s.t. \quad \mu_{g_{1j}}(x) \ge \sigma \\ \mu_{f_2}(x) \ge \sigma \\ \delta - \mu_{g_{0i}}(x) \le \beta \\ \delta - \mu_{f_2}(x) \le \beta \\ g_2(x_1, x_2) \le 0 \\ x_1, x_2 \ge 0 \end{cases} \tag{25}$$

where σ represents the approval expectation of upper decision makers.

B. Algorithm of Fuzzy Bi-level Programming

Based on the analysis in above, the solving process of fuzzy bi-level programming is given by following thoughts:

Lower decision makers give their approval expectation σ_0 ,then upper decision makers give δ_0 , and iterative computing gets approval solution $\delta^*, \sigma^*, x_1^*, x_2^*$, approval degree can be described by $\gamma = (\delta^*, \sigma^*)$. The steps of algorithm are:

Step 1 Solve mathematics programming (19) and (20), get objective value f_t^U and f_t^L , $t = 1, 2$;

Step 2 According to equation (18), construct upper decision makers and lower decision makers' membership function;

Step 3 Upper decision makers give their approval expectation δ_k, set $k = 0$;

Step 4 Lower decision makers give their approval expectation σ_t, set $\beta_0 = 0$ and iterative times $t = 0$;

Step 5 Solve mathematics programming (25);

Step 6 If $|\beta_{t+1} - \beta_t| \le \varepsilon_1$, where ε_1 is the upper decision makers' convergent precision, goto ***Step 8***; Other -wise goto ***Step 7***;

Step 7 Revise the lowers' approval expectation σ_t according to equation (9), let $t = t + 1$, goto ***Step 5***;

$$\sigma_{t+1} = \sigma_t + \theta_1(\min(\mu_{f_1^t}, \mu_{G^k}) - \delta_k) \quad (26)$$

where θ_1 is the adjustive coefficient of lowers' approval expectation σ;

Step 8 If $|\min(\mu_{f_1}, \mu_G) - \delta_k| \le \varepsilon_2$ (ε_2 is the allowance error between actual objective approval value and expectation value), then stop, the optimal solution is acceptable; Otherwise ,according to equation (10), revise the lowers' δ_k, let $k = k + 1$, goto ***Step 4***:

$$\sigma_{k+1} = \sigma_k + \theta_2(\min(\mu_{f_1^t}, \mu_{G^k}) - \delta_k) \quad (27)$$

where θ_2 is the adjustive coefficient of uppers' approval expectation δ.

IV. HIERARCHY OF SUPPLY CHAIN

A. Hierarchy of Supply Chain

Jayashankar's definition of the supply chain[22] as an independent or semi-independent economic entities, a network of systems, this network system in general is based on the "I" as the root of the two-way tree structure, in fact has gone beyond the "chain" range. From the principal point of view, the supply chain can be seen as around the core businesses, starting from procurement of raw materials, intermediate products and final products made of the final product delivered by the sales network into the hands of consumers will be raw materials suppliers, manufacturers, sub - sales, retailers, until the end-users together into a whole functional network chain structure model, in the supply chain, companies with upstream and downstream members connected to form a chain structure or network structure.

A typical supply chain hierarchy is shown as Fig 3.

In the supply chain to suppliers on the supplier down to the end user, including a number of links that may exist between the business and the chain cross. These links through information flow, logistics, capital flow controls, supply chain management operations as a whole. If considered separately in the nodes in the supply chain enterprises, closely linked to its place mainly in the upstream suppliers, downstream customers, the two closely related two nodes, the two supply chain, supply chain modeling structure is the most fundamental issues, more than two structural units constitute a complex network structure of two supply chains. From the operational level, the study of supply chain model, two supply chain is the most basic unit.

With the expansion of the supply chain system structure, the researchers found that, based on the autonomy of each node rational enterprise, supply chain agility to respond to rapid forms of organization of the supply chain is increasingly being adopted by enterprises of nodes. This is known as decentralized control structures (decentralized) supply chain system with master-slave hierarchical decision-making model features, which apply to use of master-slave hierarchical structure of decision theory to decentralized control of supply chain system modeling.

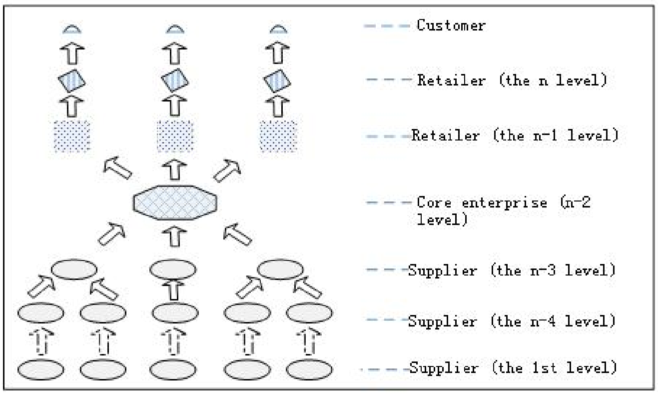

Figure 3. Hierarchy of supply chain

Douglas J (1996)[23] on the two supply chain, decentralized decision-making is divided into the buyer - the seller decentralized decision-making, the production - distribution of decentralized decision-making, inventory - three types of distribution of decentralized decision-making.

B. Measurement of Uncertainty with Bi-level Hierarchy of Supply Chain

As analyzed in section II, there exists many uncertainties in supply chain, such as transportation, logistic, need and delivery.

To customer needs, for example, frequently used fuzzy mathematics method, the customer needs to describe the semantics of estimates. In order to evaluate the uncertainty of customer demand, usually the factors that need to be considered are:

- The possible value range of customer needs and can take many different values. When the range of values increases, the value characteristics of the increasingly dispersed.
- Customer demand for different values of possibilities. When the demand for certain values of the possibilities is large, it means that the demand values scattered within the scope of these values is very likely that the demand for greater uncertainty.

Examination of these factors, access to information describing the supply chain, the degree of uncertainty in the simulation platform achieved. Literature [21] has proposed using supply chain uncertainty simulation platform - FuzzySCSim description and analysis of uncertainty in the supply chain technology. The FuzzySCSim, in regard to the description of uncertainty is used in the above modeling method based on fuzzy numbers, the main consideration of the demand side, supply-side uncertainty. The use of simulation techniques to characterize the degree of uncertainty in practice is an important link in the supply chain in this chapter mainly from the perspective of uncertain multi-planning model to consider the issue, it is no longer a measure of uncertainty in depth the issue.

C. Significance of a bi-level supply chain modeling

According to section A, for the analysis of the supply chain hierarchy, a complete supply chain includes not only many nodes in enterprises, such as a number of class suppliers, manufacturers, distributors, retailers, but also involve many industries to automobile manufacturing, for example, starting from the source, including mining, smelting, chemical, rubber cultivation, manufacture, assembly, retail, transportation and other industries. If the whole supply chain, regardless of what method, will face great difficulties, this article intends to use fuzzy theory, hierarchical master-slave fuzzy multi-planning model to study this approach in itself has a certain The mathematical complexity.

To simplify the problem, looking supply chain as a research object, namely, research links in the supply chain between two adjacent fuzzy bi-level programming model, these sectors can be Douglas J (1996)[3] decentralized decision-making suppliers, manufact- urers of the buyer, the seller cooperative relationship, or between manufacturers and distributors of production, distribution partnerships, or between the distributors and retailers, inventory, distribution, collaborative relation ship. The significance of the relationship between two levels is that for an enterprise, it occurs mainly closely linked to its upstream suppliers, downstream customers. Bi-level relationship is the most basic link in the supply chain. The following study between the manufacturers and distributors of production, distribution cooperative relations as the two master-slave hierarchical decision-making model of the object of study.

First distributors of their own inquire of consumers (customers or retailers) requirement and inform the manufacturer, so that manufacturers can be faster production, stocking, and manufacturers can quickly adjust the production line, a reasonable arrangement human and material resources. As the understanding of the demand, manufacturers can significantly reduce inventory, lower costs, timely supply, dealers can also be because the manufacturer to lower costs, making their procurement costs, and can more quickly meet consumer demand [24].

V. Fuzzy Bi-level Programming Modeling in Supply Chain

The nodes in supply chain can be classified by five departments: supplier, manufacturer, dealer, retailer and customer. Material, fund and information flow among them. Surveying the system, we find bi-level relationship is the fundamental structure of supply chain, such as producing-dealing, supplying-producing, buying-bargaining, etc. So we select a bi-level system of producing-dealing to construct the model, and fix on three elements of mathematic programming: decision vectors, objective functions, restriction conditions.

TABLE I.
SYMBOLS IN MODEL

Sets	***Explanation***	***Subscript***	***Explanation***
V	Sets of material suppliers	*v*	Material supplier
M	Set of manufacturers	*m*	Manufacturer
S	Set of dealers	*s*	Dealer
C	Set of customers	*c*	Customer
R	Set of material	*r*	Material
G	Set of production	*g*	Production

A. Decision Vector

Tab I is the symbols illumination in model; Tab II is the decision vectors in the bi-level programming; Tab III is the parameter illumination in bi-level programming.

TABLE II.
PARAMETER ILLUMINATION

Sets	***Explanation***
UIC_{sg}	Storage cost of production ***g*** in dealer ***s***
UTC_{scg}	Cost of production ***g*** carrying to customer ***c*** by dealer ***s***
UIC_{mr}	Storage cost of material ***r*** in manufacturer ***m***
UMC_{mg}	Cost of producing ***g*** in manufacturer ***m***
UTC_{msg}	Cost of production ***g*** carrying to dealer ***s*** by manufacturer ***m***
UIC_{mg}	Storage cost of production ***g*** in manufacturer ***m***

B. Objective Functions

Objective functions in bi-level supply chain is decided by the following factors: profit of dealers, manufacturers; cost of buying, stocking and delivering, material etc.

TABLE III.
DECISION VECTORS IN MODEL

Sets	***Explanation***
P_{scg}	Price of production ***g*** offered to customer ***c*** by supplier ***s***
P_{msg}	Price of production ***g*** provided to dealer ***s*** by manufacturer ***m***
P_{vmr}	Price of material ***r*** provided to manufacturer ***m*** by supplier ***v***
Q_{scg}	Amount of production ***g*** provided to customer ***c*** by dealer ***s***
Q_{msg}	Amount of production ***g*** provided to dealer ***s*** by manufacturer ***m***
Q_{vmr}	Amount of material ***r*** provided to manufacturer ***m*** by supplier ***v***
Q_{mg}	Amount of production ***g*** yielded by manufacturer ***m***
K_{sg}	Storage of production ***g*** in dealer ***s***
K_{mg}	Storage of production ***g*** in manufacturer ***m***
K_{mr}	Storage of material ***r*** in manufacturer ***m***

- upper decision makers' objective:

$$\tilde{\max_{s\in S}} \sum_{c,g} P_{scg} * Q_{scg} - \{\sum_{m,g} P_{msg} * Q_{msg} + \sum_{c,g} UTC_{scg} * Q_{scg}\} \quad (28)$$

- lower decision makers' objective:

$$\tilde{\max_{m\in M}} \sum_{m,g} P_{msg} * Q_{msg} + \sum_{c,g} P_{scg} * Q_{scg} - \{\sum_{r} P_{vmr} * Q_{vmr} + \sum_{r} UIC_{mr} * K_{mr} + \sum_{g} UIC_{mg} * K_{mg}\} \quad (29)$$

C. Restriction Conditions

Considering limitations in producing and delivering, we can construct the restriction conditions from following sides:

- flow limitation from manufacturers to dealers;
- flow and capacity limitation from dealers to customers;
- producing ability of manufacturers;
- material stocking limitation of manufacturers;
- production stocking limitation of manufacturers;
- production stocking limitation of dealers.

$$s.t.\begin{cases} \underline{Q}_{msg} \prec Q_{msg} \prec \bar{Q}_{msg} \\ \underline{Q}_{scg} \prec Q_{scg} \prec \bar{Q}_{scg} \\ Q_{mg} \prec \bar{Q}_{mg} \\ \underline{K}_{mr} \prec K_{mr} \prec \bar{K}_{mr} \\ \underline{K}_{mg} \prec K_{mg} \prec \bar{K}_{mg} \\ \underline{K}_{sg} \prec K_{sg} \prec \bar{K}_{sg} \end{cases} \quad (30)$$

Thus we get the fuzzy bi-level programming model in supply chain. The model has layered structure: upper and lower, the decision is making by Stackelberg game.

VI. Application in Numerical Example

To simplify the model, suppose there exists a dealer and a manufacturer in supply chain. Upper level (dealer) and lower level (manufacturer) control the amounts of production, that is, decision making vectors are Q_{scg} and Q_{msg}. Assume the price of production are $P_{scg} = 2$ and $P_{msg} = 1$ and ignore the other factors. Suppose fuzzy restriction allowance range is parameter d.

Where $d = (d_1, d_2, d_3, d_4) = (1.0, 3.0, 2.0, 1.5)$ .Based on the assumption above, the fuzzy bi-level programming model in producing-dealing system is:

A.Upper Decision Makers' Objective:

$$\tilde{\max} \quad 2Q_{scg} - Q_{msg} \quad (31)$$

B. Lower Decision Makers' Objective:

$$\tilde{\max} \quad Q_{scg} + 2Q_{msg} \quad (32)$$

C. Restriction Conditions:

$$s.t.\begin{cases} 3Q_{scg} + Q_{msg} \prec 27 \\ 3Q_{scg} + 4Q_{msg} \prec 45 \\ 3Q_{scg} - 5Q_{msg} \prec 15 \\ 3Q_{scg} - Q_{msg} \prec 21 \\ Q_{scg} + 3Q_{msg} \prec 30 \\ Q_{scg} \ge 0, Q_{msg} \ge 0 \end{cases} \quad (33)$$

Note objective functions of upper's and lower's are f_1 and f_2 ; x_1 and x_2 represents Q_{scg}, Q_{msg} . Applying the optimization algorithm in **section III**, results are:

TABLE IV.
ITERATIVE PROCESSING

Times	σ	μ_{f_1}	μ_{f_2}	x_1	x_2	δ	β
1	0.700	0.702	0.700	7.386	5.442	0.900	0.194
2	0.603	0.779	0.603	7.679	4.757	0.900	0.121
3	0.543	0.824	0.543	7.861	4.331	0.900	0.076
4	0.700	0.706	0.700	7.386	5.442	0.862	0.156
5	0.622	0.765	0.622	7.622	4.891	0.862	0.097
6	0.573	0.801	0.573	7.769	4.547	0.862	0.061
7	0.700	0.706	0.700	7.386	5.442	0.832	0.126
8	0.637	0.753	0.637	7.576	4.989	0.832	0.078
9	0.700	0.706	0.700	7.386	5.442	0.792	0.086
10	0.657	0.738	0.657	7.516	5.137	0.792	0.086
11	0.700	0.706	0.700	7.386	5.442	0.766	0.059
12	0.670	0.728	0.670	7.476	5.233	0.766	0.037

$$(x_{1,1}^U, x_{2,1}^U) = (8.042, 1.625), (x_{1,2}^U, x_{2,2}^U) = (4.800, 8.400)$$
$$(x_{1,1}^L, x_{2,1}^L) = (7.500, 1.500), (x_{1,2}^L, x_{2,2}^L) = (3.000, 9.000)$$
$$f_1^U = 14.46,\ f_1^L = \min\{13.500, -3.000, 1.200\} = -3.0$$
$$f_2^U = 21.60,\ f_2^L = \min\{21.000, 10.500, 11.290\} = 10.5$$

The computing process is showing by Tab IV.

So, optimal decision is

$$(Q_{scg}, Q_{msg})^* = (x_1^*, x_2^*) = (7.476, 5.233)$$

Optimal satisfactory degree is

$$\mu^* = (\mu_{f_1}, \mu_{f_2}) = (0.728, 0.670)$$

Optimal value is $f^* = (f_1^*, f_2^*) = (9.719, 17.941)$

Where $\delta_0 = 0.7, \omega_0 = 0.9, \varepsilon_1 = 0.06, \varepsilon_2 = 0.04$, $\theta_1 = \theta_2 = 0.5$,and $\mu^* = (\mu_{f_1}, \mu_{f_2}) = (0.728, 0.670)$.

VI. Conclusion

Supply Chain always have both level characteristics and fuzzy characteristics features. Fuzzy bi-level program ming is effective to modeling supply chain which has both layered and uncertain structure. How to construct the model can describe the two features is very important. In this paper, we give an actual model which describes the relationship in producing-dealing and present an algorithm to solve the model. The computing process of the numerical example testifies the validity of the algorithm.

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