

An Approximate Optimal Damping Control Method for Nonlinear Time-delay Systems with Disturbances

De-xin Gao^{1,2}

1. College of Automation and Electronic engineer Qingdao University of Science & Technology QingDao China

2. College of Communication and Control Engineering Jiangnan University WuXi China

gaodexin@qust.edu.cn.com

Jing-jing Wang

College of information Science & Technology Qingdao University of Science & Technology QingDao China

kathy1003@163.com

Abstract—The optimal damping control for nonlinear time-delay systems with persistent disturbances is considered. Based on successive approximation approach (SAA), the optimal damping control (ODC) law is achieved by solving a decoupled sequence of inhomogeneous linear two-point boundary value (TPBV) problems without time-delay and time-advance terms. The ODC law of the original problem consists of accurate state feedback term, disturbance rejection term and a nonlinear time-delay compensation term, which is the limit of the adjoint vector sequence. By using the finite-time iteration of the compensation sequence, we can obtain an approximate optimal disturbance rejection control law. The proposed algorithms not only solve optimal control problems in the nonlinear time-delay system but also reduce the computation time and improve the precision. Numerical examples are included to illustrate the procedures.

Keywords- nonlinear time-delay systems; optimal damping control; persistent disturbances; successive approximation approach

I. INTRODUCTION

Time-delays, nonlinearities and disturbances are frequently encountered in many fields of science and engineering, including rotating mechanical systems, active noise control [1], manufacturing systems, and ship autopilot control [2], etc. The analysis and synthesis of nonlinear and/or time-delay systems with disturbances has received considerable attention and some research results have been obtained. For instances, Kolmanovskiy and Maizenberg [3] investigated a finite-horizon optimal control problem for randomly varying time-delay systems. Based on a reduction method, Yue and Han [4] investigated a delayed feedback control design for uncertain systems with time-varying input delay. Michael [5] researched linear systems with time-delay in control input, and developed an optimal control law with respect to the maximum principle. Han [6] researched the stability for a class of uncertain linear neutral systems, etc.

External disturbance including stochastic disturbances and

This work was supported in part by the National Natural Science Foundation of China (60804005), and by the National Natural Science Foundation of China (60874029) and by Doctoral Found of Qingdao University of Science & Technology

deterministic disturbances widely exist in practical processes, so the problem of rejecting disturbances appears in a variety of applications. One application area of interest is on systems affected by persistent disturbances is the flight attribute control through wind shear stresses, where the disturbance forces arise from a model for wind shear stresses based on harmonic oscillations. Another application area is the active control for offshore structures, where the main disturbances affecting offshore structure performance are mainly from the wind or ocean wave forces. Other applications include the vibration damping for industry machine, the noise reduction in vehicles and transformers, the periodic disturbance reduction in disk drive, and the control of the linear course of ships, etc. In various external disturbances, sinusoidal disturbances are quite common in practice systems. Therefore it is more significant and valuable to research analysis and synthesis of nonlinear and/or time-delay systems with disturbances. The optimal control problem for nonlinear time-delay systems affecting by sinusoidal disturbances with respect to the quadratic performance index generally leads to a TPBV problem with both delay and advance terms, which is very difficult to solve precisely. So obtaining an approximate optimal control law is one of the important aims of researchers. In recent years, many better results concerning the approximate approach of optimal control for nonlinear and/or time-delay systems have been obtained [7-14].

In this paper, we will deal with the ODC problems for a class of nonlinear time-delay systems which is affected by sinusoidal disturbances. And a practical simple approach designing the ODC law is presented. Our result's contribution is to apply the SAA in [8] to the time-delay systems with disturbances, and give a design algorithm with low computational complexity, which only requires solving the Riccati equation and matrix equation groups one time, and mainly solves a recursion formula of adjoint vectors.

This paper is organized as follows. In Section 2, the external disturbances model and the nonlinear optimal control problem are presented. Two methods and their proofs are given in Section 3. The ODC law is designed and the approach is given in Section 4. In section 5, the effectiveness of the

proposed approach is demonstrated by simulation studies. Finally, some conclusions are drawn in the last section.

II. BACKGROUND

Let us now briefly summarize linear optimal control problem with disturbances. Consider linear systems with external disturbances described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Dv(t), \quad t > 0 \\ x(0) &= x_0 \end{aligned} \tag{1}$$

where $x \in R^n$, $u \in R^r$, and $v \in R^p$ are the system state vector, the control inputs, respectively, and the external disturbance vector, respectively. A , B , and D are real constant matrices of appropriate dimensions. u is unconstrained

A. Disturbances are an exosystem

Assume that the disturbance $v(t)$ is generated from an exosystem in the form

$$v^{(r)}(t) + G_r v^{(r-1)}(t) + \dots + G_1 v(t) = 0 \tag{2}$$

where $G_i \in R^{q \times q}$ ($i = 1, 2, \dots, r$) are known constant matrices of appropriate dimensions. In many practical cases the initial condition $v^{(i)}(t_0)$ ($i = 0, 1, 2, \dots, r-1$) of the exosystem (2) can be measured. If selecting the unfinite-time quadratic cost functional, we are sure to assume that all eigenvalues satisfy

$$\text{Re}(\lambda_i) \leq 0, \quad i = 1, 2, \dots, rq \tag{3}$$

Besides, eigenvalues with the zero real part are simple roots of minimum polynomial in order to guarantee that exosystem (2) is stable or asymptotically stable. Denoting $w(t)$

$$w(t) = \begin{bmatrix} w_1(t) \\ \vdots \\ w_r(t) \end{bmatrix} = \begin{bmatrix} v(t) \\ \vdots \\ v^{(r-1)}(t) \end{bmatrix} \tag{4}$$

Hence, exosystem (2) can be rewritten as

$$\begin{aligned} \dot{w}(t) &= Gw(t) \\ v(t) &= Hw(t) \end{aligned} \tag{5}$$

where

$$G = \begin{bmatrix} 0 & I_q & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & I_q \\ -G_1 & -G_2 & \dots & -G_r \end{bmatrix} \tag{6}$$

$$H = [I_q \quad 0 \quad \dots \quad 0] \tag{7}$$

The finite-time quadratic cost functional associated with (1) is given by

$$J = \frac{1}{2} \left[x^T(t_f) Q_f x(t_f) + \int_0^{t_f} [x^T(t) Q x(t) + u^T(t) R u(t)] dt \right] \tag{8}$$

where $Q_f \in R^{n \times n}$ and $Q \in R^{n \times n}$ are positive-semidefinite matrices. $R \in R^{r \times r}$ is a positive definite matrix.

This problem is minimizing the cost functional J subject to the dynamics (1) by designing an optimal control law u^* . The optimal control is given by

$$u^*(t) = -R^{-1} B^T [P(t)x(t) + \bar{P}(t)w(t)] \tag{9}$$

where the real, symmetric and positive-definite matrix $P(t)$ is the solution of the Riccati matrix differential equation

$$\begin{aligned} -\dot{P}(t) &= P(t)A + A^T P(t) - P(t)SP(t) + Q \\ P(t_f) &= Q_f \end{aligned} \tag{10}$$

The matrix $\bar{P}(t)$ is the solution of the matrix differential equation

$$\begin{aligned} -\dot{\bar{P}}(t) &= [A - SP(t)]^T \bar{P}(t) + \bar{P}(t)G + P(t)DH \\ \bar{P}(t_f) &= 0 \end{aligned} \tag{11}$$

The state vector $x(t)$ is then given by the solution of the closed-loop system

$$\begin{aligned} \dot{x}(t) &= [A - SP(t)]x(t) + [D - S\bar{P}(t)]w(t) \\ x(t_0) &= x_0 \end{aligned} \tag{12}$$

where $S = BR^{-1}B^T$.

B. Disturbances are persistent

Assume that the dynamic characteristics of external disturbance vector $v(t)$ can be expressed as follow

$$v(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_p(t) \end{bmatrix} = \begin{bmatrix} \alpha_1 \sin(\omega_1 t + \varphi_1) \\ \alpha_2 \sin(\omega_2 t + \varphi_2) \\ \vdots \\ \alpha_p \sin(\omega_p t + \varphi_p) \end{bmatrix} \tag{13}$$

The finite-time quadratic performance index associated with (1) is given by

$$J = \frac{1}{2} \left[x^T(t_f) Q_f x(t_f) + \int_0^{t_f} [x^T(t) Q x(t) + u^T(t) R u(t)] dt \right] \tag{14}$$

where $Q_f \in R^{n \times n}$ and $Q \in R^{n \times n}$ are positive semi-definite matrices. $R \in R^{r \times r}$ is a positive definite matrix. The objective is to minimize J by designing an optimal control law $u^*(t)$. The optimal control is given by

$$u^*(t) = -R^{-1} B^T [P(t)x(t) + P_1(t)v(t) + P_2(t)v_\omega(t)] \tag{15}$$

where the real, symmetric and positive semi-definite matrix $P(t)$ is the solution of the Riccati matrix differential equation

$$\begin{aligned} -\dot{P}(t) &= P(t)A + A^T P(t) - P(t)SP(t) + Q \\ P(t_f) &= Q_f \end{aligned} \tag{16}$$

The matrix $P_1(t)$ is the solution of the matrix differential equation

$$\begin{aligned} -\dot{P}_1(t) &= A^T P_1(t) - P_2(t)\Omega^2 - P(t)SP_1(t) + P(t)D \\ P_1(t_f) &= 0 \end{aligned} \tag{17}$$

$P_2(t)$ is the solution of the matrix differential equation

$$\begin{aligned} -\dot{P}_2(t) &= P_1(t) - P(t)SP_2(t) + A^T P_2(t) \\ P_2(t_f) &= 0 \end{aligned} \tag{18}$$

The state vector $x(t)$ is then given by the solution of the closed-loop system

$$\begin{aligned} \dot{x}(t) &= [A - SP(t)]x(t) + [D - SP_1(t)]v(t) - SP_2(t)v_\omega(t) \\ x(t_0) &= x_0 \end{aligned} \tag{19}$$

where

$$\begin{aligned} S &= BR^{-1}B^T \\ \Omega &= \text{Diag}(\omega_1, \omega_2, \dots, \omega_p) \\ v_\omega(t) &= \dot{v}(t) \end{aligned} \tag{20}$$

In linear systems with disturbances, we obtain the control law is optimal, because the TPBV problem is linear, which is leaded according to the maximum principle. But for nonlinear time-delay systems is difficult to solve the TPBV problem.

III. THE NONLINEAR TIME-DELAY OPTIMAL CONTROL PROBLEM WITH DISTURBANCES

Consider nonlinear time-delay systems with sinusoidal disturbances described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_1x(t-\tau) + Bu(t) + f(x) + Dv(t), \quad t > 0 \\ x(t) &= \phi(t), \quad -\tau \leq t < 0 \end{aligned} \tag{21}$$

where $x \in R^n$, $u \in R^r$, and $v \in R^p$ are the system state vector, the control inputs, respectively, and the external disturbance vector, respectively. $f(x)$ is the nonlinear function vector. A, A_1, B , and D are real constant matrices of appropriate dimensions. $\phi(t)$ is the initial state vector. $\tau > 0$ is the time delay.

Assumption 1. $f(x)$ satisfies $f(0) = 0$ and the Lipschitz condition on R^n .

$$\|f(x) - f(\hat{x})\| \leq \alpha \|x - \hat{x}\|, \quad \forall x, \hat{x} \in R^n \tag{22}$$

where α is some positive constant.

The application of the maximum principle leads to the following nonlinear TPBV problem

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_1x(t-\tau) + f(x) - S\lambda(t) + Dv(t), \quad 0 \leq t \leq t_f \\ -\dot{\lambda}(t) &= \begin{cases} Qx(t) + A^T\lambda(t) + f_x^T(x)\lambda(t) + A_1^T\lambda(t+\tau), & 0 \leq t \leq t_f - \tau \\ Qx(t) + A^T\lambda(t) + f_x^T(x)\lambda(t), & t_f - \tau < t < t_f \end{cases} \\ x(t) &= \phi(t), \quad -\tau \leq t < 0 \\ \lambda(t_f) &= Q_f x(t_f) \end{aligned} \tag{23}$$

where $f_x = \partial f(x) / \partial x^T$ is Jacobian matrix of $f(x)$ with respect to vector x . And the control law is given in the form

$$u(t) = -R^{-1}B^T\lambda(t) \tag{24}$$

Note that equations in (23) are nonlinear and the second equation is with both time-delay and time-advance terms. So obtaining the exact analytic solution is, in general, extremely difficult. So the main purpose of this paper is to apply the SAA to the TPBV problem (23), and find an approximate approach to it. Sequentially, the ODC law for the system described by (21) and (13) with the quadratic performance index (14) is obtained.

A. The translation of the nonlinear TPBV problem

In order to separate the linear part from the nonlinear part in the nonlinear TPBV problem (23), let

$$\lambda(t) = P(t)x(t) + P_1(t)v(t) + P_2(t)v_\omega(t) + g(t) \tag{25}$$

where $g(t)$ is an adjoint vector introduced to compensate for the effect of nonlinear and time-delay part upon system (23). Substitute the derivative of (25) into the second equation of (23) and compare the coefficient of $x(t)$, $v(t)$ and $v_\omega(t)$ with the help of the first equation of (23), we can obtain the Riccati matrix differential equation

$$\begin{aligned} -\dot{P}(t) &= A^T P(t) + P(t)A - P(t)SP(t) + Q \\ P(t_f) &= Q_f \end{aligned} \tag{26}$$

and the matrix differential equations

$$\begin{aligned} -\dot{P}_1(t) &= [A^T - P(t)S]P_1(t) - P_2(t)\Omega^2 + P(t)D \\ P_1(t_f) &= 0 \end{aligned} \tag{27}$$

$$\begin{aligned} -\dot{P}_2(t) &= [A^T - P(t)S]P_2(t) + P_1(t) \\ P_2(t_f) &= 0 \end{aligned} \tag{28}$$

and a new form of TPBV problem described by the following adjoint vector differential equation

$$\begin{aligned} -\dot{g}(t) &= \begin{cases} [A - SP(t)]^T g(t) + P(t)A_1x(t-\tau) + P(t)f(x) \\ \quad + f_x^T(x)\lambda(t) + A_1^T\lambda(t+\tau), & 0 \leq t \leq t_f - \tau \\ [A - SP(t)]^T g(t) + P(t)A_1x(t-\tau) + P(t)f(x) \\ \quad + f_x^T(x)\lambda(t), & t_f - \tau < t < t_f \end{cases} \\ g(t_f) &= 0 \end{aligned}$$

$$\begin{aligned}
 & \text{and a close-loop system of (21)} \\
 & \dot{x}(t) = [A - SP(t)]x(t) + [D - SP_1(t)]v(t) - SP_2(t)v_\omega(t) \\
 & \quad - Sg(t) + A_1x(t - \tau) + f(x) \\
 & x(t) = \phi(t), \quad -\tau \leq t < 0
 \end{aligned}
 \tag{29a}$$

$$\begin{aligned}
 & \text{It can be proved that } P_1 \text{ is the unique positive definite} \\
 & \text{solution of (17) and } P_2 \text{ is the unique solution of (18). We} \\
 & \text{will give the proof in the next subsection. The optimal} \\
 & \text{control law (15) can be rewritten as} \\
 & u^*(t) = -R^{-1}B^T[P(t)x(t) + P_1(t)v(t) + P_2(t)v_\omega(t) + g(t)]
 \end{aligned}
 \tag{29b}$$

Therefore, the TPBV problem (23) is transformed into the TPBV problem described by (29). Although the form is changed, it is easy to see that TPBV problem (29) has not only the time-delay term and time-advance term. And as well equation (29a) and (29b) are coupled in $x(t)$ and $g(t)$, it is known that we can not obtain the analytical solution of TPBV problem (29). Here, it should be clear that the above transformation is developed for the convenience of the introduction of our method. We will give the detailed design process in next subsection.

B. Proof of the global convergence

Since the problem has no analytical solution, a great deal of effort will be devoted to the construction of a new numerical method based on the successive approximation. The purpose of this subsection is to give a design strategy of approximate approach for solving the optimal control problem. We first prefer to give the following available lemmas in order to obtain the main results in the sequel.

Lemma 1 Consider the nonlinear time-delay system described by

$$\begin{aligned}
 \dot{z}(t) &= G(t)z(t) + A_1z(t - \tau) + f(z(t), t) + Fw(t), \quad t > t_0 \\
 z(t) &= \phi(t), \quad -\tau \leq t \leq t_0
 \end{aligned}
 \tag{31}$$

where $x \in R^n$ is the state vector, $w \in R^p$ is the input vector, $\phi(t)$ is a known initial state vector; $G \in R^{n \times n}$, $f: C^1(R^n \times R_{t_0}) \rightarrow R^n$ which satisfies the Lipschitz conditions on $R^n \times R_{t_0}$, $R_{t_0} = (t_0, \infty)$.

According to (31), construct the following sequence

$$\begin{aligned}
 \dot{z}^{(k)}(t) &= G(t)z^{(k)}(t) + A_1z^{(k-1)}(t - \tau) + f(z^{(k-1)}(t), t) + Fw(t), \quad t > t_0 \\
 z^{(k)}(t_1) &= \phi(t), \quad -\tau \leq t \leq t_0
 \end{aligned}
 \tag{32}$$

The solution $\{z^{(k)}(t)\}$ of the vector function sequence is as follows

$$\begin{aligned}
 z^{(0)}(t) &= \Phi(t, t_0)\phi(t_0) + \int_{t_0}^t \Phi(t, r)Fw(r)dr \\
 z^{(k)}(t) &= \Phi(t, t_0)\phi(t_0) \\
 &+ \int_{t_0}^t \Phi(t, r) \left[A_1z^{(k-1)}(r - \tau) + f(z^{(k-1)}(r), r) + Fw(r) \right] dr \\
 &, \quad t > t_0 \\
 z^{(k)}(t) &= \phi(t), \quad -\tau \leq t \leq t_0; \quad k = 1, 2, \dots
 \end{aligned}
 \tag{33}$$

where $\Phi(t, t_0)$ is state-transition matrix corresponding to $G(t)$. Then the sequence $\{z^{(k)}(t)\}$ uniformly converges to the solution of system (31).

Proof. Consider $\{z^{(k)}(t)\}$ as a sequence of $C^N(R^n \times R_{t_0})$, from (33) we can obtain

$$z^{(1)}(t) - z^{(0)}(t) = \int_{t_0}^t \Phi(t, r) \left[A_1z^{(0)}(r - \tau) + f(z^{(0)}(r), r) \right] dr, \quad t > t_0
 \tag{34}$$

Because f satisfies the Lipschitz conditions, there exist some positive constants α_1 and β_1 such that

$$\begin{aligned}
 \|f(z, t)\| &\leq \alpha_1 \|z\|, \\
 \|f(z, t) - f(y, t)\| &\leq \beta_1 \|z - y\|,
 \end{aligned}
 \tag{35}$$

From(33), we can obtain

$$\|\Phi(t)\| \leq M_1
 \tag{36}$$

where M_1 is a positive constant, $\|\cdot\|$ denotes any appropriate vector or matrix norm. Assume that $\|A_1\| = N_1$, $\|\phi(t)\| \leq \bar{L}$. We first consider the situation of $t \in (t_0, T]$, where T is some sufficiently large constant. From (34) we can obtain

$$\begin{aligned}
 \|z^{(1)}(t) - z^{(0)}(t)\| &\leq \int_{t_0}^t \left(M_1^2 \bar{L} N_1 + M_1 \alpha_1 \|z^{(0)}(r)\| \right) dr \\
 &\leq M_1^2 \bar{L} (N_1 + \alpha_1) (T - t_0), \quad t \in (t_0, T]
 \end{aligned}
 \tag{37}$$

Moreover, from (33) we have

$$\begin{aligned}
 \|z^{(2)}(t) - z^{(1)}(t)\| &\leq M_1 (N_1 + \beta_1) \int_{t_0}^t \|z^{(1)}(r) - z^{(0)}(r)\| dr \\
 &\leq M_1 (N_1 + \beta_1) M_1^2 \bar{L} (N_1 + \alpha_1) \int_{t_0}^t r dr \\
 &= M_1^3 \bar{L} (N_1 + \beta_1) (N_1 + \alpha_1) \frac{1}{2!} (T - t_0)^2, \\
 &t \in (t_0, T]
 \end{aligned}
 \tag{38}$$

By the mathematical induction, we have

$$\|z^{(k)}(t) - z^{(k-1)}(t)\| \leq M_1^{k+2} \bar{L} (N_1 + \beta_1)^k (N_1 + \alpha_1) \frac{(T-t_0)^{k+1}}{(k+1)!},$$

$$t \in (t_0, T], k=1, 2, \dots \tag{39}$$

According to the trigonometry inequality, for any positive integer j and k

$$\|z^{(k+j)}(t) - z^{(k)}(t)\| \leq \bar{L} (N_1 + \alpha_1) \sum_{i=k}^{k+j-1} \frac{M_1^{i+2} (N_1 + \beta_1)^i (T-t_0)^{i+1}}{(i+1)!}$$

$$\leq \frac{\bar{L} (N_1 + \alpha_1) M_1^{k+2} (N_1 + \beta_1)^k (T-t_0)^{k+1}}{(k+1)!} \exp[M_1 (N_1 + \beta_1) (T-t_0)],$$

$$t \in (t_0, T], k=0, 1, 2, \dots \tag{40}$$

Inequality (40) implies

$$\lim_{k \rightarrow \infty} \|z^{(k+j)}(t) - z^{(k)}(t)\| = 0, \quad \forall j > 0, t \in (t_0, T] \tag{41}$$

Thus $\{z^{(k)}(t)\}$ is a Cauchy sequence. This sequence is uniformly convergent and the limit of the sequence is the solution of system (31).

Obviously, the constant T may be as large as possible. In practical control systems, we may consider $T \rightarrow \infty$ when T is large enough. The proof is complete.

C. Design of the finite time ODC law

In this subsection, we will discuss the design of the ODC law for system (21) with quadratic performance index (14), and the ODC law will be presented in the following theorem.

Theorem 1. Consider the optimal control problem described by (21), (22) and (13) with quadratic performance index (14). Assume that assumptions (1) and (2) all hold. Then the ODC law $u^*(t)$ is existent and unique, and its form is as follows:

$$u^*(t) = -R^{-1} B^T [P(t)x(t) + P_1(t)v(t) + P_2(t)v_\omega(t) + \lim_{k \rightarrow \infty} g^{(k)}(t)] \tag{42}$$

where $P(t)$ is the unique positive semidefinite solution of the Riccati matrix differential equation(26), $P_1(t)$ and $P_2(t)$ are the unique solutions of the matrix differential equations (27) and (28); $g^{(\infty)}(t) = \lim_{k \rightarrow \infty} g^{(k)}(t)$ is the solution of the TPBV problem sequence

$$-\dot{g}^{(1)}(t) = [A - SP(t)]^T g^{(1)}(t) + f_x^T(0)[P_1(t)v(t) + P_2(t)v_\omega(t)], \quad 0 \leq t \leq t_f$$

$$-\dot{g}^{(k)}(t) = \begin{cases} [A - SP(t)]^T g^{(k)}(t) + P(t)A_1 x^{(k-1)}(t - \tau) + P(t)f(x^{(k-1)}(t)) \\ \quad + f_x^T(x^{(k-1)}(t))\lambda^{(k-1)}(t) + A_1^T \lambda^{(k-1)}(t + \tau), \quad 0 \leq t \leq t_f - \tau \\ [A - SP(t)]^T g^{(k)}(t) + P(t)A_1 x^{(k-1)}(t - \tau) + P(t)f(x^{(k-1)}(t)) \\ \quad + f_x^T(x^{(k-1)}(t))\lambda^{(k-1)}(t), \quad t_f - \tau < t < t_f \end{cases}$$

$$g^{(k)}(t_f) = 0, \quad k=2, 3, \dots \tag{43}$$

where $x^{(k)}(t)$ is the solution to the following state sequence

$$\dot{x}^{(1)}(t) = [A - SP(t)]x^{(1)}(t) + [D - SP_1(t)]v(t) - SP_2(t)v_\omega(t) - Sg^{(1)}(t), \quad 0 \leq t \leq t_f$$

$$\dot{x}^{(k)}(t) = [A - SP(t)]x^{(k)}(t) + [D - SP_1(t)]v(t) - SP_2(t)v_\omega(t) - Sg^{(k)}(t) + A_1 x^{(k-1)}(t - \tau) + f(x^{(k-1)}(t)), \quad 0 \leq t \leq t_f$$

$$x^{(k)}(t) = \phi(t), \quad -\tau \leq t < 0; \quad k=2, 3, \dots \tag{44}$$

in which

$$\lambda^{(k)}(t) = P(t)x^{(k)}(t) + P_1(t)v(t) + P_2(t)v_\omega(t) + g^{(k)}(t) \tag{45}$$

Proof. Because equations (29a) and (29b) is coupling. In order to decouple equations to obtain the solution of this TPBV problem, we introduce a successive approximation process by constructing a sequence of adjoint vector differential equations (43) and state equations (44). Correspondingly, the control sequence is given in the form

$$u^{(k)}(t) = -R^{-1} B^T [P(t)x^{(k)}(t) + P_1(t)v(t) + P_2(t)v_\omega(t) + g^{(k)}(t)] \tag{46}$$

$$k=1, 2, \dots$$

Here, k in equations (43), (44), (45) and (46) has the same meaning as that of equation (33) in Lemma 1.

It has been pointed out that $g^{(0)}(t) = 0, x^{(0)}(t) = 0, f(x^{(0)}) = 0, x^{(0)}(t - \tau) = 0$, and $\lambda^{(0)}(t + \tau) = 0$ for any $t \in [0, \infty)$. So we can solve $g^{(1)}(t)$ from the following equation

$$-\dot{g}^{(1)}(t) = [A - SP(t)]^T g^{(1)}(t) + f_x^T(x^{(0)}(t))[P_1(t)v(t) + P_2(t)v_\omega(t)]$$

$$g^{(1)}(t_f) = 0 \tag{47}$$

It is a linear differential equation without time-delay and time-advance terms, $g^{(1)}(t)$ can be easily obtained by reverse integration. In the following equation, $g^{(1)}(t)$ is known

$$\begin{aligned} \dot{x}^{(1)}(t) &= [A - SP(t)]x^{(1)}(t) + [D - SP_1(t)]v(t) - SP_2(t)v_\omega(t) \\ &\quad - Sg^{(1)}(t), \quad t_0 \leq t \leq t_f \\ x^{(1)}(t) &= \phi(t), \quad -\tau \leq t < t_0 \end{aligned} \tag{48}$$

$x^{(1)}(t)$ can be obtained by integration, then $f(x^{(1)})$, $\lambda^{(1)}(t)$, $x^{(1)}(t - \tau)$ and $g^{(1)}(t + \tau)$ became known, for every term is obtained from previous iteration in the following equation

$$\begin{aligned} -\dot{g}^{(2)}(t) &= \begin{cases} [A - SP(t)]^T g^{(2)}(t) + P(t)Ax^{(1)}(t - \tau) + P(t)f(x^{(1)}(t)) \\ \quad + f_x^T(x^{(1)}(t))\lambda^{(1)}(t) + A_1^T \lambda^{(1)}(t + \tau), \quad t_0 \leq t \leq t_f - \tau \\ [A - SP(t)]^T g^{(2)}(t) + P(t)Ax^{(1)}(t - \tau) + P(t)f(x^{(1)}(t)) \\ \quad + f_x^T(x^{(1)}(t))\lambda^{(1)}(t), \quad t_f - \tau < t < t_f \end{cases} \\ g^{(2)}(t_f) &= 0, \quad k=1, 2, \dots \end{aligned} \tag{49}$$

so $g^{(2)}(t)$ is solved, then $x^{(2)}(t)$ is obtained from the following equation

$$\begin{aligned} \dot{x}^{(2)}(t) &= [A - SP(t)]x^{(2)}(t) + [D - SP_1(t)]v(t) - SP_2(t)v_\omega(t) \\ &\quad - Sg^{(2)}(t) + A_1x^{(1)}(t - \tau) + f(x^{(1)}), \quad t_0 \leq t \leq t_f \\ x^{(2)}(t) &= \phi(t), \quad -\tau \leq t < t_0 \end{aligned} \tag{50}$$

It is readily evident that solving the sequences of the equations (43) and (44) is an iterative process. Hence, we can easily get the values of $g^{(k)}(t)$ and $x^{(k)}(t)$ in equations (43) and (44) when $k \geq 1$ by using the approximation approaches of matrix ordinary differential equations, such as Euler's methods and Runge-Kutta methods, together with known initial and terminal conditions. This is because $g^{(k-1)}(t)$, $x^{(k-1)}(t)$, $x^{(k-1)}(t - \tau)$, $x^{(k-1)}(t + \tau)$, and $g^{(k-1)}(t + \tau)$ in (43) and (44) have been obtained from the previous iteration. So equations (43) and (44) have been transformed into linear inhomogeneous vector differential equations. So, the control law (46) can be got by solving (43) and (44) step by step. A conclusion can be made that the coupled TPBV problem (29) with both time-delay and time-advance terms has been transformed into a sequence of TPBV problem, which is decoupling and without time-delay and time-advance terms. We emphasize that vector differential equations (43) and (44) can be guaranteed to be unilaterally decoupling by applying this SAA.

Now let us discuss the convergence of this numerical iteration strategy. Let $\{x^{(k)}(t)\}$ and $\{u^{(k)}(t)\}$ denote solution sequences of equations (43) and (44), respectively.

Then, according to Lemma 1, solutions to the sequence of adjoint vector differential equations in (43) uniformly converge to the solution of equation (29a). Similarly, solutions to the sequence of the state equations in (33) uniformly converge to the solution of equation (29b), i.e.,

$$\lim_{k \rightarrow \infty} g^{(k)}(t) = g(t), \quad \lim_{k \rightarrow \infty} x^{(k)}(t) = x(t). \tag{51}$$

Since the control sequence $\{u^{(k)}(t)\}$ is only related to $\{x^{(k)}(t)\}$ and $\{g^{(k)}(t)\}$, the control sequence $\{u^{(k)}(t)\}$ uniformly converges to the ODC law $u^*(t)$, i.e.

The proof is complete.

Remark 1. In practical designs of the ODC law, the limit $g^{(\infty)}(t)$ of the solution sequence $\{g^{(k)}(t)\}$ to (21) cannot, in general, be obtained precisely. We can approximate the precise solution by using the k th iteration of the adjoint equations and obtain the k th approximate ODC law

$$u_k(t) = -R^{-1}B^T[P(t)x(t) + P_1(t)v(t) + P_2(t)v_\omega(t) + g^{(k)}(t)] \tag{52}$$

Remark 2. Because $x(t)$ in the first term of (44) is the precise solution to the state vector, the approximate ODC law $u_k(t)$ is better than the k th approximate ODC law defined in (42).

Remark 3. It is shown that the term $P_1(t)v(t)$ and $P_2(t)v_\omega(t)$ in (52) is used to cancel the disturbances, and the term $g^{(\infty)}(t)$ in (52) is introduced to compensate the bad effect caused by the time-delay and the nonlinear part. When $A_1 = 0$ and $f(x) = 0$ hold in systems (21), the adjoint vector $g^{(k)}(t)$ will always be zero as $k = 1, 2, \dots$ and the optimal control law will become the equation (15). When the disturbances is the exosystem, and the optimal control law will become the equation (9).

In practice, we can select k according to the control precision of the quadratic performance index. We give a practical algorithm calculating the k th approximate ODC law.

Algorithm 1.

Step 1: Obtain $P(t)$, $P_1(t)$, and $P_2(t)$ from (15), (16) and (17), respectively; Let $k = 1$, Give a constant $\sigma > 0$ and a large enough positive number J_0 .

Step 2: Solve $g^{(k)}(t)$ from (29a); Obtain $u_k(t)$ from (52).

Step 3: Substituting $u_k(t)$ into system (21), the closed loop system is found. And calculate J_k from the following formula

$$J = \frac{1}{2} \left[x^T(t_f) Q_f x(t_f) + \int_0^{t_f} [x^T(t) Q x(t) + u_k^T(t) R u_k(t)] dt \right] \tag{53}$$

Step 4: If $|(J_{k-1} - J_k)/J_k| < \sigma$, then $N = k$, output $u_N(t)$ and stop calculating.

Step 5: Otherwise, solve $x^{(k)}(t)$ from (29b); Let $k = k + 1$ and go to step 2.

IV. SIMULATION AND DISCUSSION

Consider a nonlinear time-delay system with sinusoidal disturbances; system parameters are as follows

$$A = \begin{bmatrix} 0 & 1 \\ -0.2 & 3 \end{bmatrix}, A_1 = \begin{bmatrix} 0 & 1 \\ -0.1 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{54}$$

$$f(x) = \begin{bmatrix} x_1 x_2 \\ x_1^2 - x_2^2 \end{bmatrix}, x(t) = 0, \tau \leq t < 0$$

The sinusoidal disturbances is express as

$$v(t) = \begin{bmatrix} 0.1 \sin \pi/2 t \\ 0.4 \sin \pi/3 t \end{bmatrix} \tag{55}$$

The parameters of quadratic performance index are as follows

$$Q_f = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, R = 1, t_f = 30 \tag{56}$$

We choose the control precision $\sigma = 0.05$. When $|(J_{k-1} - J_k)/J_k| < \sigma$, the control law $u_k(t)$ is approximately considered as the ODC law.

Case 1. When time delay $\tau = 1$, the simulation curves of the state $x_1(t)$, $x_2(t)$, and the control law $u(t)$ are presented in figure 1, figure 2, and figure 3. The values of performance index at different iterative times are listed in table 1.

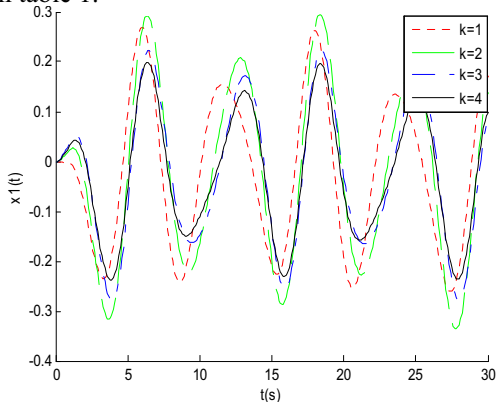


Fig. 1 Simulation curves of the state $x_1(t)$ when $\tau = 1$.

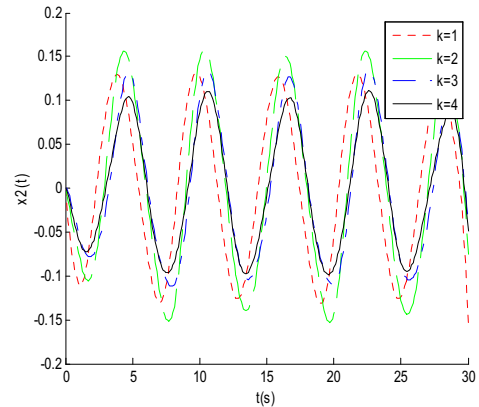


Fig. 2 Simulation curves of the state $x_2(t)$ when $\tau = 1$.

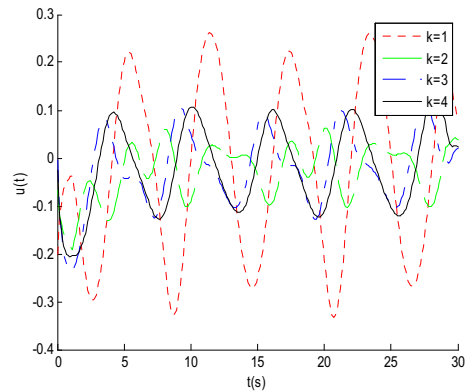


Fig. 3 Simulation curves of the control law $u(t)$ when $\tau = 1$

TABLE 1. QUADRATIC PERFORMANCE INDEX VALUES AND CONTROL PRECISIONS WHEN $\tau = 1$

k	1	2	3	4
J_k	0.0856	0.0295	0.0274	0.0267
$ (J_k - J_{k-1})/J_k $	/	0.6554	0.0712	0.0255

Table 1 shows that $J_1 > J_2 > J_3 > J_4$. This means the quadratic performance index values decrease as iterative times increase, and tend to a stable optimal criterion J^* . Table 1 also shows that the relative errors of the quadratic criterion decrease with the increase of iterative times. When $k = 4$, it satisfies the control precision and $u_4(t)$ can be considered as the approximate ODC law.

Case 2. When time delay $\tau = 4$, the simulation curves of the state $x_1(t)$, $x_2(t)$, and the control law $u(t)$ are presented in figure 4, figure 5, and figure 6. The values of performance index at different iterative times are listed in table 2.

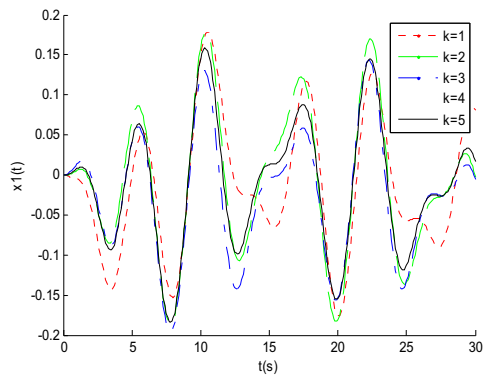


Fig. 4 Simulation curves of the state $x_1(t)$ when $\tau = 4$.

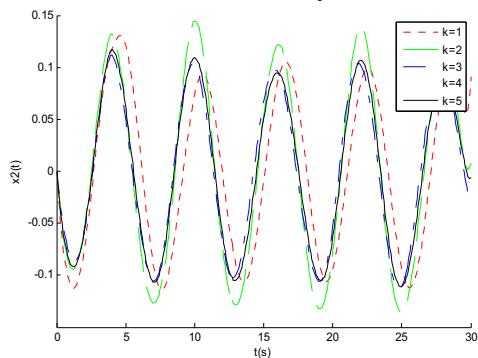


Fig. 5 Simulation curves of the state $x_2(t)$ when $\tau = 4$.

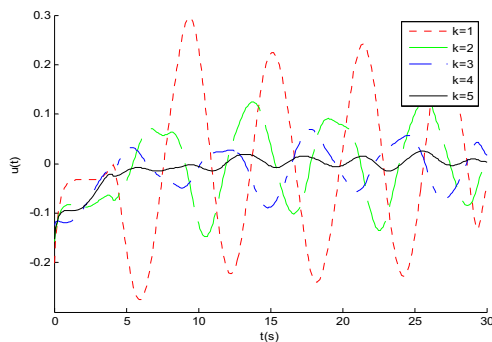


Fig. 6 Simulation curves of the control law $u(t)$ when $\tau = 4$.

TABLE 2. QUADRATIC PERFORMANCE INDEX VALUES AND CONTROL

PRECISIONS WHEN $\tau = 4$

迭代次数 k	1	2	3	4	5
性能指标 J_k	0.0567	0.0207	0.0110	0.0082	0.0080
$ (J_k - J_{k-1})/J_k $	/	0.6349	0.4686	0.2545	0.0244

From Table2, it can be seen that, it satisfies the control precision when $k = 5$, and $u_5(t)$ can be considered as the approximate ODC law. Compared with Table 1, the iterative times k increases at long time delay, but the quadratic performance index still tends to a stable optimal criterion J^* .

From the above numerical example, we can conclude that the proposed algorithm is effective at different time delays. And for long time-delay systems the algorithm still has small computational complexity.

V. CONCLUSIONS

A systematic ODC algorithm is presented in this paper for nonlinear time-delay systems with sinusoidal disturbances. The SAA has been developed to avoid solving the TPBV problem with both delay and advance terms directly. It is important to notice that in the proposed algorithm only a few iteration-steps are required in order to get the approximate ODC law. Simulation results show that the ODC law is high in efficiency, and robust with respect to external disturbances.

REFERENCES

- [1] M.Bodson, J. S. Jensen and S. C. Douglas, "Active noise control for periodic disturbances", IEEE Transactions on Control Systems Technology, 2001,9(1), 200-205.
- [2] T. Cimen, S. P. Banks, "Nonlinear optimal tracking control with application to super-tankers for autopilot design", Automatica, 2004,40 (11), 1845-1863.
- [3] I. V. Kolmanovsky and T. L. Maizenberg, "Optimal control of continuous-time linear systems with time-varying, random delay," Systems and Control Letters, 2001,40(2), 119-126.
- [4] D. Yue, Q.-L. Han, "Delayed feedback control of uncertain systems with time-varying input delay," Automatica, 2005, 41(2), 233-240.
- [5] B. Michael, R. G. Jesus and M. Z. Rodolfo, "Optimal control for linear systems with time delay in control input", Journal of the Franklin Institute, 2004, 341(3), 267-278.
- [6] Q.-L. Han, "Robust stability of uncertain delay-differential systems of neutral type," Automatica, 2002, 38(4), 719-723.
- [7] G.-Y. Tang, P. Liu and N. Xie, "Feedforward and feedback optimal control for time-delay systems with persistent disturbances", Control and Decision, 2005, 20(5), 505-510.
- [8] G.-Y. Tang, "Suboptimal control for nonlinear systems: a successive approximation approach", Systems and Control Letters, 2005, 54(5): 429-434.
- [9] R. W. Beard, T. W. McLain, "Successive Galerkin approximation algorithms for nonlinear optimal and robust control", International Journal of Control, 1998, 71(5): 717 -743.
- [10] A. P. Afanasev, S. M. Dzyuba, S. M. Lobanov, and A. V. Tyutyunnik, "Successive approximation and suboptimal control of the systems with separated linear part", Applied and Computational Mathematics, 2003, 2(1), 48-56.
- [11] G.-Y. Tang, D.-X. Gao, "Feedforward and feedback optimal control for linear systems with persistent disturbances", Journal of System Simulation, 2005, 17(6), 1518-1521.
- [12] Z Aganovic, Z Gajic, "The Successive Approximation Procedure for Finite-time Optimal Control of Bilinear Systems," J. IEEE Trans on Automatic Control, vol.39,1994, pp.1932-1935.
- [13] Zasadzinski, E Magarotto, H Rafaralahy, et al." Residual Generator Design for Singular Bilinear Systems Subjected to Unmeasurable Disturbances: An LM Approach," J. Automatica, vol.39 ,no.4, 2003, :pp.703-713.
- [14] No T S, Min B M, R H Stone, K C Wong," Control and simulation of arbitrary trajectory-tracking," J. Control Engineering Practice, vol.13,no.5,2005,pp. 601-612.

A Receipt-free Coercion-resistant Remote Internet Voting Protocol without Physical Assumptions through Deniable Encryption and Trapdoor Commitment Scheme

Bo Meng

School of Computer, South-Center University for Nationalities, Wuhan, China
Email: mengscuec@gmail.com

Zimao Li and Jun Qin

School of Computer, South-Center University for Nationalities, Wuhan, China
Email: {lizm@sdu.edu.cn, wrj_qj@hotmail.com}

Abstract—The secure remote Internet voting protocol play an important role in Internet voting system. The direction of development of remote Internet voting protocol is that implementation of receipt-freeness and coercion-resistance is from with strong physical assumptions to with weak physical assumptions. The final purpose is that receipt-freeness and coercion-resistance is implemented without physical assumptions. In this paper firstly, a receipt-free coercion-resistant remote Internet voting protocol based on MW deniable encryption scheme and BCP commitment scheme is developed. To our best knowledge the proposed remote Internet voting protocol, which has receipt-freeness and coercion-resistance, is the first remote Internet voting protocol implemented without physical assumptions. Secondly, we analyze receipt-freeness and coercion-resistance of the proposed remote Internet voting protocol. Finally, we compare security properties of several typical protocols with our present protocol.

Index Terms—physical assumptions, remote Internet voting, deniable encryption, trapdoor commitment scheme, protocol security

I. INTRODUCTION

With the progress of society and development of democracy of nation, people can use the election to express their opinions. Owing to the popularity of Internet and information technology, many traditional transactions are processed through Internet. People may want to use the personal computer at their home to vote in election, which is called remote Internet voting. Thus the secure remote Internet voting system plays an important role in remote Internet voting. The secure Internet voting protocol is the base of the remote Internet voting system.

The secure and practical remote Internet voting

protocol should have the following properties:

- ✿ Basic properties: privacy, completeness, soundness, unreusability, fairness, eligibility, and invariableness.

- ✿ Expanded properties: universal verifiability, receipt-freeness[1,2], coercion-resistance [3]

Receipt-freeness: The voter can not produce a receipt to prove that he votes a special ballot. Its purpose is to protect against vote buying. Notion of receipt-freeness was introduced by Benaloh and Tuinstra [1]. They propose a receipt-free scheme with strong physical assumptions: voting-booth. Hirt and Sako in [4] point out that their scheme is not receipt-free.

Coercion-resistance: A coercion-resistant voting protocol should offer not only receipt-freeness, but also it can prevent randomization attack, forced-abstention attack and simulation attack.

The direction of development of remote Internet voting protocol is implementation of receipt-freeness and coercion-resistance without physical assumptions and constraints. The final purpose is that receipt-freeness and coercion-resistance is implemented without physical assumptions. People have developed a lot of Internet voting protocols with receipt-freeness and coercion-resistance. But according to our analysis we found that the weakest physical assumption among implementations of receipt-freeness and coercion-resistance is one way anonymous channel. To our best knowledge up to now the remote Internet voting protocol with receipt-freeness and coercion-resistance implemented without physical assumptions does not exist. Motivated by this in this paper we apply deniable encryption and trapdoor commitment scheme to implement the receipt-freeness and coercion-resistance in the remote Internet voting protocol without physical assumptions.

The main contributions of this paper are summarized as follows.

- ✿ A receipt-free coercion-resistant remote Internet voting protocol is introduced.

Corresponding author: Bo Meng, School of Computer, South-Center University for Nationalities, Wuhan, China, 430074

This work was supported by National Natural Science Foundation of China (60603008) and Natural Science Foundation of Hubei Province (2007ABA342)