Fuzzy Linguistic Hybrid Harmonic Mean Operator and Its Application to Software Selection

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***Abstract*—With respect to multiple attribute group decision making problem with triangular fuzzy linguistic information, in which the attribute weights and expert weights take the form of real numbers, and the preference values take the form of triangular fuzzy linguistic variables, some operators for aggregating triangular fuzzy linguistic variables, such as the fuzzy linguistic harmonic mean (FLHM) operator, fuzzy linguistic weighted harmonic mean (FLWHM) operator, fuzzy linguistic ordered weighted harmonic mean (FLOWHM) operator, and fuzzy linguistic hybrid harmonic mean (FLHHM) operator are proposed. Based on the FLWHM and FLHHM operators, a practical method is developed for group decision making with triangular fuzzy linguistic variables. Finally, an illustrative example about software selection is given to verify the developed approach.**

***Index Terms*—Multiple Attribute Group Decision Making; Triangular Fuzzy Linguistic Variables; Fuzzy Linguistic Ordered Weighted Harmonic Mean (FLOWHM) Operator; Fuzzy Linguistic Hybrid Harmonic Mean (FLHHM) Operator**

I. INTRODUCTION

In the real world, human beings are constantly making decisions under linguistic environment [1-12, 20-30]. For example, when evaluating the “comfort” or “design” of a car, linguistic terms like “good”, “fair”, “poor” are usually be used [1]. Sometimes, however, the decision makers are willing or able to provide only triangular fuzzy linguistic information because of time pressure, lack of knowledge, or data, and their limited expertise related to the problem domain [8]. Thus, Xu[8] developed some operators for aggregating triangular fuzzy linguistic variables, such as the fuzzy linguistic averaging (FLA) operator, fuzzy linguistic weighted averaging (FLWA) operator, fuzzy linguistic ordered weighted averaging (FLOWA) operator, and induced FLOWA (IFLOWA) operator, etc.

The aim of this paper is to develop some harmonic aggregation operators for aggregating triangular fuzzy linguistic information, such as the fuzzy linguistic harmonic mean (FLHM) operator, fuzzy linguistic weighted harmonic mean (FLWHM) operator, fuzzy linguistic ordered weighted harmonic mean (FLOWHM) operator, and fuzzy linguistic hybrid harmonic mean (FLHHM) operator are proposed. Based on the FLWHM and FLHHM operators, a practical method is developed for group decision making with triangular fuzzy linguistic variables.

In order to do so, this paper is set out as follows. Section 2 we introduce the concept of triangular fuzzy linguistic variable and some operational laws of triangular fuzzy linguistic variables, and a formula for comparing triangular fuzzy linguistic variables. Section 3 develop some harmonic aggregation operators for aggregating triangular fuzzy linguistic information, such as the fuzzy linguistic harmonic mean (FLHM) operator, fuzzy linguistic weighted harmonic mean (FLWHM) operator, fuzzy linguistic ordered weighted harmonic mean (FLOWHM) operator, and fuzzy linguistic hybrid harmonic mean (FLHHM) operator. Section 4 we develop an approach to for group decision making with triangular fuzzy linguistic variables based on the FLWHM and FLHHM operators, which is straightforward and has no loss of information. Section 5 we give an illustrative example about software selection to verify the developed approach and to demonstrate its feasibility and practicality. Section 6 concludes the paper.

II. TRIANGULAR FUZZY LINGUISTIC VARIABLES

Let $S=\{s_i \mid i=1,2,\cdots,t\}$ be a linguistic term set with odd cardinality. Any label, s_i represents a possible value for a linguistic variable, and it should satisfy the following characteristics: ①The set is ordered: $s_i > s_j$, if $i > j$; ② There is the reciprocal operator: $rec(s_i)=s_j$ such that $i=t+1-j$; ③Max operator: $\max(s_i,s_j)=s_i$, if $s_i \ge s_j$; Min operator: $\min(s_i,s_j)=s_i$, if $s_i \le s_j$. For example, S can be defined as [5]

$$S=\{s_1=extremely\ poor, s_2=very\ poor,\\ s_3=poor, s_4=medium, s_5=good,\\ s_6=very\ good, s_7=extremely\ good\}$$

To preserve all the given information, we extend the discrete term set S to a continuous term set $\overline{S}=\{s_a \mid s_1 \le s_a \le s_q, a\in[1,q]\}$, where q is a sufficiently large positive integer. If $s_a \in S$, then we call s_a the original linguistic term, otherwise, we call s_a the

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virtual linguistic term. In general, the decision maker uses the original linguistic term to evaluate attributes and alternatives, and the virtual linguistic terms can only appear in calculation [5].

Definition 1. Let $s_\alpha, s_\beta \in \bar{S}$, then we call

$$d\left(s_\alpha, s_\beta\right) = \left|\alpha - \beta\right| \tag{1}$$

the distance between s_α and s_β [8].

In the following we introduce the concept of triangular fuzzy linguistic variable.

Definition 2. Let $\tilde{s} = [s_\alpha, s_\beta, s_\gamma] \in \tilde{S}$, where $s_\alpha, s_\beta, s_\gamma \in \bar{S}$, s_α, s_β and s_γ are the lower, modal and upper values of $\tilde{s}$, respectively, then we call $\tilde{s}$ a triangular fuzzy linguistic variable, which is characterized by the following member function [8]

$$\mu_{\tilde{s}}(\theta) = \begin{cases} 0, & s_1 \le s_\theta \le s_\alpha \\ d\left(s_\theta, s_\alpha\right) / d\left(s_\beta, s_\alpha\right), & s_\alpha \le s_\theta \le s_\beta \\ d\left(s_\theta, s_\gamma\right) / d\left(s_\beta, s_\gamma\right), & s_\beta \le s_\theta \le s_\gamma \\ 0, & s_\gamma \le s_\theta \le s_q \end{cases} \tag{2}$$

Clearly, s_β gives the maximal grade of $\mu_{\tilde{s}}(\theta)\left(\mu_{\tilde{s}}(\theta) = 1\right)$, s_α and s_γ are the lower and upper bounds with limit the field of the possible evaluation. Especially, if $s_\alpha = s_\beta = s_\gamma$, then $\tilde{s}$ is reduced to a linguistic variable.

Let $\tilde{S}$ be the set of all triangular fuzzy linguistic variables. Consider any three triangular fuzzy linguistic variables $\tilde{s} = \left(s_\alpha, s_\beta, s_\gamma\right)$, $\tilde{s}_1 = \left(s_{\alpha_1}, s_{\beta_1}, s_{\gamma_1}\right)$, $\tilde{s}_2 = \left(s_{\alpha_2}, s_{\beta_2}, s_{\gamma_2}\right) \in \tilde{S}$, and suppose that $\lambda \in [0,1]$, then we define their operational laws as follows:

(1) $\tilde{s}_1 \otimes \tilde{s}_2 = \left(s_{\alpha_1}, s_{\beta_1}, s_{\gamma_1}\right) \otimes \left(s_{\alpha_2}, s_{\beta_2}, s_{\gamma_2}\right)$
$= \left(s_{\alpha_1\alpha_2}, s_{\beta_1\beta_2}, s_{\gamma_1\gamma_2}\right)$;

(2) $\tilde{s}^\lambda = \left(s_\alpha, s_\beta, s_\gamma\right)^\lambda = \left(s_{\alpha^\lambda}, s_{\beta^\lambda}, s_{\gamma^\lambda}\right)$.

(3) $\tilde{s}^{-1} = \left(s_\alpha, s_\beta, s_\gamma\right)^{-1} = \left(s_{1/\gamma}, s_{1/\beta}, s_{1/\alpha}\right)$

In the following, we introduce a formula for comparing triangular fuzzy linguistic variables.

Definition 3.

Let $\tilde{s}_1 = \left(s_{\alpha_1}, s_{\beta_1}, s_{\gamma_1}\right)$, $\tilde{s}_2 = \left(s_{\alpha_2}, s_{\beta_2}, s_{\gamma_2}\right) \in \tilde{S}$, then the degree of possibility of $\tilde{s}_1 \ge \tilde{s}_2$ is defined as[8]

$$\begin{aligned} p(\tilde{s}_1 \ge \tilde{s}_2) &= \lambda \max\left\{1 - \max\left[d\left(s_{\beta_2}, s_{\alpha_1}\right) / \left(d\left(s_{\beta_1}, s_{\alpha_1}\right) + d\left(s_{\beta_2}, s_{\alpha_2}\right)\right), 0\right], 0\right\} + \\ &(1-\lambda)\max\left\{1 - \max\left[d\left(s_{\gamma_2}, s_{\beta_1}\right) / \left(d\left(s_{\gamma_1}, s_{\beta_1}\right) + d\left(s_{\gamma_2}, s_{\beta_2}\right)\right), 0\right], 0\right\} \end{aligned} \tag{3}$$

where the value λ is an index of rating attitude. It reflects the decision maker's risk-bearing attitude. If $\lambda < 0.5$, the decision maker is risk lover. If $\lambda = 0.5$, the decision maker is neutral to risk. If $\lambda > 0.5$, the decision maker is risk avertor.

From Definition 3, we can easily get the following results easily:

(1) $0 \le p\left(\tilde{s}_1 \ge \tilde{s}_2\right) \le 1, 0 \le p\left(\tilde{s}_2 \ge \tilde{s}_1\right) \le 1$;

(2) $p\left(\tilde{s}_1 \ge \tilde{s}_2\right) + p\left(\tilde{s}_2 \ge \tilde{s}_1\right) = 1$. Especially, $p\left(\tilde{s}_1 \ge \tilde{s}_1\right) = p\left(\tilde{s}_2 \ge \tilde{s}_2\right) = 0.5$.

III. Some Harmonic Aggregation Operators with Triangular Fuzzy Linguistic Information

Based on the well-known harmonic mean[13-15], in the following, we shall develop some harmonic aggregating operators to deal with triangular fuzzy linguistic information.

Some operators are proposed as follows:

Definition 4. Let $\mathrm{FLWHM}: \tilde{S}^n \to \tilde{S}$, if

$$\begin{aligned} &\mathrm{FLWHM}_\omega\left(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n\right) \\ &= \left(\omega_1\left(\tilde{s}_1\right)^{-1} \oplus \omega_2\left(\tilde{s}_2\right)^{-1} \oplus \cdots \oplus \omega_n\left(\tilde{s}_n\right)^{-1}\right)^{-1} \end{aligned} \tag{4}$$

where $\omega = \left(\omega_1, \omega_2, \cdots, \omega_n\right)$ is the weighting vector of triangular fuzzy linguistic variables $\tilde{s}_i\left(\tilde{s}_i \in \tilde{S}, i = 1, 2, \cdots, n\right)$ with $\omega_i \in [0,1]$, $\sum_{i=1}^n \omega_i = 1$, then function FLWHM is called the fuzzy linguistic weighted harmonic mean (FLWHM) operator of dimension n. Especially, if $\omega = (1/n, 1/n, \cdots, 1/n)$, then FLWHM operator is reduced to a fuzzy linguistic harmonic mean (FLHM) operator.

Definition 5. Let $\tilde{s}_i\left(i = 1, 2, \cdots, n\right)$ be a set of triangular fuzzy linguistic variables, and

$$\begin{aligned} &\mathrm{FLOWHM}_w\left(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n\right) \\ &= \left(w_1\left(\tilde{r}_1\right)^{-1} \oplus w_2\left(\tilde{r}_2\right)^{-1} \oplus \cdots \oplus w_n\left(\tilde{r}_n\right)^{-1}\right)^{-1} \end{aligned} \tag{5}$$

where $w = \left(w_1, w_2, \cdots, w_n\right)$ is the associated weighting vector, with $w_j \in [0,1]$, $\sum_{j=1}^n w_j = 1$, and $\tilde{r}_j$ is the j-th largest element in the triangular fuzzy linguistic variables set $\left(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n\right)$, then the function FLOWHM is called the fuzzy linguistic variables ordered weighted harmonic mean (FLOWHM) operator of dimension n .

To rank these triangular fuzzy linguistic variables $\tilde{s}_j\left(j = 1, 2, \cdots, n\right)$, we first compare each argument $\tilde{s}_i$ with all the triangular fuzzy linguistic variables $\tilde{s}_j\left(j = 1, 2, \cdots, n\right)$ by using Eq. (3). For simplicity, we let $p_{ij} = p\left(\tilde{s}_i \geq \tilde{s}_j\right)$, then we develop a complementary matrix as $P = \left(p_{ij}\right)_{n\times n}$, where $p_{ij} \geq 0$, $p_{ij} + p_{ji} = 1$, $p_{ii} = 0.5$, $i, j = 1,2,\cdots,n$. Summing all the elements in each line of matrix P, we get

$$p_i = \sum_{j=1}^{n} p_{ij}, i = 1,2,\cdots,n.$$

Then we rank the arguments $\tilde{s}_j$ in descending order in accordance with the values of $p_i\left(i = 1,2,\cdots,n\right)$.

From Definitions 4 and 5, we know that the FLWHM operator weights the triangular fuzzy linguistic arguments while the FLOWHM operator weights the ordered positions of the triangular fuzzy linguistic arguments instead of weighting the arguments themselves. Therefore, weights represent different aspects in both the FLWHM and FLOWHM operators. However, both the operators consider only one of them. To solve this drawback, in the following we shall propose a fuzzy linguistic hybrid harmonic mean (FLHHM) operator.

Definition 6. A fuzzy linguistic hybrid harmonic mean (FLHHM) operator is defined as follows:

$$\begin{aligned}&\mathrm{FLHHM}_{w,\omega}\left(\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_n\right)\\ &= \left(w_1\left(\tilde{r}_1\right)^{-1} \oplus w_2\left(\tilde{r}_2\right)^{-1} \oplus \cdots \oplus w_n\left(\tilde{r}_n\right)^{-1}\right)^{-1}\end{aligned} \qquad (6)$$

where $w = \left(w_1, w_2, \cdots, w_n\right)$ is the associated weighting vector, with $w_j \in [0,1]$, $\sum_{j=1}^{n} w_j = 1$, and $\tilde{r}_j$ is the j-th largest element of the triangular fuzzy linguistic weighted argument $\tilde{s}_i'\left(\tilde{s}_i' = \dfrac{\tilde{s}_i}{n\omega_i}\right)$, $\omega = \left(\omega_1, \omega_2, \cdots, \omega_n\right)$ is the weighting vector of triangular fuzzy linguistic variables $\tilde{s}_i\left(i = 1,2,\cdots,n\right)$, with $\omega_i \in [0,1]$, $\sum_{i=1}^{n} \omega_i = 1$, and n is the balancing coefficient, then the function FLHHM is called the fuzzy linguistic hybrid harmonic mean (FLHHM) operator of dimension n. Especially, if $w = \left(1/n, 1/n, \cdots, 1/n\right)^T$, then FLHHM is reduced to the fuzzy linguistic weighted harmonic mean (FLWHM) operator; if $\omega = \left(1/n, 1/n, \cdots, 1/n\right)$, then FLHHM is reduced to the fuzzy linguistic variables ordered weighted harmonic mean (FLOWHM) operator.

IV. An Approach to Group Decision Making under Triangular Fuzzy Linguistic Environment

In this section, we shall develop an approach based on the FLWHM and FLHHM operators to group decision making under triangular fuzzy linguistic environment as follows.

Let $A = \left\{A_1, A_2, \cdots, A_m\right\}$ be a discrete set of alternatives, $G = \left\{G_1, G_2, \cdots, G_n\right\}$ be the set of attributes, $\omega = \left(\omega_1, \omega_2, \cdots, \omega_n\right)$ is the exponential weighting vector of the attributes $G_j\left(j = 1,2,\cdots,n\right)$, where $\omega_j \in [0,1]$, $\sum_{j=1}^{n} \omega_j = 1$, $D = \left\{D_1, D_2, \cdots, D_t\right\}$ be the set of decision makers, and $\nu = \left(\nu_1, \nu_2, \cdots, \nu_t\right)$ be the weighting vector of decision makers, where $\nu_k \in [0,1]$, $\sum_{k=1}^{t} \nu_k = 1$. Suppose that $\tilde{R}_k = \left(\tilde{r}_{ij}^{(k)}\right)_{m\times n}$ is the fuzzy linguistic decision matrix, where $\tilde{r}_{ij}^{(k)} \in \tilde{S}$ is a preference value, which take the form of triangular fuzzy linguistic variable, given by the decision maker $D_k \in D$, for the alternative $A_i \in A$ with respect to the attribute $G_j \in G$.

Step 1. Utilize the decision information given in matrix $\tilde{R}_k$, and the FLWHM operator

$$\tilde{r}_i^{(k)} = FLWHM_{\omega}\left(\tilde{r}_{i1}^{(k)}, \tilde{r}_{i2}^{(k)}, \cdots, \tilde{r}_{in}^{(k)}\right)$$
$$i = 1,2,\cdots,m, k = 1,2,\cdots,t.$$

to derive the individual overall preference value $\tilde{r}_i^{(k)}$ of the alternative A_i.

Step 2. Utilize the FLHHM operator:

$$\tilde{r}_i = FLHHM_{\nu,w}\left(\tilde{r}_i^{(1)}, \tilde{r}_i^{(2)}, \cdots, \tilde{r}_i^{(t)}\right), i = 1,2,\cdots,m$$

to derive the collective overall preference values $\tilde{r}_i\left(i = 1,2,\cdots,m\right)$ of the alternative A_i, where $\nu = \left(\nu_1, \nu_2, \cdots, \nu_n\right)$ be the weighting vector of decision makers, with $\nu_k \in [0,1]$, $\sum_{k=1}^{t} \nu_k = 1$; $w = \left(w_1, w_2, \cdots, w_n\right)$ is the associated weighting vector of the FLHHM operator, with $w_j \in [0,1]$, $\sum_{j=1}^{n} w_j = 1$.

Step 3. To rank these collective overall preference values $\tilde{r}_i\left(i = 1,2,\cdots,m\right)$, we first compare each $\tilde{r}_i$ with all the $\tilde{r}_j\left(j = 1,2,\cdots,m\right)$ by using Eq. (3). For simplicity, we let $p_{ij} = p\left(\tilde{r}_i \geq \tilde{r}_j\right)$, then we develop a complementary matrix as $P = \left(p_{ij}\right)_{m\times m}$, where $p_{ij} \geq 0$, $p_{ij} + p_{ji} = 1$, $p_{ii} = 0.5$,

$i, j = 1, 2, \cdots, m$.

Summing all the elements in each line of matrix P, we have

$$p_i = \sum_{j=1}^{m} p_{ij}, i = 1, 2, \cdots, m.$$

Then we rank the collective overall preference values $\tilde{r}_i \left(i = 1, 2, \cdots, m\right)$ in descending order in accordance with the values of $p_i \left(i = 1, 2, \cdots, m\right)$.

Step 4. Rank all the alternatives $A_i \left(i = 1, 2, \cdots, m\right)$ and select the best one(s) in accordance with the collective overall preference values $\tilde{r}_i \left(i = 1, 2, \cdots, m\right)$.

Step 5. End.

V. ILLUSTRATIVE EXAMPLE

In this section, an example is provided. A software selection problem can be calculated as a multiple attribute decision making problem in which alternatives are the software packages to be selected and criteria are those attributes under consideration. A computer center in a university desires to select a new information system in order to improve work productivity. After preliminary screening, five alternatives $A_i \left(i = 1, 2, \cdots, 5\right)$ have remained in the candidate list. Three decision makers (experts) $D_k \left(k = 1, 2, 3\right)$ form a committee to act as decision makers. The investment company must take a decision according to the following four attributes: G_1 is the costs of hardware/software investment; G_2 is the contribution to organization performance; G_3 is the effort to transform from current system; G_4 is the outsourcing software developer reliability. The five possible alternatives $A_i \left(i = 1, 2, \cdots, 5\right)$ are to be evaluated using the linguistic term set S by the three decision makers (whose weighting vector $v = \left(0.4, 0.5, 0.1\right)$) under the above four attributes, and construct, respectively, the decision matrices as follows $\tilde{R}_k = \left(\tilde{r}_{ij}^{(k)}\right)_{5\times 4} \left(k = 1, 2, 3\right)$:

$$\tilde{R}_1 = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{pmatrix} (s_2, s_3, s_4) & (s_3, s_4, s_5) & (s_1, s_2, s_4) & (s_1, s_2, s_3) \\ (s_1, s_2, s_3) & (s_4, s_5, s_7) & (s_3, s_4, s_5) & (s_2, s_3, s_4) \\ (s_2, s_4, s_5) & (s_1, s_3, s_4) & (s_4, s_5, s_6) & (s_3, s_4, s_7) \\ (s_4, s_5, s_6) & (s_1, s_2, s_3) & (s_2, s_3, s_4) & (s_2, s_3, s_5) \\ (s_3, s_4, s_5) & (s_4, s_6, s_7) & (s_1, s_4, s_5) & (s_3, s_4, s_6) \end{pmatrix}$$

$$\tilde{R}_2 = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{pmatrix} (s_1, s_2, s_3) & (s_1, s_4, s_5) & (s_1, s_2, s_3) & (s_3, s_5, s_6) \\ (s_1, s_3, s_4) & (s_2, s_4, s_5) & (s_2, s_3, s_4) & (s_2, s_3, s_5) \\ (s_2, s_4, s_6) & (s_1, s_2, s_3) & (s_1, s_3, s_5) & (s_2, s_3, s_4) \\ (s_2, s_3, s_5) & (s_3, s_4, s_5) & (s_3, s_5, s_6) & (s_1, s_2, s_3) \\ (s_2, s_4, s_5) & (s_4, s_5, s_6) & (s_1, s_3, s_4) & (s_3, s_4, s_5) \end{pmatrix}$$

$$\tilde{R}_3 = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{pmatrix} (s_2, s_4, s_5) & (s_3, s_5, s_6) & (s_1, s_2, s_3) & (s_1, s_2, s_4) \\ (s_1, s_2, s_3) & (s_2, s_3, s_4) & (s_3, s_4, s_6) & (s_2, s_3, s_4) \\ (s_3, s_4, s_5) & (s_2, s_3, s_4) & (s_4, s_5, s_7) & (s_5, s_6, s_7) \\ (s_4, s_5, s_7) & (s_2, s_3, s_5) & (s_4, s_5, s_6) & (s_2, s_3, s_4) \\ (s_1, s_2, s_3) & (s_4, s_5, s_6) & (s_3, s_4, s_5) & (s_5, s_6, s_7) \end{pmatrix}$$

Then, we utilize the approach developed to get the most desirable alternative(s).

Step 1. Utilize FLWHM operator (Let $\omega = \left(0.3, 0.1, 0.2, 0.4\right)$), we get

$$\tilde{r}_1^{(1)} = \left(s_{1.28}, s_{2.35}, s_{3.59}\right), \tilde{r}_2^{(1)} = \left(s_{1.69}, s_{2.83}, s_{3.93}\right)$$
$$\tilde{r}_3^{(1)} = \left(s_{2.31}, s_{4.03}, s_{5.70}\right), \tilde{r}_4^{(1)} = \left(s_{2.11}, s_{3.23}, s_{4.69}\right)$$
$$\tilde{r}_5^{(1)} = \left(s_{2.18}, s_{4.14}, s_{5.53}\right), \tilde{r}_1^{(2)} = \left(s_{1.36}, s_{2.82}, s_{3.95}\right)$$
$$\tilde{r}_2^{(2)} = \left(s_{1.54}, s_{3.08}, s_{4.44}\right), \tilde{r}_3^{(2)} = \left(s_{1.54}, s_{3.08}, s_{4.48}\right)$$
$$\tilde{r}_4^{(2)} = \left(s_{1.54}, s_{2.74}, s_{4.05}\right), \tilde{r}_5^{(2)} = \left(s_{1.97}, s_{3.82}, s_{4.84}\right)$$
$$\tilde{r}_1^{(3)} = \left(s_{1.28}, s_{2.53}, s_{4.11}\right), \tilde{r}_2^{(3)} = \left(s_{1.62}, s_{2.73}, s_{3.87}\right)$$
$$\tilde{r}_3^{(3)} = \left(s_{3.57}, s_{4.65}, s_{5.86}\right), \tilde{r}_4^{(3)} = \left(s_{2.67}, s_{3.75}, s_{5.10}\right)$$
$$\tilde{r}_5^{(3)} = \left(s_{2.12}, s_{3.49}, s_{4.68}\right)$$

Step 2. Utilize the FLHHM operator (let $v = \left(0.3, 0.4, 0.3\right)$, $w = \left(0.4, 0.3, 0.3\right)$), we get

$$\tilde{r}_1 = \left(s_{1.320}, s_{2.599}, s_{3.938}\right), \tilde{r}_2 = \left(s_{1.630}, s_{2.914}, s_{4.127}\right)$$
$$\tilde{r}_3 = \left(s_{2.213}, s_{3.825}, s_{5.284}\right), \tilde{r}_4 = \left(s_{2.011}, s_{3.214}, s_{4.608}\right)$$
$$\tilde{r}_5 = \left(s_{2.104}, s_{3.867}, s_{5.069}\right)$$

Step 3. Suppose that the decision maker's attitude neutral to the risk, i.e., $\rho = 0.5$, then by using Eq. (3), and develop a complementary matrix:

$$P = \begin{bmatrix} 0.500 & 0.390 & 0.087 & 0.251 & 0.095 \\ 0.610 & 0.500 & 0.178 & 0.357 & 0.187 \\ 0.913 & 0.822 & 0.500 & 0.685 & 0.521 \\ 0.749 & 0.643 & 0.315 & 0.500 & 0.330 \\ 0.905 & 0.813 & 0.479 & 0.670 & 0.500 \end{bmatrix}$$

Summing all the elements in each line of matrix P, we have

$$p_1 = 1.323, p_2 = 1.831, p_3 = 3.442$$
$$p_4 = 2.538, p_5 = 3.367.$$

Then we rank the collective overall preference values $\tilde{r}_i\left(i=1,2,3,4,5\right)$ in descending order in accordance with the values of $p_i\left(i=1,2,\cdots,5\right)$: $\tilde{r}_5 \succ \tilde{r}_3 \succ \tilde{r}_4 \succ \tilde{r}_2 \succ \tilde{r}_1$.

Step 4. Rank all the alternatives $A_i\left(i=1,2,\cdots,5\right)$ in accordance with the overall preference values $\tilde{r}_i\left(i=1,2,\cdots,5\right)$: $A_5 \succ A_3 \succ A_4 \succ A_2 \succ A_1$, and thus the most desirable alternative is A_5.

VI. CONCLUSION

In this paper, we have investigated the MAGDM problems with triangular fuzzy linguistic variables. We have introduced the concept and some operational laws of triangular fuzzy linguistic variables and proposed some new harmonic aggregation operators for aggregating fuzzy linguistic variables. We have proved both FLWHM and FLOWHM operators are the special case of the FLHHM operator. The FLHHM operator generalizes both the FLWHM and FLOWHM operators, and reflects the importance degrees of both the given arguments and their ordered positions. Based on the FLWHM and FLHHM operators, we have proposed an approach to MAGDM under triangular fuzzy linguistic environment. We have also applied the proposed approach to the practical problem for software selection.

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