Research on Spatial Location Unit Distribution Model in Pervasive Computing

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***Abstract*—How to obtain location information of an unknown node precisely is a key problem of locating service under ubiquitous computing environment. The paper proposes and proves the theorem of spatial location unit distribution model according to the analysis of the location error produced during location using the polygon location method in the three-dimensional space. On this basis, the location unit selection (LUS) algorithm is proposed by improving on the traditional polygon location algorithm. The simulation results indicate that the location unit distribution model theorem and the LUS algorithm can meet the requirements of a ubiquitous terminal's real-time location and possess a preferable precision in location.**

***Index Terms*—pervasive computing, location error, location unit distribution model, location unit selection (LUS) algorithm**

I. INTRODUCTION

Location service is one of the major context-aware services in pervasive computing environment[1,2]. Getting accurate location information and providing corresponding information services currently have become the high priority issue needed to be solved in pervasive computing. The most common procedures of existing location algorithms can be divided as follows.

1) Measure the distance between the reference nodes and the unknown mobile node;

2) Estimate the unknown node's position;

3) Optimize and update the unknown node's position through iteration [3-5].

These algorithms can obtain the node's position with a certain degree of accuracy. However, if ranging errors exist in the first step, an accumulation of errors will occur in the succeeding steps. How do we reduce the accumulated error and obtain the smallest possible location error? The related studies and algorithms for reducing location errors are few. Reference [6-7] presented a location unit distribution theorem in two-dimensional space. In this paper, the three-dimensional spatial location errors are discussed, and location unit distribution model theorem in three-dimensional space is proved in mathematics. On this basis, the location unit selection (LUS) algorithm is presented based on the polygon dynamic geometry.

II. SPATIAL LOCATION UNIT DISTRIBUTION MODEL

A. The Principle of Polygons Location

An unknown node needs to use at least four reference nodes that is one location unit, to locate itself in three-dimensional space[8]. The distance between the nodes is calculated under the assumption that the measurement is reliable; however, measurement errors are really inevitable. When there was a measurement error, the four spheres no longer intersect at one point .In the following statements, we will assume that the errors are in the range (0, ε), in which ε >0. That is, if the actual distance between the two nodes is r, the actual measurement is between [$r-\varepsilon, r+\varepsilon$]. In the case of existing errors, the four spheres will form a small area. As shown in Fig.1, marked with A_p .Then the volume of A_p shows the degree of the location error.

In order to identify the location node P's coordinate (*x*, *y*, *z*), we should first find the location unit four nodes' coordinates $(x_i, y_i, z_i), i=1,2,3,4$, then measure the distance between the unknown node and the reference node is r_i, and get the error of distance measurement ε_i. Finally, we can obtain the following equations:

$$A_{p_i}=\left\{(x,y,z)\,|\,(x-x_i)^2+(y-y_i)^2+(z-z_i)^2\le(r_i+\varepsilon_i)^2,\right.$$
$$\left.(x-x_i)^2+(y-y_i)^2+(z-z_i)^2\ge(r_i-\varepsilon_i)^2\right\} \quad (1)$$

$$A_p=\left\{(x,y,z)\,|\,x\in\bigcap_{i=1}^{4}A_{P_i},\, y\in\bigcap_{i=1}^{4}A_{P_i},\, z\in\bigcap_{i=1}^{4}A_{P_i}\right\} \quad (2)$$

$$R_p=\left\{(x,y,z)\,|\,x^2+y^2+z^2=\varepsilon^2, \varepsilon>0\right\} \quad (3)$$

Positioning technology of the smart space is used as an indoor technology, it requires high accuracy by using high-precision positioning technologies, such as ultrasound and so on, which are with smaller positioning error, generally between a few centimeters to a dozen centimeters; In terms of the positioning errors between each two nodes, as the measurement for any two nodes is independent and the indoor environment is relatively stable, so the jamming signal in all directions have a smaller margin of error; To simplify the analysis, it is assumed, the distance measurement errors are the same. In this way, if $\varepsilon\equiv 0$, the point set A_p will become one point in Eq. (2). However, when considering errors, that is, $\varepsilon>0$, the volume of A_p means value of location error.

Suppose the beeline through the point P and $P_i(i=1,2,3,4)$ crossed the sphere R_p in the Eq. (3) with two points. Through the two points, the tangents $S_{ij}(j=1,2)$ of R_p are made respectively, and the tangent S_{i1} is paralleled to S_{i2}. All the tangent planes are intersected into an octahedron, and then the area $\tilde{A}_{p_i}$ will be between the tangent S_{i1} and S_{i2}. Set $\tilde{A}_p = \tilde{A}_{p_1} \cap \tilde{A}_{p_2} \cap \tilde{A}_{p_3} \cap \tilde{A}_{p_4}$. As the measurement error is tiny, the edge area of A_p can be planarized, and estimated as $\tilde{A}_p$. Thus, the following issue will be converted into the one that under what circumstances, the location error $V(\tilde{A}_p)$ is the minimum one.

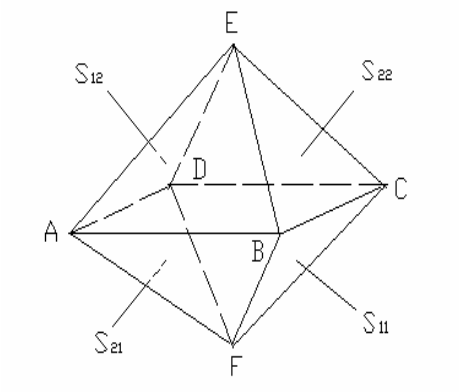

Figure 2. Analysis of location error

Considering the octahedron as shown in Fig.2, $S_{11}//S_{12}$ and $S_{21}//S_{22}$, the separate analysis on the four spheres may be easier to prove $AD//BC$, and likely, $AB//CD$. Therefore, it is obtained that $ABCD$ is on the same plane, and forming a parallelogram.

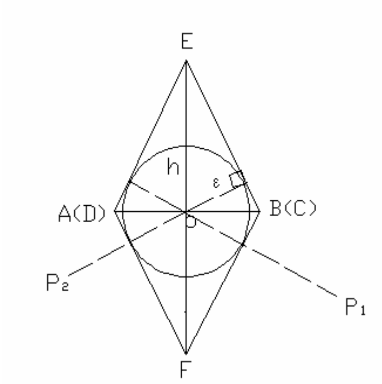

Figure 3. Side-view of the positioning error analysis

Looking in the direction of BC, as $AD//BC$, both AD and BC become a point, and S_{11}, S_{12}, S_{21} and S_{22} become a beeline, as shown in Fig. 3. As they are tangent with the sphere R_p, the distance of S_{11} and S_{12} is 2ε, the same to that of S_{21} and S_{22}. In this direction, S_{11}, S_{12}, S_{21} and S_{22} form a diamond, with its sphere center on the plane $ABCD$. Suppose the distance of EO is h. It can be obtained that the distance of AD and BC is $2h\varepsilon\big/\sqrt{h^2-\varepsilon^2}$.

Likely, looking in the direction of AB, and the distance of AB and CD is also $d = 2h\varepsilon\big/\sqrt{h^2-\varepsilon^2}$, so $ABCD$ is a diamond. Looking in the two directions of AB and BC, $OE \perp ABCD$ is always correct, and so OE and the plane $ABCD$ is vertical in the spatial relations. On this basis, the octahedron is formed with two rectangular pyramids, with diamond as their bottoms. The connection line between the top and the diamond center is vertical to the bottom.

From E, the vertical line EG and EH are made towards AB and CD. As $OE \perp AB$ and $EG \perp AB$, it can be get $AB \perp EGFH$. Then $AB \perp GH$, and $GH = d$ can be get as shown in Fig. 4.

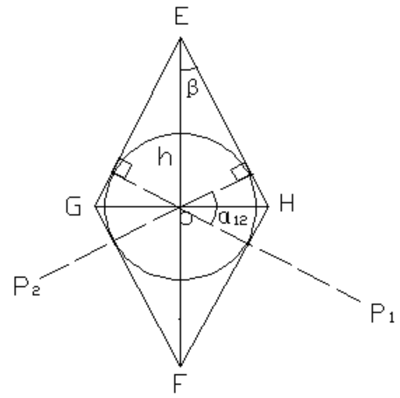

Figure 4. Section of the angle op_i and op_j

B. Location unit Distribution Model Theorem

Theorem 1. Suppose α_{ij} is the angle (acute angle) between the vector op_i and op_j. As $\alpha_{12} = \alpha_{13} = \alpha_{14} = \alpha_{23} = \alpha_{24} = \alpha_{34} \approx 70.5264°$, $V(\tilde{A}_p)$ reaches its minimum value. That is, as any two reference nodes of location unit have an angle (acute angle) with the connection line of the unknown node of 70.5264°, the positioning node's location error will be the smallest . At the same time, the location unit also form the smallest spatial location unit in the 3-D environment.

Proof. As shown in Fig.4, set the angle of OE and EH as β, then

$$h = \frac{\varepsilon}{\sin\beta}, \quad d = \frac{2\varepsilon}{\cos\beta}$$

The volume of the octahedron $V(\tilde{A}_p) = 2 \times \frac{1}{3} S_{\square ABCD} h$

Set the angle of AB and BC as γ, then

$$S_{\square ABCD} = \frac{d^2}{\sin\gamma},$$

$$V(\tilde{A}_p) = \frac{2}{3}\frac{d^2}{\sin\gamma}h = \frac{8\varepsilon^3}{3\sin\gamma\cos^2\beta\sin\beta}$$

Now the problem is how to choose γ and β to make the minimal volume of the octahedron.

Set $f(\gamma) = \frac{1}{\sin\gamma}$,

$$g(\beta) = \frac{1}{\cos^2\beta\sin\beta} = \frac{1}{(1-\sin^2\beta)\sin\beta}$$

Then as $\gamma = \frac{\pi}{2}, \beta = 2\arcsin(\frac{\sqrt{3}}{3}) \approx 35.2632^\circ$, $V(\tilde{A}_p)$ gets its minimal value.

$$V(\tilde{A}_p) = \frac{8}{3}\varepsilon^3 f_{\min}(\gamma) g_{\min}(\beta) \quad (4)$$

As $\gamma = \frac{\pi}{2}$ and $\sin\beta = \frac{\sqrt{3}}{3}$, it can be obtained that the octahedron have the same side length. As $ABCD$ is a square, the octahedron is a regular octahedron. Then it can be get:

$$\alpha_{12} = \alpha_{13} = \alpha_{14} = \alpha_{23} = \alpha_{24} = \alpha_{34} \approx 70.5264^\circ \quad (5)$$

That is, as the unknown nodes and the connected line of any two of the location unit have an angle (acute angle) are all equivalent to 70.5264 °, location error is minimal.

C. Convergence Analysis of Location Error

Theorem 2 n (integer $n > 1$) positioning units are used to compute the positioned nodes' position. As n increases, the positioned node location error will decrease. If n makes an unlimited increase which tends to be infinite, the location error tends to be a constant $\frac{4}{3}\pi\varepsilon^3$ (ε for the ranging error). So the location error is convergent.

Proof

1) If $n = 1$, by Theorem 1, the positioned error is about $V(\tilde{A}_{p0}) = 4\sqrt{3}\varepsilon^3$.

2) If $n = 2$, around the positioned node, a new error space is formed and denoted as A_{p1} ,and A_{p1} intersecting with A_{p0}. The sphere R_p is a common part of both. Outside of the R_p, the twelve small regions of A_{p0} separated by R_p are re-intersected by A_{p1}. Suppose that outside the sphere, the common region obtained through each small region intersection is $1/c$ of the original A_{p0} 's small region, where $c > 1$ and c is a constant. By deduction, the volume of $A_p = A_{p0} \cap A_{p1}$, named as $V(\tilde{A}_{p1})$, can be obtained as follows:

$$V(\tilde{A}_{p1}) = \frac{1}{c}(4\sqrt{3}\varepsilon^3 - \frac{4}{3}\pi\varepsilon^3) + \frac{4}{3}\pi\varepsilon^3 = \frac{1}{c}V(\tilde{A}_{p0}) + \frac{c-1}{3c}4\pi\varepsilon^3$$

3) On the analogy of what are mentioned above, when $n = 3$, $A_p = A_{p0} \cap A_{p1} \cap A_{P2}$, can be obtained as follows:

$$V(\tilde{A}_{p2}) = \frac{1}{c}(V(\tilde{A}_{p1}) - \frac{4}{3}\pi\varepsilon^3) + \frac{4}{3}\pi\varepsilon^3 = \frac{1}{c}V(\tilde{A}_{p1}) + \frac{c-1}{3c}4\pi\varepsilon^3$$

Similarly, we can deduce the following iterative relationship

$$\begin{cases} V(\tilde{A}_{P0}) = 4\sqrt{3}\varepsilon^3 \\ V(\tilde{A}_{P_n}) = \frac{1}{c}V(\tilde{A}_{P_{n-1}}) + \frac{c-1}{3c}4\pi\varepsilon^3 \end{cases}$$

Solve the difference equations

$$V(\tilde{A}_{P_n}) = \frac{4\sqrt{3}\varepsilon^3}{c^n} + \frac{c-1}{3c^n}4\pi\varepsilon^3 + \frac{c-1}{3c^{n-1}}4\pi\varepsilon^3 + \cdots + \frac{c-1}{3c}4\pi\varepsilon^3) \quad (6)$$

If n tends to infinity, limit Eq. (6), we will get

$$\lim_{n\to\infty} V(A_{P_n}) = \lim_{n\to\infty}(\frac{4\sqrt{3}\varepsilon^3}{c^n} + \frac{c-1}{3c^n}4\pi\varepsilon^3 + \frac{c-1}{3c^{n-1}}4\pi\varepsilon^3 + \cdots + \frac{c-1}{3c}4\pi\varepsilon^3) = \frac{\frac{c-1}{3c}}{1-\frac{1}{c}}4\pi\varepsilon^3 = \frac{4}{3}\pi\varepsilon^3 \quad (7)$$

The physical meaning of Eq.(7) is that after the introduction of countless positioning units, contributions to the error region are concentrated in the exterior sphere R_p until six small error regions are all eliminated due to iteration. But owing to the ranging error ε, no matter how many reference nodes are added, there would be no contribution to the interior of the sphere R_p. Therefore, the error tends to be a constant $\frac{4}{3}\pi\varepsilon^3$. That is to say, the location error converges.

III. THE LOCATING UNIT SELECTION ALGORITHM

In pervasive computing, the location algorithms aim at how to use the existing reference nodes to the position, to track a mobile terminal and obtain its movement track. Ordinary pervasive terminals work under a resource-constrained environment, which means limited computing power, storage capacity and communications capability. However, the traditional polygon location algorithm requires much time and space costs, so it is extremely difficult to apply to a resource-constrained pervasive terminal. Based on the distribution model of locating unit, the LUS algorithm is presented to replace traditional algorithms.

A. The traditional polygon location algorithm

To compare with the LUS algorithm presented in this paper, we will describe a traditional polygon location algorithm as follows.

1) Each reference node sends broadcasting packets, including location information denoted as $\{ID, T_{send}, (a,b,c)\}$. ID is identification of the reference node. T_{send} is time when the message is sent and (a,b,c) is its coordinate;

2) In virtue of the received multiple broadcasting packets, the parameters T_{send} and T_{rece} can be updated. Then, the distance between the two nodes can be computed according to $T_{diff} = T_{send} - T_{rece}$. Consequently, the positioned node's position can be approximated with the first four packets;

3) For all N received packets, take 4 packets out of N, we will get C_N^4 combinations. Calculate the distance between the two nodes respectively and decide whether they are in a plane or not;

4) For each locating units which are not in a plane, computing the positioned node's position respectively, we will get estimated positions $\{L_1, L_2, \cdots\}$;

5) Average elements of the estimated positions set, get estimated position of the positioned node L_{avg}. Then, the algorithm comes to an end.

Obviously, the traditional polygon location algorithm roughly calculates the positioned node's position at first, and then improves the locating precision through multiple similar iterative processes. With the interests of the locating units' amount, the algorithm will exponentially increase. In conclusion, the traditional algorithm is unable to meet the requirements of real-time positioning.

B. The locating units selection algorithm(LUS)

Based on theorems presented above, it is essential to place locating unit according to the theorem 1, The LUS algorithm is described as follows.

1) Each reference node sends broadcasting packets, including location information denoted as $\{ID, T_{send}, (a,b,c)\}$, and ID is identification of the reference node. T_{send} is time when the message is sent, and (a,b,c) is its coordinates;

2) The positioned node receives multiple broadcasting packets. For N received packets, combine C_N^4 times. During each combination, compute the distance between two nodes respectively using $T_{diff} = T_{send} - T_{rece}$, and then judge whether the locating unit can meets the theorem 1 conditions;

3) Roughly compute the positioned node's position using the initial locating unit;

4) As for each locating unit, meeting requirements, we compute the positioned node's position respectively and get a location set $\{L_1, L_2, \cdots\}$;

5) Average elements in the estimation positions set, termed as L_{avg}, then L_{avg} is the estimated position of the positioned node. The algorithm comes to an end.

The LUS algorithm has several advantages, such as fewer calculations, high real-time performance, and small location errors. In this way, it can be used to carry out real-time tracking to mobile users and meet location needs under the pervasive computing environment perfectly.

IV. SIMULATION EXPERIMENTS AND EVALUATION

A. Analysis and validation of theorem 1

The location error of the positioned node is affected by factors such as distance measurement error ε and the number of location unit participating in location n. Consider that the location errors vary with ε and n, values respectively by MATLAB. As we know, indoor distance measurement error varies from several centimeters to dozens of centimeters[9]. To discuss, we select four different measurement errors: 0.1m, 0.15m, 0.2m, and 0.3m. The test results are shown in Fig.5. Seen from Fig.5, the distance measurement error greatly influences the location error region. In addition, they follow a cube relationship. So during actual location computing, we should do our best to improve the accuracy of the distance.

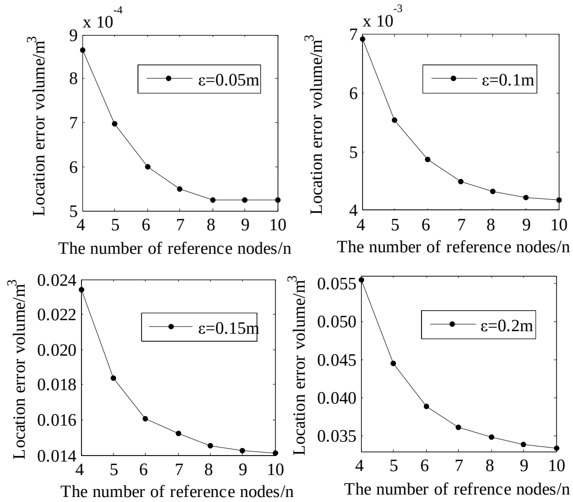

Figure 5. The effects of *N* and *ε* to location errors

Also, the number of location unit has a great influence on the location error. When the location unit increases, the location error decreases exponentially. If the number of location unit is larger than 6, the error decreases slowly. It is worth noting that the location error tends to be a constant if the number of location unit increases unlimitedly.

B. Real-time performance

Webit5.0 is a ubiquitous terminal. It is developed independently by the embedded technology laboratory of Liaoning province. Its master control chip is 8-bits micro controller AVR Atmega128. On the platform of the Webit5.0, we can compare the performance of the two algorithms with various numbers of reference nodes n, especially the real-time performance. In this experiment, only 12 reference nodes are used because of the capacity limitation of Webit5.0. The result is shown in Fig. 6.

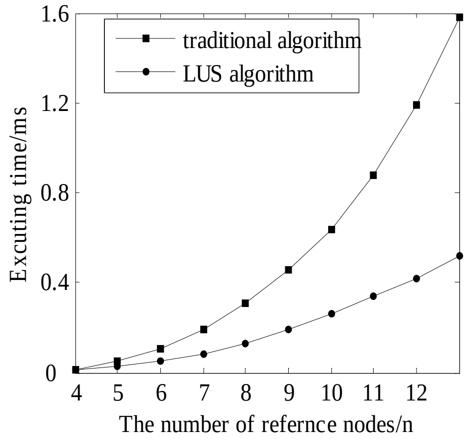

Figure 6. Reference nodes vs. cost positioning time

From the experimental results, when the number of reference nodes is about 7, the time cost of LUS algorithm is about 120ms, and the traditional polygon location algorithm is about 270ms. Especially, along with the increasing of reference nodes, the time cost of the latter will increase according to power law, while the time cost of LUS algorithm tends to be linear. In other words, to guarantee the real-time positioning with a small location error in an indoor environment, we shouldn't take many into location computing. Once the optimal computing unit is formed, location costs would only be tens of milliseconds. Thus, the algorithm meets the requirements of real-time positioning.

C. Location error

For simulation, 21 nodes have been built randomly by ns-2, the relationship between the location error and the number of reference nodes is shown in Fig. 7.

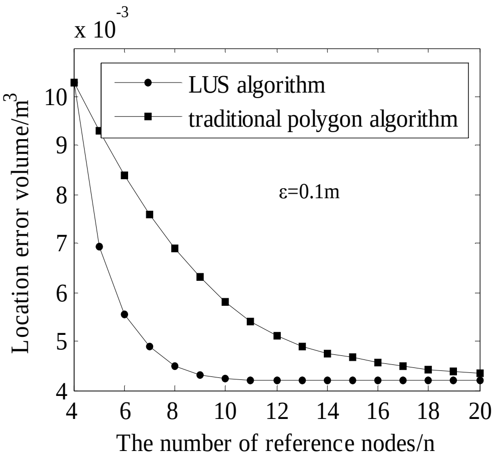

Figure 7. Location error area compared

As seen from Fig.7, with the increase of reference nodes, the location error of an unknown node decreases. The decrease seems to be smart in the beginning and slows down later. When the number of reference node is about 7, there is a great difference of the location error area between the two algorithms, and the LUS algorithm performs better. When the number of reference nodes number goes beyond 8, the traditional polygon location algorithm behaves better. Why? Because the number of location unit that meet theorem 2.1 grows slower than ordinary locating unit with the increasing of reference nodes, and the number of the new locating unit participating in the location is less. At the same time, the calculations of the traditional polygon location algorithm increases exponentially, which means getting a higher location precision by increasing computation.

D. Positioning track

To compare performances between the proposed LUS algorithm and the traditional polygons locating algorithm, we use ns-2 to simulate a 15.0m x 15.0m x 15.0m indoor environment. We place 21 reference nodes and one mobile unknown node. Therefore, we can do some comparisons between the positioning tracks by the two algorithms and the real mobile track. Error is on the level of centimeter. Easy to display, we only give part of the results in Fig. 8. Clearly, the proposed LUS algorithm is more accurate than the traditional polygon algorithm.

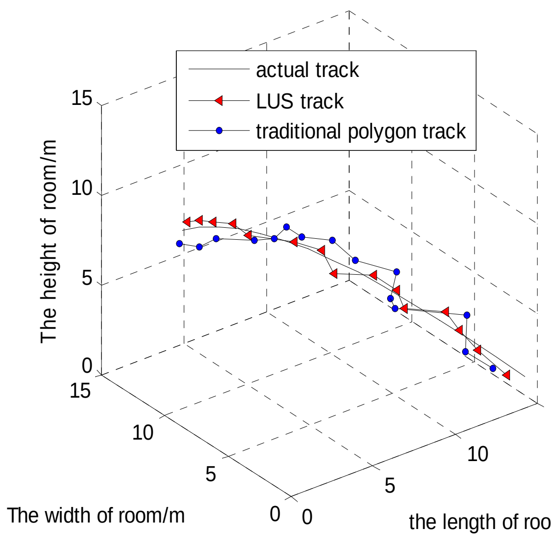

Figure 8. Comparisons between location tracks

V. CONCLUSION

It is obvious that location of an unknown node accurately is a key problem of location services in pervasive computing. To sum up, the paper is organized in the following way. First, reducing the location error is analyzed. Also,the model theorem of location unit distribution is presented and proved.The location unit distribution model theorem provide a theoretical basis for location unit distribution in three-dimensional space. Secondly, the location unit selection algorithm is presented on the basis of the improvement to the polygons location algorithm according to the theorem 1. It is proved that it cannot only reduce location error but meet the real-time location requirements. Finally, the theorem of location unit distribution and the location location unit selection algorithm are validated through simulation.

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