

# A Robust $k$ -Means Type Algorithm for Soft Subspace Clustering and Its Application to Text Clustering

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**Abstract**—Soft subspace clustering are effective clustering techniques for high dimensional datasets. Although several soft subspace clustering algorithms have been developed in recently years, its robustness should be further improved. In this work, a novel soft subspace clustering algorithm RSSKM are proposed. It is based on the incorporation of the alternative distance metric into the framework of  $k$ -means type algorithm for soft subspace clustering and can automatically calculates the feature weights of each cluster in the clustering process. The properties of RSSKM are also investigated. Experiments on real world text datasets are conducted and the results show that RSSKM outperformed some popular clustering algorithms for text mining, while still maintaining efficiency of the  $k$ -means clustering process.

**Index Terms**— $k$ -means, soft subspace clustering, text clustering

## I. INTRODUCTION

Recently, subspace clustering has become an effective data mining tool for high dimensional text data. It pursues two tasks, locating the subspaces in which clusters can be found and discovering the clusters from different subspaces. According to the ways in which the subspaces are identified, subspace clustering can be classified into two categories. The first category, which is called hard subspace clustering, is to locate the exact subspaces of different clusters [1][2][3][4]. In the algorithms of this category, the membership of a feature belonging to one cluster is identified by a binary value. The second category is called soft subspace clustering. The methods in this category cluster data objects in the entire data space but assign different weights to different features of clusters in the clustering process, based on the importance of the features in identifying the corresponding clusters [5][6].

Subspace clustering techniques need to compute the cluster memberships of data objects and the subspace of each cluster simultaneously [14], which throws a key challenge to researchers. Up to now, many subspace

clustering algorithms have been developed and have become effective and powerful methods in clustering high dimensional text data. However, most of them are still sensitive to noisy data. In this study, we will develop a novel robust  $k$ -means type clustering algorithm under the framework of soft subspace clustering. By incorporating the alternative distance metric to compute dissimilarities of data items along with each feature [7], the robust statistics is incorporated into the soft subspace clustering algorithms, which makes the algorithm suitable for the high dimensional sparse data and insensitive to the noise in the dataset.

## II. RELATED WORKS

Recently, many soft subspace clustering algorithms have been proposed. Generally speaking, most of them can be unified as the problem of finding the local minimum of the objective function

$$J(\mathbf{U}, \mathbf{W}, \mathbf{V}) = f\left(\sum_{h=1}^s w_{ih}^\alpha (x_{kh} - v_{ih})^2\right) + H \quad (1)$$

under the constraints  $\sum_{i=1}^c u_{ik} = 1$  and  $\sum_{h=1}^d w_{ih} = 1$ . For most existing algorithms, the first term

$f\left(\sum_{h=1}^s w_{ih}^\alpha (x_{kh} - v_{ih})^2\right)$  is interpreted as the total weighted distance of each data object to cluster centers and often computed as  $\sum_{i=1}^c \sum_{k=1}^n u_{ik} \sum_{h=1}^s w_{ih}^\alpha (x_{kh} - v_{ih})^2$ .

The second term  $H$  is a penalty term which is used to enhance the performance of the clustering algorithm. According to different forms of  $H$ , many soft subspace clustering algorithms are proposed in literatures, typical representatives of them are AWA [5], FWKM [8], FSC [9][10], EWKM[11], and COSA [12].

By inspecting these algorithms, it is clear that all the cluster centers along with each feature are computed as

$$\mathbf{v}_{ih}^* = \frac{\sum_{k=1}^n u_{ik} \mathbf{x}_{kh}}{\sum_{k=1}^n u_{ik}}, I=1, 2, \dots, c, h=1, 2, \dots, s. \quad (2)$$

Eq.(2) implies that each data point belonging to cluster  $i$  has equal weight 1 even though a data point is far away from other data points, which makes the cluster center heavily affected by the noisy data. In order to make the cluster centers more robust, we should give a smaller weight to those noisy data and a large weight to those compact data in the dataset. In order to achieve this goal, we use the following Eq.(3) as the distance function:

$$d = \sqrt{1 - \exp(-\beta \|\mathbf{x}_1 - \mathbf{x}_2\|^2)} \quad (3)$$

Obviously, it satisfies the following conditions [13]:

- (1)  $d(x,y) > 0, \forall x \neq y, d(x,x) = 0;$
- (2)  $d(x,y) = d(y,x);$
- (3)  $d(x,y) \leq d(x,z) + d(z,y), \forall z.$

After incorporating Eq.(3) into the framework of soft subspace clustering, Eq.(2) can be modified as:

$$\mathbf{v}_{ih}^* = \frac{\sum_{k=1}^n u_{ik} \mathbf{x}_{kh} \exp(-\beta_h (x_{kh} - v_{ih})^2)}{\sum_{k=1}^n u_{ik} \exp(-\beta_h (x_{kh} - v_{ih})^2)} \quad I=1, 2, \dots, c, h=1, 2, \dots, s \quad (4)$$

As we expect, it assigns larger weights to the data objects which are closer to cluster centers  $\mathbf{v}_i$  and smaller weights to those far away from  $\mathbf{v}$ . Thus algorithm will be more robust if the distance function Eq.(3) is utilized.

### III ALGORITHM RSSKM

By incorporating Eq.(2) into the framework of soft subspace clustering, we consider a novel algorithm named RSSKM with the following objective function:

$$J(\mathbf{U}, \mathbf{W}, \mathbf{V}) = \sum_{i=1}^c \sum_{k=1}^n u_{ik} \sum_{h=1}^s w_{ih}^\alpha \left( 1 - \exp(-\beta_h (x_{kh} - v_{ih})^2) \right) \quad (5)$$

under the following constraints

$$u_{ik} \in \{0,1\}, i=1, 2, \dots, c, k=1, 2, \dots, n \quad (6a)$$

$$\sum_{i=1}^c u_{ik} = 1, k=1, 2, \dots, n \quad (6b)$$

$$0 < \sum_{k=1}^n u_{ik} < n, I=1, 2, \dots, c \quad (6c)$$

and

$$0 \leq w_{ih} \leq 1, i=1, 2, \dots, c, h=1, 2, \dots, s \quad (7a)$$

$$\sum_{h=1}^s w_{ih} = 1, I=1, 2, \dots, c. \quad (7b)$$

in which  $c \geq 2$  is the cluster number,  $n$  is the number of data points. In Eq.(5),  $\sqrt{1 - \exp(-\beta_h (x_{kh} - v_{ih})^2)}$  is used to compute the distance between data point  $\mathbf{x}_k$  and cluster center  $\mathbf{v}_i$  along with the  $h$ th feature.

Similarly, the objective function of RSSKM can be minimized by iteratively solving the following three minimization problems:

(a) Problem P1: Fix  $\mathbf{W}=\mathbf{W}^*, \mathbf{V}=\mathbf{V}^*$ , solve the reduced problem that minimizes  $J_m(\mathbf{W}^*, \mathbf{V}^*, \mathbf{U})$  under the constraint Eq.(6).

(b) Problem P2: Fix  $\mathbf{V}=\mathbf{V}^*, \mathbf{U}=\mathbf{U}^*$ , solve the reduced problem that minimizes  $J_m(\mathbf{W}, \mathbf{V}^*, \mathbf{U}^*)$  under the constraint Eq.(7);

(c) Problem P3: Fix  $\mathbf{W}=\mathbf{W}^*, \mathbf{U}=\mathbf{U}^*$ , solve the reduced problem that minimizes  $J_m(\mathbf{W}^*, \mathbf{V}, \mathbf{U}^*)$ ;

By using Lagrange multipliers, Problem P1 is solved by

$$\mathbf{v}_{ih}^* = \frac{\sum_{k=1}^n u_{ik} \mathbf{x}_{kh} \exp(-\beta_h (x_{kh} - v_{ih})^2)}{\sum_{k=1}^n u_{ik} \exp(-\beta_h (x_{kh} - v_{ih})^2)} \quad i=1, 2, \dots, c, h=1, 2, \dots, s \quad (8)$$

In order to compute the cluster centers  $\mathbf{V}$  with Eq.(8), the fix-point iteration should be employed. It is an inefficient and time-consuming process. However, according to our experiment, we observe that it is enough to estimate the cluster centers  $\mathbf{V}$  with one step, which can also achieve ideal results. This improvement makes RSSKM convergent to its local minimum value quickly.

Similar with Problem P1, Problem P2 is solved by

$$w_{ih}^* = \frac{1}{\sum_{i=1}^s \left( \frac{D_{ih}}{D_{il}} \right)^{\frac{1}{\alpha-1}}} \quad (9)$$

$i=1, 2, \dots, c, h=1, 2, \dots, s,$

where  $D_{ih} = \sum_{k=1}^n u_{ik} \left( 1 - \exp(-\beta_h (x_{kh} - v_{ih})^2) \right),$

and Problem P3 is solved by

$$u_{ik}^* = \begin{cases} \sum_{h=1}^s w_{ih}^\alpha \left( 1 - \exp(-\beta_h (x_{kh} - v_{ih})^2) \right) & \\ 1 & \\ \leq \sum_{h=1}^s w_{ih}^\alpha \left( 1 - \exp(-\beta_h (x_{kh} - v_{ih})^2) \right), I=1, 2, \dots, c & \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

Now, the proposed algorithm RSSKM is given as follows:

Algorithm RSSKM
Input: The number of clusters $c$ , parameter $\gamma$
Randomly select $c$ cluster centers and set all initial weights to $1/s$ ;
REPEAT
Update the partition matrix $\mathbf{U}$ with Eq.(10);
Update the feature weights matrix $\mathbf{W}$ with Eq.(9);
Update the cluster centers $\mathbf{V}$ with Eq.(8) using the fixed-point iteration;
UNTIL (the objective function obtains its local minimum value);
Output: The partition matrix $\mathbf{U}$ and feature weights matrix $\mathbf{W}$ .

After a finite number of iterations, RSSKM algorithm converges to the local minimal of the objective function. Using Eq.(8), Eq.(9) and Eq.(10), we can show that the sequence  $J^{(l)}(\mathbf{U}, \mathbf{W}, \mathbf{V})$  generated by Eq.(5) decreases

strictly. Meanwhile, we can also observe that each possible partition  $\mathbf{U}$  only occurs once in the clustering process. Thus, RSSKM algorithm converges in a finite number of iterations [10].

Assuming  $s$  is the number of features,  $n$  is the number of data objects and  $c$  is the number of clusters, the computational complexity of RSSKM per iteration is  $O(snc)$ . On the other hand, we need  $O(ns)$  space to store  $n$  data points,  $O(cs)$  space to store  $c$  cluster centers  $\mathbf{V}$ ,  $O(cs)$  space to store the feature weight matrix  $\mathbf{W}$  and  $O(cn)$  space to store the partition matrix  $\mathbf{U}$ . That is to say, both the computational complexity and the storage complexity of RSSKM are linearly dependent on the number of data objects when the number of features is fixed. Thus, the proposed RSSKM algorithm is well suitable for high dimensional datasets and to large scaled datasets.

The proposed RSSKM is different from some previous work on soft subspace clustering. In this work, we incorporate the alternative distance into the framework of  $k$ -mean type algorithms for soft subspace clustering, which makes the data points far away from the cluster centers have smaller weights. Thus RSSKM will be more robust and the performance of RSSKM on noisy dataset is improved.

IV EXPERIMENTS

In this section, we present the clustering results obtained by RSSKM on the well-known text datasets *20 Newsgroups*, which was a publicly available dataset from website <http://kdd.ics.uci.edu/databases/20newsgroups/20newsgroups.html>. In our experiment, the original text data was first preprocessed to strip the news messages from the e-mail headers and special tags and eliminate the stop words and stem words to their root forms. Then, 1000 features were selected according to their IDF (inverse document frequency) values. In order to improve the

performance of the tested algorithms, the dataset was further processed using the following *tf-idf* formulas:

$$tf_{ij} = \frac{n_{ij}}{\sum_k n_{kj}}$$

$$idf_i = \log \frac{|D|}{|\{d : d \ni t_i\}|}$$

$$tfidf_{ij} = tf_{ij} \cdot idf_i \tag{11}$$

where  $n_{ij}$  denoted the term frequency of term  $t_i$  in document  $d_j$ ,  $|D|$  denoted the total number of the documents in dataset,  $|\{d : d \ni t_i\}|$  denoted the number of documents in which the term  $t_i$  appeared.

In our experiment, 6 datasets were established from the *20 Newsgroups*. As can be seen from Table 1, these datasets are divided into two series: series A and series B. The categories in series A are more semantically different than that in series B. The datasets of each series are generated incrementally by adding two more categories to the former dataset, resulting to name them NG20-A2 (B2), NG20-A4 (B4) and NG20-A6 (B6) accordingly. The number after the series code shows the number of categories in this dataset. For example, A2 denotes a dataset in series A with two categories in it. Each category contains  $n_{doc}$  documents which were chosen randomly from the original *20 Newsgroups* dataset.

In our experiments, we evaluate them by the performance index *RandIndex*, which was computed as follows:

$$RI = \frac{a + d}{a + b + c + d} \tag{12}$$

in which the value of  $a+b$  can be interpreted as the total pairs predicted in the same cluster and the value of  $a+d$  can be interpreted as the total pairs in the in the same class.

TABLE I.  
TWO SERIES OF NEWSGROUP DATASETS

NG20-A2	$n_{doc}$	NG20-B2	$n_{doc}$
comp.sys.ibm.pc.hardware (4)	500	comp.sys.ibm.pc.hardware (4)	500
talk.politics.guns (17)	500	comp.sys.mac.hardware (5)	500
NG20-A4	$n_{doc}$	NG20-B4	$n_{doc}$
comp.sys.ibm.pc.hardware (4)	500	comp.os.ms-windows.misc (3)	500
rec.autos (8)	500	comp.sys.ibm.pc.hardware (4)	500
sci.electronics (13)	500	comp.sys.mac.hardware (5)	500
talk.politics.guns (17)	500	comp.windows.x (6)	500
NG20-A6	$n_{doc}$	NG20-B6	$n_{doc}$
comp.sys.ibm.pc.hardware (4)	500	comp.os.ms-windows.misc (3)	500
rec.autos (8)	500	comp.sys.ibm.pc.hardware (4)	500
rec.sport.baseball (10)	500	comp.sys.mac.hardware (5)	500
sci.electronics (13)	500	comp.windows.x (6)	500
soc.religion.christian (16)	500	talk.politics.guns(17)	500
talk.politics.guns (17)	500	talk.politics.mideast(18)	500

TABLE II.  
PARAMETER SETTINGS OF THE ALGORITHMS

Algorithms	Parameter settings
RSSKM	$\alpha=1.25, 1.5, 1.75, 2, 2.25, 2.5, 2.75, 3$
W- $k$ -means	$\beta = 2; 5; 10; 50; 100; 1000; 10^4; 10^5$

AWA	$\alpha=1.25, 1.5, 1.75, 2, 2.25, 2.5, 2.75, 3$
FWKM	$\sigma = \frac{\sum_{k=1}^{\hat{n}} \sum_{h=1}^s (x_{kh} - v_{oh})^2}{\hat{n} \cdot s}$

TABLE III.  
RI PERFORMANCE FOR DIFFERENT ALGORITHMS

Datasets	RSSKM	<i>k</i> -means	W- <i>k</i> -means	FWKM	AWA
NG20-A2	<b>0.7136±0.0034</b>	0.5099±0.0215	0.4993±0.0001	0.6201±0.0317	0.6037±0.0097
NG20-A4	<b>0.7297±0.0005</b>	0.5000±0.0010	0.4993±0.0002	0.6304±0.0480	0.6002±0.0020
NG20-A6	<b>0.5367±0.0376</b>	0.3491±0.0684	0.3546±0.0101	0.4511±0.0201	0.4437±0.0738
NG20-B2	<b>0.5856±0.0293</b>	0.4121±0.0372	0.2608±0.0040	0.5201±0.1102	0.4994±0.0976
NG20-B4	<b>0.5620±0.0957</b>	0.3685±0.0694	0.3391±0.1130	0.5104±0.0976	0.4896±0.1279
NG20-B6	<b>0.5802±0.0377</b>	0.4489±0.0475	0.2877±0.0038	0.5211±0.1201	0.4990±0.1050

In the experiment, the performance of RSSKM was compared with four *k*-means type clustering algorithms, namely *k*-means, W-*k*-means [14], AWA [5] and FWKM [8]. The parameters used in these algorithms are tabulated in Table 2. Since the clustering results can be easily affected by the initial cluster centers, the random selection method for initial centers was used. In our experiments, each algorithm was repeated ten times. The clustering results were tabulated in Table 3, from which it can be easily observed that RSSKM outperforms its rivals in most cases. This indicates that the utilization of Eq.(3) as the distance function can make RSSKM more robust and improve its performance greatly.

V CONCLUSION

In this study, by integrating a novel distance metric into the learning criterion of the algorithm, a novel robust soft subspace clustering algorithm RSSKM is proposed. This work involves the following aspects: (1) a novel objective function integrating the alternative distance metric and soft subspace clustering is proposed based on the *k*-means type clustering algorithms; (2) a novel robust soft subspace clustering RSSKM is developed and its properties are investigated; (3) experiments on high dimensional text datasets are carried out to verify the performance of the RSSKM algorithm on high dimensional datasets.

It is necessary to find a suitable value for parameter  $\alpha$  in RSSKM. In this study, we only set these parameters empirically. Our future work involves further theoretical study on the parameters, which will be of great importance in providing useful and convenient guidelines for the real-world applications of the soft subspace clustering algorithms.

This study will be further extended to improve its performance by extending the *k*-means type clustering algorithms into the fuzzy clustering version. In addition, other ideas such as entropy weighting as well as multi-

view learning can also be integrated. In this way, the performance of the robust soft subspace clustering will be further improved.

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