Variational Image Decomposition Model OSV with General Diffusion Regularization

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Abstract—Image decomposition technology is a very useful tool for image analysis. Images contain structural component and textural component which can be decomposed by variational methods such as VO (Vese-Osher) and OSV (Osher-Sole-Vese) models. OSV model is a powerful tool for image decomposition but the minimization is a hard problem because of solving the 4th order partial differential equations with complex finite difference scheme for Laplacian of curvature. In this paper we proposed an improved OSV model with general diffusion regularization. The general diffusion terms can be TV (Total Variation), Nonlinear diffusion (Perona and Malik) and Charbonnier regularizers. Additionally, we also use L1 norm as data term inspired by TV-L1 method. We also use Split Bregman method for the easy implementation of the improved OSV model. Experiments show the proposed method is a valid method for image decomposition.

Index Terms—image decomposition, OSV model, Split Bregman method, TV-L1, general diffusion regularization

I. INTRODUCTION

Image decomposition technology can decompose the image into structural component, textural component, noise and other image components. The decomposed texture part is very useful in image analysis such as texture segmentation, texture discrimination and other applications.

The variational image decomposition methods are the popular ones. Total Variation (TV) model [1] is the basic nonlinear variational model of image diffusion. It laid the foundation for variational method of image processing and computer vision. Although it can separate noise from image, the texture part can't be decomposed by it. Meyer [2] established modeling the texture component as having a small norm in a suitably defined Banach space [3]. But Meyer didn't give the realization method. Le [4] proposed Besov space to describe the oscillation part of the image. Vese [5] proposed a VO model which approximates Meyer's theoretical model, that is, they proposed an $L^1$ approximation to the norm $\|\cdot\|_0$, meanwhile, they gave the corresponding Euler-Lagrange equations. Osher [6] extended VO model, and presented a variational model for image decomposition which based on the total variation and the norm $H^{-1}$. The authors show that this new model is simpler than VO model, however, the decomposition model based on this function suffers from low running time because the Euler-Lagrange equation is a fourth-order nonlinear PDE, its difference format is complex. Aujol [7] proposed two norms of Sobolev and Besov norms and split the image into three components, they are structure, texture and noise parts. Chan [8] introduced high order diffusion term to reduce the staircase effect and introduced the dual variables which can rapidly implement the decomposition of the image texture and structure information of the OSV model. Ng [9] introduced a decomposition model to restore blurred images with missing pixels. They used the total variation norm and its dual norm to regularize the cartoon and texture respectively. Then they recommended an efficient numerical algorithm based on the splitting version of augmented Lagrangian method to solve the problem.

In addition, to enhance the quality of image diffusion for classical ROF model, Osher [10] extended the ROF model to an iterative regulation method based on the Bregman Distance. They added the noise after diffusion to the original image to image diffusion again and repeated this process. This algorithm improved the quality of regularized solutions in terms of texture preservation, and reduced the influence of penalty parameter in the diffusing process. Although the computational efficiency has been greatly improved, it was still complex for implementation. To simplify implementation and improve computational efficiency, Wang [11] splitted the classical TV model into an alternating iterative process by simple divergence.
operation and shrinkage operator of soft threshold formula through the introduction of auxiliary variable. Goldstein [12] proposed Split Bregman method of ROF model by combining split algorithm [11] and Bregman iteration [10]. Zhao [12] proposed using Split Bregman method for solving OSV model. There were many other methods and applications for signal and image decomposition [17, 18, 19].

In this paper, we propose a general diffusion regularization of OSV method and using Split Bregman method by introducing auxiliary variables and Bregman iteration parameter for solving the equation. We devote to decompose an image $f$ into two well-structured component $u$ and oscillating patterns (both textures and noise) $v$.

The organization of this paper goes as follows. In Section 2, we will introduce the original OSV model briefly. Then the Split Bregman method of OSV model with general diffusion regularization is designed in Section 3. Then some numerical examples are shown in Section 4. Section 5 is concluding remarks.

II. ORIGINAL OSV MODEL

In [2], Meyer proposed the Banach space $G$ as:

$$G = \left\{ v = \partial_x g_1(x,y) + \partial_y g_2(x,y) : g_1, g_2 \in L^\infty(\Omega) \right\}$$

(1)

The norm is:

$$\left\| v \right\| = \inf_{g_1, g_2} \left\{ \sqrt{\left\| g_1^2 + g_2^2 \right\|} : v = \partial_x g_1 + \partial_y g_2 \right\}$$

(2)

Here, $\Omega$ is an open and bounded domain. Given an image $f$ defined on $\Omega$, Meyer’s decomposition model becomes:

$$\min_u \left\{ E(u) = \int_{\Omega} |\nabla u| + \lambda \left\| u - f \right\|, \cdot, f = u + v \right\}$$

(3)

In the model, $u$ is structural component or smooth part of the image, $v$ is oscillating component containing texture and noise information. But in practice, model (3) is difficult for implementation. Vese [6] overcome this difficulty by proposing an $L^p$ approximation to the norm $\left\| v \right\|$:

$$\min_{g_1, g_2} \left\{ \int_{\Omega} |\nabla u| \, dx \, dy + \lambda \int_{\Omega} |f - u - \nabla \tilde{g} \cdot \nabla \tilde{f}| \, dx \, dy \right\}$$

$$+ \mu \int_{\Omega} \left( \sqrt{g_1^2 + g_2^2} \right)^{\frac{1}{p}} \, dx \, dy$$

(4)

where, $\tilde{g} = (g_1, g_2)$, $\tilde{f} = \sqrt{g_1^2 + g_2^2}$, $v(x,y) = \partial_x g_1(x,y) + \partial_y g_2(x,y)$, $g_1, g_2 \in L^\infty\left( \mathbb{R}^2 \right)$. By experiments, the authors use the value $p=1$, and they show there are no obvious difference for different values of $p$, with $1 \leq p \leq 10$. Osher [7] used $L^p$ approximation to the norm $\left\| g_1^2 + g_2^2 \right\|$ and chose $p=2$ which corresponds to the space $H^1(\Omega)$. Then he proposed the famous image decomposition model called OSV model based on the negative norm $H^{-1}$ follows:

$$\min_u \left\{ E(u) = \int_{\Omega} |\nabla u| \, dx \, dy + \frac{1}{2\lambda} \int_{\Omega} \left( \nabla (f - u) \right)^2 \, dx \, dy \right\}$$

(5)

Equation (5) is complex because of the fourth order term of partial differential equations and low efficiency of computation. Aujol [8] used the dual variable $p$, and defined the dual form of the TV norm in the following:

$$\int_{\Omega} |\nabla u| \, dx \, dy = \max_{r \in H^1(\Omega)} \left\{ u \cdot \text{div}(p) \, dx \, dy \right\}$$

Hence, OSV model can be transformed as:

$$\min_{u, p} \left\{ E(u, p) = \int_{\Omega} u \cdot \text{div}(p) \, dx \, dy \right\}$$

$$\left. + \frac{1}{2\lambda} \int_{\Omega} \left( \nabla (f - u) \right)^2 \, dx \, dy \right\}$$

(7)

where $p = (p_1, p_2)$.

This problem can be solved by the method in [8]. Thus, the final iterative method is:

$$p^k - \tau \left( \frac{\nabla u^k}{\lambda} + \nabla (\text{div}(p)_{j+1}) \right)$$

$$p^0 = 0, p^{k+1} = \frac{p^k - \tau \left( \frac{\nabla u^k}{\lambda} + \nabla (\text{div}(p)_{j+1}) \right)}{1 + \tau \frac{\nabla u^k}{\lambda} + \nabla (\text{div}(p)_{j+1})}$$

(8)

$$u^{k+1} = f + \lambda \text{div} p^{k+1}$$

The convergence condition for (8) is proved in [9].

III. OSV MODEL WITH GENERAL DIFFUSION REGULARIZATION AND SPLIT BREGMAN ALGORITHM

The OSV model using total variation as diffusion term. There are many other diffusion term such as PM [14] diffusion term and Charbonnier term [15], they also give good metric in edge preserving and noise removing. So we proposed a general diffusion term for OSV model. Additional, the famous ROF model has some drawbacks such as staircase effects, loss of geometric characteristics, and so on. Chan [16] proposed TV-L1 model by changed
the data term from L2 norm to L1 norm as \( \int_{\Omega} |u - f| \, dx \, dy \).

This model can be effectively reduce the loss of contrast and geometric feature, but its calculations are more complex than the traditional TV model. In this paper, we also use TV-L1 term as data term for our general OSV model. Thus, the proposed decomposition model becomes:

\[
\min_u \left\{ E(u) = \frac{1}{2\alpha} \int_{\Omega} |\nabla u| \, dx \, dy + \frac{1}{2\beta} \int_{\Omega} \left( w_i - \nabla u - b^{i-1} \right)^2 \, dx \, dy \right\} \tag{1}
\]

The diffusion term of \( \varphi(\nabla u) \) has several patterns.

When diffusion term is TV norm, then

\[
\varphi(\nabla u) = |\nabla u| \tag{11}
\]

When diffusion term is PM norm, then

\[
\varphi(\nabla u) = \mu^2 \log \left( 1 + \frac{\|\nabla u\|}{\mu} \right) \tag{12}
\]

When diffusion term is Charbonnier, then

\[
\varphi(\nabla u) = 2\mu^2 \left( \sqrt{1 + \frac{\|\nabla u\|^2}{\mu^2}} - 1 \right) \tag{13}
\]

We use Split Bregman method for solving equation (10). With alternating optimization method, we introduce the auxiliary variables \( w_1 = (w_{11}, w_{12}) \), \( w_2 = (w_{21}, w_{22}) \) and Bregman iteration parameters \( b = (b_1, b_2) \), when the following energy functional gets its minimization,

\[
w_1 \approx \nabla u \quad \text{and} \quad w_2 \approx (\nabla^2) (u - f) \Rightarrow \Delta w_2 = u - f \quad \text{and} \quad w_3 \approx \nabla w_2 .
\]

Then equation (10) became the following form

\[
\min_{w_1, w_2, w_3} \left\{ E(u, w, w_1, w_2) = \frac{1}{2\alpha} \int_{\Omega} |\nabla u| \, dx \, dy + \frac{1}{2\beta} \int_{\Omega} \left( w_i - \nabla u - b^{i-1} \right)^2 \, dx \, dy \right\} \tag{14}
\]

where

\[
b^{i+1} = b^i + \nabla u^i - w^i \tag{15}
\]

One way of minimizing (14) amounts to solving the following minimization problems:

when \( u, w_1 \) and \( w_2 \) being fixed, we search for \( w_3 \) as solution of:

\[
\min_{w_3} \left\{ E_3(w_3) = \frac{1}{2\beta} \int_{\Omega} \left( w_i - \nabla u - b^{i-1} \right)^2 \, dx \, dy \right\} \tag{17}
\]

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\]

With variational method, the corresponding Euler-Lagrange equations respectively are:

\[
u^{k+1} = f + \Delta w_2^k + \frac{\theta_2}{\theta_1} \left( \Delta u^k + \nabla \cdot b^{k+1} - \nabla \cdot w_1^k \right) \tag{20}
\]

\[
w_1^{k+1} = \nabla u^{k+1} + b^{k+1} - \theta_1 \frac{\partial \varphi(|w_1|)}{\partial w_1} \tag{21}
\]

\[
\Delta (\Delta w_2^k) - \frac{\theta_2}{\theta_1} \Delta w_2^k = \Delta u^{k+1} - f - \theta_1 \nabla \cdot w_1^k \tag{22}
\]

\[
w_3^{k+1} = \nabla w_2^{k+1} - \frac{\theta_2}{\theta_1} \frac{\nabla w_1^{k+1}}{\|\nabla w_1^{k+1}\|} \tag{23}
\]

For TV term, equation (21) becomes:

\[
w_1^{k+1} = \nabla u^{k+1} + b^{k+1} - \theta_1 \frac{\nabla w_1^{k+1}}{\|\nabla w_1^{k+1}\|} \tag{24}
\]

For PM term, equation (21) becomes:

\[
w_1^{k+1} = \nabla u^{k+1} + b^{k+1} - \theta_1 \frac{\nabla w_1^{k+1}}{\|\nabla w_1^{k+1}\|} \tag{25}
\]

For Charbonnier term, equation (21) becomes:

\[
w_1^{k+1} = \nabla u^{k+1} + b^{k+1} - \theta_1 \frac{\nabla w_1^{k+1}}{\|\nabla w_1^{k+1}\|} \tag{26}
\]

In this paper, (20) uses the form of explicit iterative, (22) uses the form of semi-implicit iterative, (23) and (25) use a generalized shrinkage formula:

\[
w_1^{k+1} = \max \left( \nabla u^{k+1} + b^{k+1} - \theta_1, \varphi(w_1^k) \right) \frac{\nabla u^{k+1} + b^{k+1}}{\|\nabla u^{k+1} + b^{k+1}\|} \tag{27}
\]
For different diffusion terms, we replace the corresponding terms.

For TV term, equation (27) becomes:

$$w_i^{k+1} = \text{Max} \left( \nabla u_i^{k+1} + b_i^{k+1} \right) - \theta_1 \frac{\nabla u_i^{k+1} + b_i^{k+1}}{\nabla u_i^{k+1} + b_i^{k+1}}$$

(28)

For PM term, equation (27) becomes:

$$w_i^{k+1} = \text{Max} \left( \nabla u_i^{k+1} + b_i^{k+1} \right) - \theta_2 \frac{2 w_i^j}{(1 + \mu)^2} \frac{\nabla u_i^{k+1} + b_i^{k+1}}{\nabla u_i^{k+1} + b_i^{k+1}}$$

(29)

For Charbonnier term, equation (27) becomes:

$$w_i^{k+1} = \text{Max} \left( \nabla u_i^{k+1} - \frac{\theta_3}{\lambda} \frac{\nabla w_i^{k+1}}{\nabla w_i^{k+1}} \right)$$

(30)

(31)

The following describes the algorithm:

1. Initialization: \( u^0 = f \),

\[ w_i^0 = w_{i1}^0 = w_{i2}^0 = w_{i3}^0 = w_{i4}^0 = 0 \]

2. Iterations:

\[ b^{k+1} = b^k + \nabla u^k + w^k \]

\[ u^{k+1} = f + \Delta w_2^k + \frac{\theta_3}{\theta_1} \left( \Delta u^k + \nabla \cdot b^{k+1} - \nabla \cdot w_3^k \right) \]

\[ \Delta (\Delta w_2^k) - \frac{\theta_3}{\theta_1} \Delta w_2^k = \Delta u^{k+1} - \Delta f - \frac{\theta_3}{\theta_1} \nabla \cdot w_3^k \]

$$w_1^{k+1} = \text{Max} \left( \nabla u_1^{k+1} + b_1^{k+1} \right) - \theta_1 \phi(w_1^k, 0) \frac{\nabla u_1^{k+1} + b_1^{k+1}}{\nabla u_1^{k+1} + b_1^{k+1}}$$

$$w_2^{k+1} = \text{Max} \left( \nabla w_2^{k+1} - \theta_2 \frac{\nabla w_2^{k+1}}{\nabla w_2^{k+1}} \right)$$

(3) Stopping criterion: we stop if \( \max |u^{k+1} - u^k| \leq \varepsilon \)

IV. NUMERICAL EXPERIMENTS

All results of experiments in this paper are implemented on PC (Intel(R) Core 2 Duo, CPU E7400 2.80GHz, Memory 2.00GB) using matlab 7.0. To verify the effect of the proposed model, we test our method on three gray images shown in Fig.1. The first is a gray image which is a combination of four different Brodatz textures, the second a famous image called 'Barbala' and the third is the famous image "Lena" corrupted by a Gauss noise ( \( \sigma = 15 \) ).

Results are shown in Fig.2, Fig.3 and Fig.4. Texture image \( v \) is obtained from \( f - u \) (the gray value of \( v \) is too small to display clearly, so the corresponding figure shows the decomposed texture added by 150). The choice of \( \lambda \) controls the texture components: the bigger the parameter \( \lambda \) is, the less the texture contained in \( u \); the smaller the parameter \( \lambda \) is, the less the texture contained in \( v \). In the experiments, we set \( \lambda = 0.2 \).

Figure 1: Three original images for image decomposition: (a) Brodatz textures image. (b) Clean Barbara gray image containing structure and texture components. (c) Noisy observed Lena image, \( \sigma = 15 \).

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From Fig.2, we can see the original OSV model has less decomposition ability than our proposed methods. The texture information in Fig. 2 (d), Fig.2 (g) and Fig.2 (h) are more clarity than that in Fig.2 (c).

At the edge of the turban and scarf at the bottom of the chin in Fig.3 (a) are still remained the texture, yet, the texture in Fig.3 (b), Fig.3 (e) and Fig.3 (f) are all disappeared.
In the experiments of Fig. 4, the noise is also decomposed in the process of decomposing textures. The denoising effect of our proposed methods is better than the original OSV model. From the details of textural part, we can see our proposed models have superiority in the decomposition of the texture and other details.

From above experiments, we can see that the proposed general diffusion terms for OSV model have achieved the goal of texture decomposition. The general diffusion terms with TV norm, PM norm and Charbonnier have small difference but are all superior than the original OSV model in the ability of texture decomposition. The decomposed texture part using our methods are much more richer than the original one.

To verify the efficiency of our proposed methods, we compare the run time of each experiments. Results of time trials for the experiments mentioned above are shown in Table 1, and all the units are second(s).

<table>
<thead>
<tr>
<th>Solving method</th>
<th>CPU time of each iteration</th>
<th>Total CPU time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Fig 1(a)</td>
</tr>
<tr>
<td>Original-OSV model</td>
<td>0.125</td>
<td>9.406</td>
</tr>
<tr>
<td>OSV model with TV norm</td>
<td>0.043</td>
<td>2.131</td>
</tr>
<tr>
<td>OSV model with PM norm</td>
<td>0.051</td>
<td>3.250</td>
</tr>
<tr>
<td>OSV model with Charbonnier</td>
<td>0.057</td>
<td>3.631</td>
</tr>
</tbody>
</table>

It can be shown from Table 1, Split Bregman method of our proposed method has less time consuming than original method.

V. CONCLUSION

In this paper, we propose modified OSV models with general diffusion terms. Then the Split Bregman method is introduced to enhance the computational efficiency. Experiments show the validity and efficiency of proposed method. However, there are lots of parameters in the model and the parameter value influences the test results largely, so the next step is to establish a parameter adaptive algorithm to eliminate the influence of man-made factors on the parameter values.

ACKNOWLEDGMENT

This work was supported by National Natural Science Foundation of China (No.61305045 and No.61170106), National "Twelfth Five-Year" development plan of science and technology (No.2013BAI01B03), Qingdao science and technology development project (No. 13-1-4-190-jch).

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