

Rasch GSP Toolbox for Assessing Academic Achievement

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Abstract— The main purpose of this research is to computerize GSP chart and Rasch GSP methods. GSP chart and Rasch GSP are effective and useful methods which used for analyzing and evaluating academic achievement. GSP chart provides information for diagnosing the learning status of students and the quality of questions while Rasch GSP provides information for assessing the quality of classes and test quality. Therefore, this research has constructed Rasch GSP toolbox that is a support tool for teachers and researchers in using and studying GSP chart and Rasch GSP methods. The research has also proposed a number of improvements in Rasch GSP method and made supplements in order to increase the application range of the toolbox. Through specific examples, the ways of using this toolbox and applying Rasch GSP method are explained in detail in this paper. Rasch GSP toolbox can quickly calculate and give exact results for users, beside, GSP graphs and Rasch GSP graphs are also presented clearly. Hence, it helps users easily view the visual image of assessment result. In addition, a graphical user interface (GUI) is also designed to be easy for use. In practice, many researches that applied Rasch GSP toolbox have showed that it is a useful evaluation tool. It not only can supply big support to researchers in their research process but also can provide feedbacks about teaching and learning to teachers and students. So students can make reasonable adjustments for learning activity and teachers have an effective reference for learning guidance.

Index Terms— Academic assessment, GRA, GSP chart, MATLAB, Rasch GSP, S-P chart.

I. INTRODUCTION

In 1982, Deng first proposed grey system theory whose research object is the "poor information" and "small sample" systems [1, 2]. This theory is developed rapidly and applied in many fields such as economics, meteorology, ecology, biology, military, medicine, education, etc. Grey relational analysis (GRA) is one effective mathematical tool of grey system theory. GRA measures the degree of similarity or difference between

two sequences based on the grade of relationship [3, 4]. In GRA, the global comparison between two sets of data is used instead of local comparison by measuring the distance between two points in topological space. Thus, it avoids the side effect of subjective setting of parameters within the model [5].

Nagai proposed Grey Student - Problem chart (GSP chart) in 2010 by combining GRA with Student - Problem chart (S-P chart). This new analytical method can overcome the defects that suffered by the S-P chart in the test data evaluation [6]. GSP chart is used in many studies, is suitable for researching on uncertain and unclear factors, for dealing with multivariable, discrete and incomplete data. Additionally, Nagai also proposed using logistic model to evaluate data sets in GSP chart, thereby created Rasch GSP method. Rasch GSP method can give visual representation and characteristic values of set of students and set of questions. Many studies proved the effectiveness of Rasch GSP in educational evaluation [6-8].

The main purpose of this research is to study how to computerize GSP and Rasch GSP methods. This will help to reduce complex mathematical calculations for users, provide student assessment results and question assessment results quickly and precisely. This also opens the possibility of combining Rasch GSP method with other methods of assessment, thereby to form an effective assessment system for evaluating academic achievement of students. Rasch GSP toolbox which is the result of this research can quickly calculate and provide exact, visual results. A graphical user interface is also designed to help users to quickly become familiar and use the toolbox easily. Especially, this research also proposed and executed some improvements in using GSP and Rasch GSP methods, so expanded the application range of these methods. Rasch GSP toolbox is actually useful for evaluating, diagnosing the achievement of students.

II. BASIC THEORY

Rasch GSP toolbox is designed for using and studying GSP chart and Rasch GSP methods. The related theories are shown below:

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A. Student - Problem Chart

Student - Problem chart, which was proposed by Takahiro Sato in 1969, is a useful tool for diagnosing the student's learning state and the item quality [9-11].

Definition 1: Student-Problem Chart

Let item response matrix of students as follow:

$$Y = [y_{ij}]_{m \times n} \quad (1)$$

$$\text{where } y_{ij} = \begin{cases} 1 & \text{if answer is right} \\ 0 & \text{if answer is wrong} \end{cases},$$

Student number $S_i, i = 1, 2, \dots, m$

Problem number $P_j, j = 1, 2, \dots, n$

Caution Index for Student (CS): Let caution index for student

$$CS_i = 1 - \frac{\sum_{j=1}^n (y_{ij})(y_{\cdot j}) - (y_{i\cdot})(\bar{y})}{\sum_{j=1}^l (y_{\cdot j}) - (y_{i\cdot})(\bar{y})} \quad (2)$$

$$\text{where } \bar{y} = \frac{1}{n} \sum_{j=1}^n y_{\cdot j} \text{ and } l = y_{i\cdot} = \sum_{j=1}^n y_{ij}$$

Caution Index for Problem (CP): Let caution index for problem

$$CP_j = 1 - \frac{\sum_{i=1}^m (y_{ij})(y_{i\cdot}) - (y_{\cdot j})(\bar{y}')}{\sum_{i=1}^{l'} (y_{i\cdot}) - (y_{\cdot j})(\bar{y}')} \quad (3)$$

$$\text{where } \bar{y}' = \frac{1}{m} \sum_{i=1}^m y_{i\cdot} \text{ and } l' = y_{\cdot j} = \sum_{i=1}^m y_{ij}$$

TABLE I.
S-P CHART

	Problem number $P_j, j = 1, 2, \dots, n$	Total score	CS
Student number $S_i, i = 1, 2, \dots, m$	$Y = [y_{ij}]_{m \times n}$	High $\updownarrow SS_i$ Low	CS_i
Number of correct answer	More \longleftrightarrow Less PP_j	$\sum_{i=1}^m SS_i = \sum_{j=1}^n PP_j$	
CP	CP_j		

Note. Adapted from "Analysis of Test Score Patterns: The Student - Problem (S - P) Technique", D. L. McArthur, 1983, *CSE Report No 218*, 2.

Based on CS and rate of problems answered correctly by student, students are divided into six patterns. Based on CP and rate of students answering problem correctly, problems are divided into four patterns (as shown in Fig. 1 and Fig. 2).

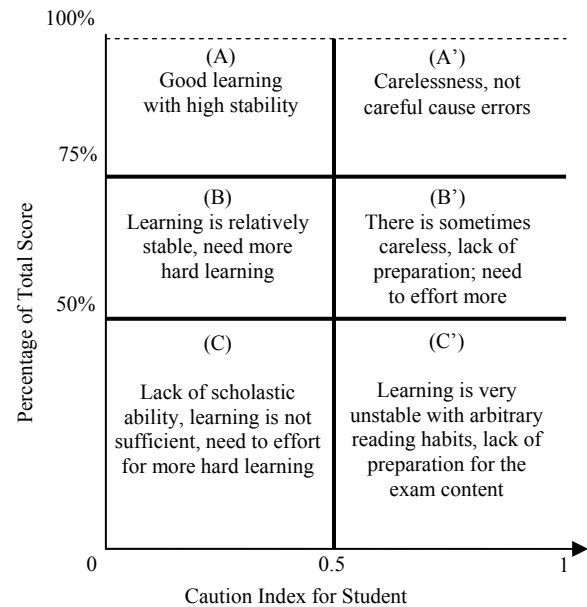


Figure 1. Student diagnostic analysis

Note. Adapted from *Educational Testing and Assessment* (p. 378), by M. N. Yu, 2011, Taipei: Psychology Publisher.

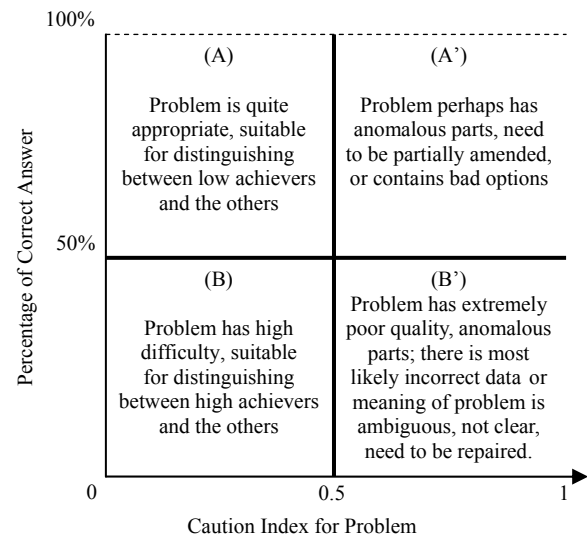


Figure 2. Item diagnostic analysis

Note. Adapted from *Educational Testing and Assessment* (p. 375), by M. N. Yu, 2011, Taipei: Psychology Publisher.

B. Grey Relational Analysis

This paper refers to localized grey relational analysis that is proposed by Nagai [12]. Its procedures are shown as follows:

Establish the reference vector y_0 and inspected vectors of the raw data y_{ij} , $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$, listed as follows [13]:

$$\begin{aligned} y_0 &= (y_{01}, y_{02}, \dots, y_{0n}) \\ y_1 &= (y_{11}, y_{12}, \dots, y_{1n}) \\ y_2 &= (y_{21}, y_{22}, \dots, y_{2n}) \\ &\vdots \\ y_m &= (y_{m1}, y_{m2}, \dots, y_{mn}) \end{aligned} \quad (4)$$

Generations of grey relation: The established series has to satisfy three conditions: non-dimension, scaling, polarization. Grey relational generation has three ways: larger-the-better (the expected goal is bigger the better), smaller-the-better (the expected goal is smaller the better), and nominal-the-better (the expected goal is between maximum and minimum) [12].

Definition 2: Localized Grey Relational Grade (LGRG)

$$\Gamma_{0i} = \Gamma(y_0, y_i) = \frac{\bar{\Delta}_{\max} - \bar{\Delta}_{0i}}{\bar{\Delta}_{\max} - \bar{\Delta}_{\min}} \quad (5)$$

$$\text{where } \bar{\Delta}_{0i} = \|y_0 - y_i\|_\rho = \left(\sum_{j=1}^n [\Delta_{0i}(j)]^\rho \right)^{\frac{1}{\rho}}$$

$\bar{\Delta}_{0i}$ is the Minkowski distance between target point which is characterized by reference vector y_0 and considered point which is characterized by inspected vector y_i .

Grey Relational Ordinal: The whole decision-making is made by the comparison of the grey relation Γ_{0i} . Through the ordinal, different causes can be identified, and the most important influence can be found, becoming the relational standard in the system.

C. Rasch Model

The Rasch model is one of the effective models for educational and psychological measurement. In 1960, Georg Rasch first announced this model for hoping through the response of the answerers obtain an objective interval scale that can measure the latent trait of answerer [14]. In the Rasch model, the correct response probability of a student is a logistic function of the difference between that student's ability and the item difficulty [15]. The relation between latent trait (theta) and correct response probability is described by an item characteristic curve (ICC). The Rasch model has descriptive function and predictive function. In descriptive function, this model can clearly explain the relationship between student's ability and item difficulty, the difference between students and the difference between items. In

predictive function, this model can predict the probability of a student who has a specified ability to answer a specified item correctly [14].

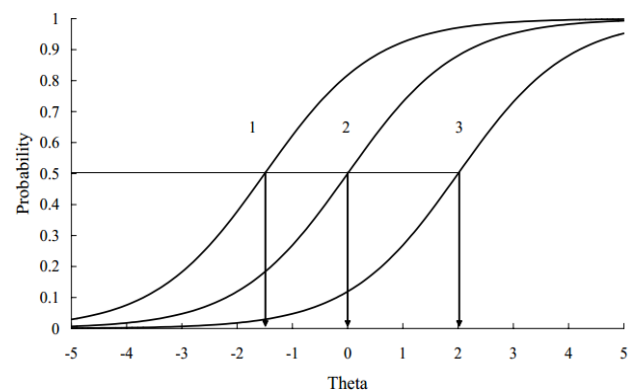


Figure 3. ICC for three different items in Rasch model

Note. Adapted from "Rasch Measurement Theory and Application in Education and Psychology," by W. C. Wang, 2004, *Journal of Education & Psychology* 27(4), p. 644.

D. GSP Chart and Rasch GSP

GSP chart was first proposed by Nagai in 2010. Essentially, GSP chart is a combination of GRA and S-P chart. The GSP chart was developed in order to overcome the weaknesses of the S-P chart. GSP chart can make the analysis more concrete and accurate, and the uncertain factors in the studies can also be analyzed. The weight or ordinal numbers between the discrete data can be evaluated [16, 17]. In this method, the relative level of an object (in a set of defined objects) with a target is considered based on the combination of the difference between each attribute of the object and the corresponding attribute of the target. The value that is used to assess an object is the relative level of this object (with the target) compared with all relative levels of other objects in the set. The selected target depends on the purpose of evaluation.

TABLE II.
GSP CHART

	Problem number P_j , $j = 1, 2, \dots, n$	Total score	LGRG-S	CS
Student number S_i , $i = 1, 2, \dots, m$	$Y = [y_{ij}]_{m \times n}$	SS_i	High $\updownarrow GS_i$ Low	CS_i
Number of correct answer	PP_j	$\sum_{i=1}^m SS_i = \sum_{j=1}^n PP_j$		
LGRG-P	More \longleftrightarrow Less GP_j			
CP	CP_j			

$$GS_i = \Gamma_{S0i} = \Gamma(y_{S0}, y_{Si}) = \frac{\bar{\Delta}_{S \max} - \bar{\Delta}_{S0i}}{\bar{\Delta}_{S \max} - \bar{\Delta}_{S \min}} \quad (6)$$

where

$$y_{Si} = (y_{i1}, y_{i2}, \dots, y_{in})$$

$$y_{S0} = (\max_{\forall i}(y_{i1}), \max_{\forall i}(y_{i2}), \dots, \max_{\forall i}(y_{in}))$$

$$GP_j = \Gamma_{P0j} = \Gamma(y_{P0}, y_{Pj}) = \frac{\bar{\Delta}_{P \max} - \bar{\Delta}_{P0j}}{\bar{\Delta}_{P \max} - \bar{\Delta}_{P \min}} \quad (7)$$

where

$$y_{Pj} = (y_{1j}, y_{2j}, \dots, y_{mj})$$

$$y_{P0} = (\max_{\forall j}(y_{1j}), \max_{\forall j}(y_{2j}), \dots, \max_{\forall j}(y_{mj}))$$

Rasch GSP method uses logistic regression to study and analyze the relationship between two sets of data: the set of the order value of students (or the order of item difficulty) and set of the local grey relational grade. Its purpose is to find a function that represent all the data [18]. This function is called Rasch GSP function, the graph of this function is called Rasch GSP graph.

The logistic regression model used in this study is the three - parameters logistic model. Its mathematical formula is described as follows:

$$y = f(x) = \gamma + \frac{1 - \gamma}{1 + e^{-\alpha(x - \beta)}} \quad (8)$$

Where, x is the order value of student (or the order of item difficulty), y is the local grey relational grade α , β and γ are parameters.

III. BUILDING TOOLBOX

A. Software Requirements Specification

- 1) Windows XP, Windows 7 or upgrade versions.
- 2) Screen resolution 1024×768.
- 3) MATLAB version 7.10 or upgrade versions.

B. Progress for Building Toolbox

Step 1: Determine the purpose of the program design

The program was designed to assist teachers in evaluating academic achievement of student based on the Rasch GSP method.

Step 2: Determine the requirements that the program must satisfy

After completion, the program must be accurate, fast and easy to use.

Step 3: Construct the flowcharts of the program

The algorithms of the overall program and each subroutine are specifically built and represented by the flowcharts.

Step 4: Test the flowcharts of the program

This study uses Crocodile ICT 605 software to test the accuracy of the algorithm of the program. Crocodile ICT 605 is powerful software for drawing flowcharts, and

testing result of algorithms.

Step 5: Write the program based on MATLAB language

MATLAB is a high-level language and interactive environment for numerical computation, visualization, and programming. MATLAB can be used for a range of applications, including signal processing and communications, image and video processing, control systems, test and measurement, computational finance, and computational biology, etc. [19-22]. In this study, the program is developed by MATLAB 7.10 (2010a) software.

Step 6: Test the program

The program will be tested several times to ensure it satisfies the requirements determined in step 2. If not satisfied, the program continues to be tested, repaired and improved.

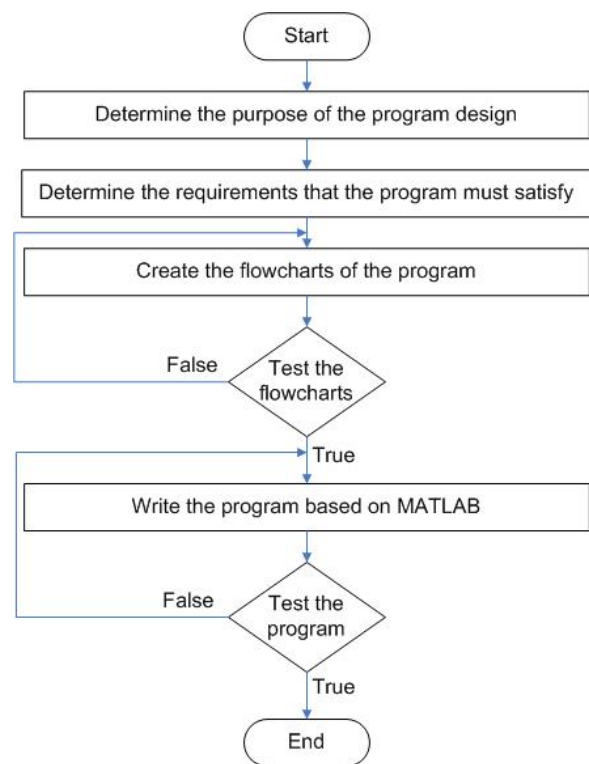


Figure 4. The progress of designing the toolbox

C. Algorithms for Program

1. Main Algorithm

The program has 4 basic steps.

Step 1: Input data

Data is the response matrix of students that is numerical and written in *.csv file.

Step 2: Calculate GSP chart

Firstly, the program calculates LGRG-S and LGRG-P. Then, based on LGRG-S values and LGRG-P values, the program arranges students from high LGRG-S to low LGRG-S and arranges items from more LGRG-P to less LGRG-P. Finally, the program calculates all percentage, caution indices and determine student patterns and item patterns.

Step 3: Draw graphs

In this stem, based on the choice of user, the program will draw GSP graph, Rasch GSP graph.

Step 4: Choose for new data.

If new data is selected, the program will continue and back to step 1, or else the program will be ended.

The main algorithms of the program are described in figures 5, 6, 7 and 8. The most important parts of the program are described in detail in section 2, section 3 and section 4.

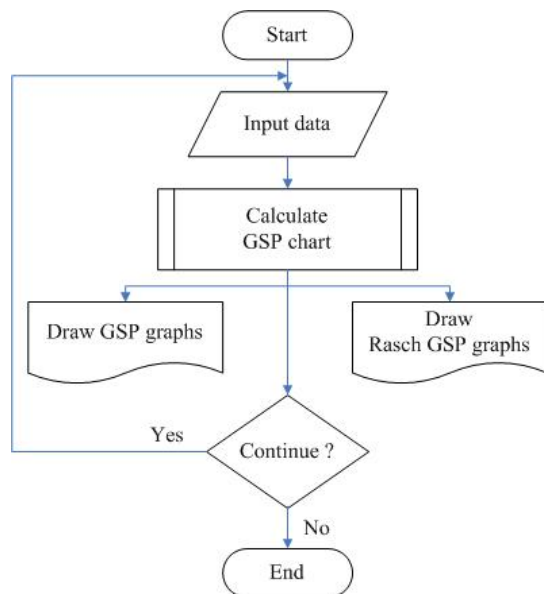


Figure 5. The flowchart for the main procedures

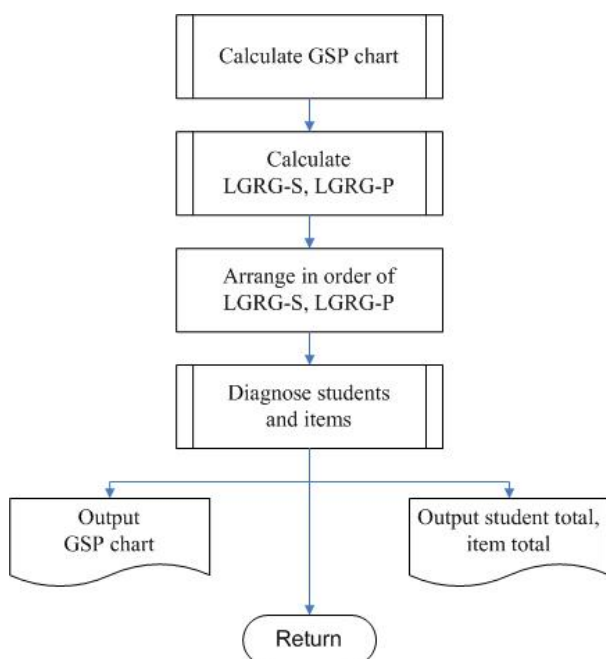


Figure 6. The flowchart for calculating GSP chart

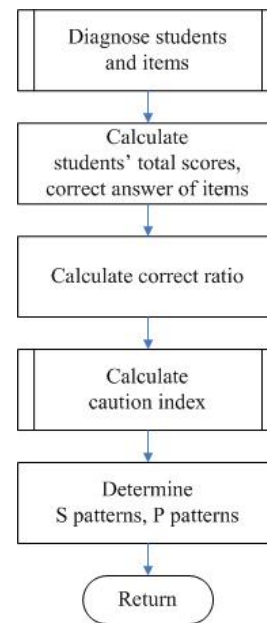


Figure 7. The flowchart for diagnosing student and item

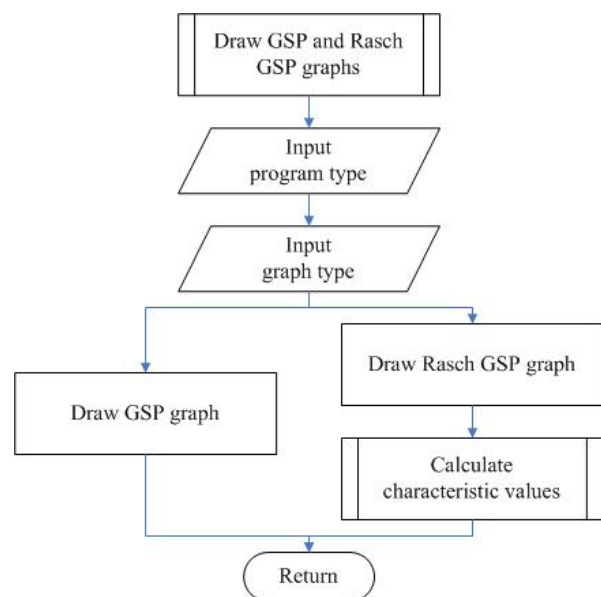


Figure 8. The flowchart for drawing GSP graph and Rasch GSP graph

2. Algorithm for Calculating LGRG

This toolbox proposes new LGRG calculation method for calculating LGRG-S and LGRG-P. For calculating LGRG-S by this method, the highest score set $y_{S \max}$ and the lowest score set $y_{S \min}$, which students can achieve at each item, have to be determined. The reference vector is $y_{S \max}$. The set of the inspected vectors includes: $y_{S \max}$, the set of the scores vectors of all students and

$y_{S\min}$. Thus, the achievement of a student not only is assessed in relation to the achievement of other students in the class, but also is referred to the best achievement. LGRG-S obtained through this method not only helps to compare the achievement of students in a class, but also help to compare different classes. Calculating LGRG-P is also performed in similar manner.

The algorithm for calculating LGRG is shown as follows:

- 1) Determine the highest score vector y_{\max} and the lowest score vector y_{\min} .
- 2) Establish set of inspected vectors include: y_{\max} , answer response of students and y_{\min} .
- 3) Set y_{\max} as reference vector.
- 4) Calculating LGRG according to (5).

This study proposes to use matrix operations for calculating LGRG. This makes the calculation of the program becomes simple and faster. LGRG is calculated by function *HieuLGRA* that described in Fig. 9.

```
function [gamma] = HieuLGRA(data,zeta)
% data is a matrix that its first row is
the reference vector and the other rows
are comparative vectors.
% zeta is the distinguish coefficient.
% gamma is the vector of localized grey
relational grades.
TK=data(1,:);
MTXX=data(2:end,:);
MTTK= repmat( TK, size(MTXX,1),1);
MTabs=abs( (MTTK'-MTXX')' );
b=(sum(MTabs.^zeta,2)).^(1/zeta);
m=max(b); n=min(b);
gamma=(m-b)./(m-n);
end;
```

Figure 9. The code for function *HieuLGRA*

3. Algorithm for Calculating Caution Index

The most complex task in calculating GSP chart is determining caution indices. In this task, the caution index of each student is determined according to (2), while the caution index of each item is determined according to (3). However, the calculation done for each object makes the program become more complex. This research proposes to calculate caution indices by matrix operations. Thereby, it is possible to calculate the caution indices of all students at the same time. This way is also applied to determine the caution indices of items. The algorithm proposed by this research is shown in MATLAB language as Fig. 10.

```
% Score_M is the students' response
matrix that obtained after rearranging
S-P chart.
% sumI is the vector that shows the
total score of each student.
% sumS is the vector that shows the
correct answer number for each item.
% CS is the vector that shows the
caution index of each student.
% Calculate real response matrix:
ReaS=Score_M*sumS';
% Calculate perfect response matrix:
TMS=sort(Score_M,2,'descend');
PerS=TMS*sumS';
% Calculate expected response
matrix:
HopS=(sum(sumS,2)/size(sumI,1))*sumI;
% Calculate caution indices
CS=(PerS-ReaS)./(PerS-HopS);
```

Figure 10. The code for calculating caution indices

4. Algorithm for Finding Characteristic Values

In Rasch GSP method, each set of students (or items) has two characteristic values: partition b and discrimination a . Let the value of LGRG is γ . Value of b is the abscissa of the intersection of the line $\gamma = 0.5$ and the logistic regression line. Value b divides set of students (or items) into two groups: the first group includes all objects whose $\gamma < 0.5$ and the second group includes all objects whose $\gamma > 0.5$. Value a is the slope of the tangent of logistic regression line at the point $(b, 0.5)$. Value a represent the discrimination between group of high γ and group of low γ . The algorithms for calculating them are shown as follows:

- 1) Find the equation of the logistic regression line according to (8).
- 2) Set $y = 0.5$, find the intersection of line $y = 0.5$ with the regression line.
- 3) Set b equal the abscissa of the intersection.
- 4) Determine the equation of first derivative of the function y . The form of this equation is:

$$y'(x) = \frac{(1-\gamma) \alpha e^{-\alpha(x-\beta)}}{(1+e^{-\alpha(x-\beta)})^2} \quad (9)$$

- 5) Find the value of the function y' at $x = b$. Set a equal the value of the function y' at $x = b$.

$$a = \frac{(1-\gamma) \alpha e^{-\alpha(b-\beta)}}{(1+e^{-\alpha(b-\beta)})^2} \quad (10)$$

This study proposes the algorithm in MATLAB language as Fig. 11.


```

% Heso is the coefficient vector of
Rasch model GSP function.
% a and b are the characteristic values.
binhquan=solve(f3(t,Heso)-0.5);
daoham=diff(f3(t,Heso));
phanbiet=subs(daoham,t,binhquan);
b=double(binhquan);
a=double(phanbiet);

function f = f3(x, Heso)
f = Heso(1)+(1-Heso(1))*exp(Heso(2)...
...*(x-Heso(3)))/(1+exp(Heso(2)...
...*(x-Heso(3))));
end;

```

Figure 11. The code for calculating values of a and b

IV. RESEARCH RESULT AND APPLICATION

A. Working with the Toolbox

The whole process of working with this toolbox is executed with a graphical user interface (GUI). This GUI can be divided into three areas. The left area includes button “Input data”, button “START”, GSP chart and the

buttons for selecting graph type. The central area has boxes to show the total of students, the total of problems, Cronbach's alpha coefficient, graphs to display total score distribution and item correct answer distribution. The right area contains the area for displaying graphs and boxes show characteristic values of the Rasch GSP graph.

Data in a selected folder is imported through button “Input data”. Then, user clicks the button “START” to start the program. All results of GSP chart will be shown in “GSP chart”. GSP graphs and Rasch GSP graphs will be plotted if user finishes selecting graph type and clicking button “Draw”. Depending on the choice of user, the toolbox can plot one of the following type: GSP graph for student, Rasch GSP graph for student, GSP graph for problem, and Rasch GSP graph for problem. This toolbox can also plot different graph types at the same time. “Save graph” button allows users to save graph in JPEG format to a selected folder.

Fig. 12 is an example for the result output by the program. Raw data that suggested by the study is the response result of 14 students for 9 items.

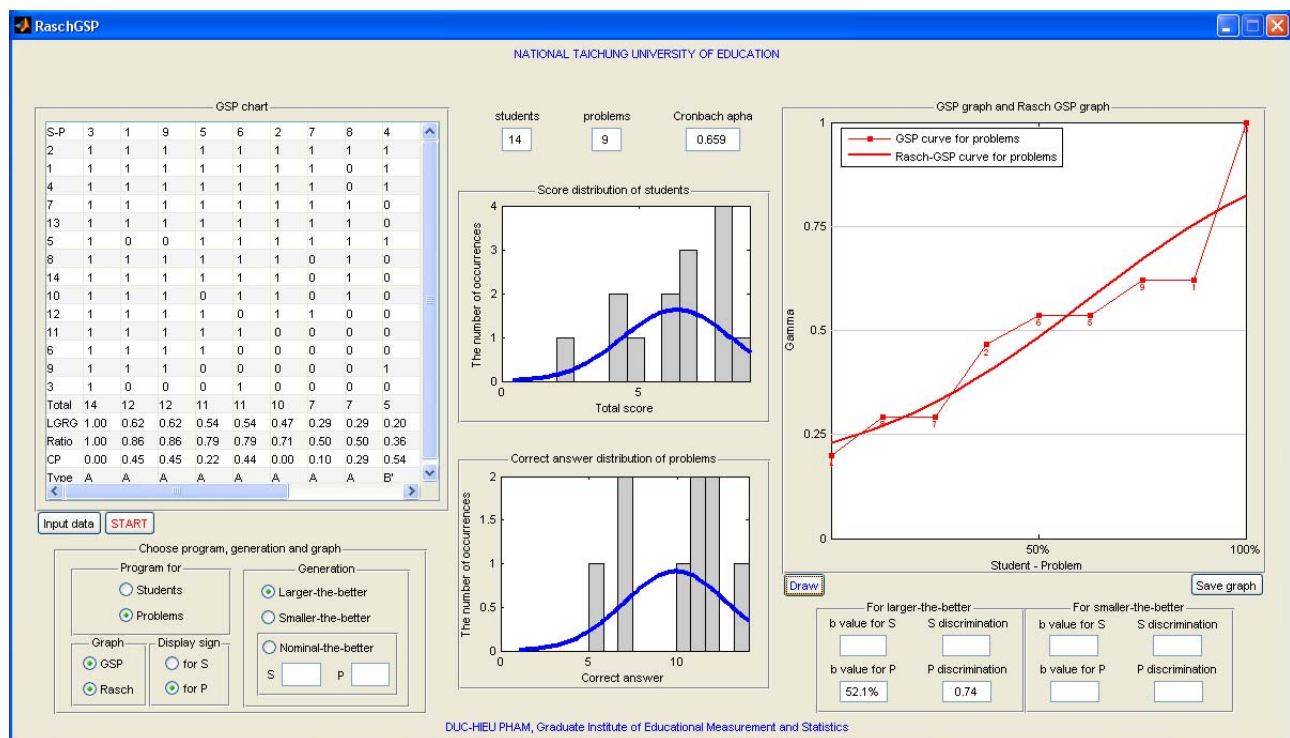


Figure 12. The graphical user interface of the toolbox

B. Application Example

This study uses the test results of the two classes of high school students in Taichung, Taiwan for instance using the program in academic assessment. Class A consists of 33 students and class B consists of 30 students. Both classes participated in a 25-item test. The Cronbach's alpha coefficient of the test result of class A is 0.864 and the Cronbach's alpha coefficient of the test result of class B is 0.879.

1. Student Diagnosis and Item Diagnosis

The toolbox is built according to GSP chart which was developed by combining S-P chart analysis method with grey relational analysis method. The GSP chart provided by this toolbox also contains all the necessary information of S-P chart. Thus, through this toolbox, users can obtain the information of S-P chart, especially information for student diagnosis and item diagnosis. In

addition this toolbox can provide diagnostic information in case of partial credit scoring which S-P chart cannot. The results from GSP chart analysis method applying for class A are shown in Table III.

TABLE III.
DIAGNOSIS RESULT BY GSP CHART FOR CLASS A

For student					For problem				
Student	Total	Ratio	CS	Type	Item	Total	Ratio	CP	Type
2	25	1.00	0.00	A	11	29	0.88	0.36	A
7	25	1.00	0.00	A	13	28	0.85	0.23	A
6	23	0.92	1.14**	A'	10	26	0.79	0.55*	A'
18	22	0.88	0.36	A	1	25	0.76	0.20	A
24	22	0.88	0.36	A	7	25	0.76	0.22	A
30	22	0.88	0.29	A	2	24	0.73	0.23	A
10	19	0.76	0.28	A	6	24	0.73	0.52*	A'
14	19	0.76	0.17	A	22	24	0.73	0.12	A
21	19	0.76	0.42	A	24	22	0.67	0.52*	A'
17	18	0.72	0.50*	B	9	21	0.64	0.18	A
22	18	0.72	0.12	B	19	21	0.64	0.33	A
16	17	0.68	0.54*	B'	12	20	0.61	0.26	A
25	17	0.68	0.39	B	18	20	0.61	0.40	A
33	16	0.64	0.25	B	17	19	0.58	0.26	A
1	14	0.56	0.41	B	14	18	0.55	0.17	A
9	14	0.56	0.27	B	4	17	0.52	0.70*	A'
20	14	0.56	0.30	B	20	17	0.52	0.38	A
27	14	0.56	0.68*	B'	16	16	0.48	0.19	B
29	14	0.56	0.44	B	23	16	0.48	0.61*	B'
31	13	0.52	0.52*	B'	21	15	0.45	0.40	B
13	11	0.44	0.66*	C'	25	12	0.36	0.74*	B'
15	11	0.44	0.48	C	3	11	0.33	0.85**	B'
28	11	0.44	0.62*	C'	5	10	0.30	0.34	B
32	11	0.44	0.42	C	8	10	0.30	0.12	B
3	10	0.40	0.70*	C'	15	10	0.30	0.32	B
8	10	0.40	0.50*	C'					
5	9	0.36	0.52*	C'					
11	9	0.36	0.50*	C					
12	8	0.32	0.78**	C'					
26	8	0.32	0.37	C					
23	7	0.28	0.62*	C'					
19	6	0.24	1.34**	C'					
4	4	0.16	0.32	C					

In Table III, the student whose $CS \geq 0.5$ (marked by * or **) have unusual answer response, especially the student having $CS \geq 0.75$ (marked by **). The item whose $CP \geq 0.5$ (marked by * or **) is unusual item, especially the item having $CP \geq 0.75$ (marked by **). These items need to be checked, adjusted or even removed.

2. Visualization of Achievement and Difficulty

In GSP chart, the achievement of student and the difficulty of question are expressed by the corresponding LGRG-S or LGRG-P. GSP graphs visually describe achievement of all students as well as the difficulty of all questions. On the GSP graph for students, each student is represented by a point that its abscissa is determined by the position of student on ordinal scale of achievement and its ordinate is determined by corresponding LGRG. On the GSP graph for items, each item is represented by a point that its abscissa is determined by the position of item on ordinal scale of difficulty and its ordinate is determined by corresponding LGRG. The GSP graphs for class B are presented in Fig. 13.

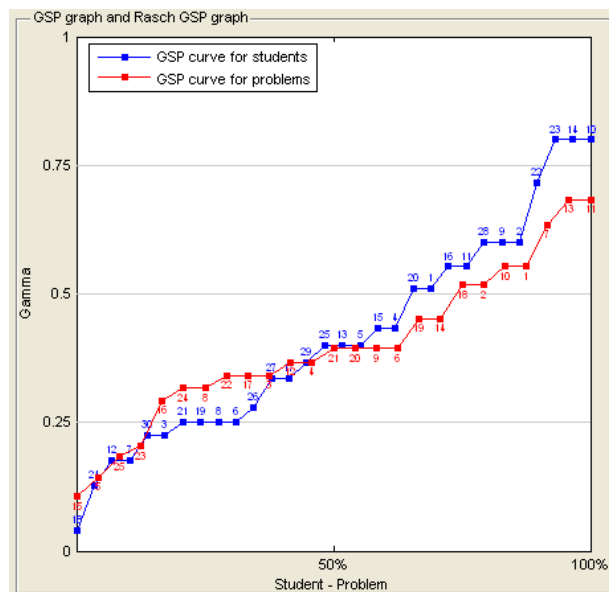


Figure 13. GSP graphs for class B

The graphs of total score distribution and item correct answer distribution are shown in Fig. 14 and Fig. 15.

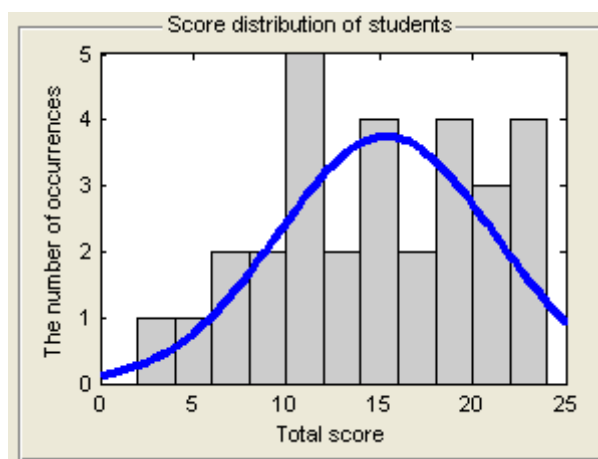


Figure 14. Total score distribution of class B

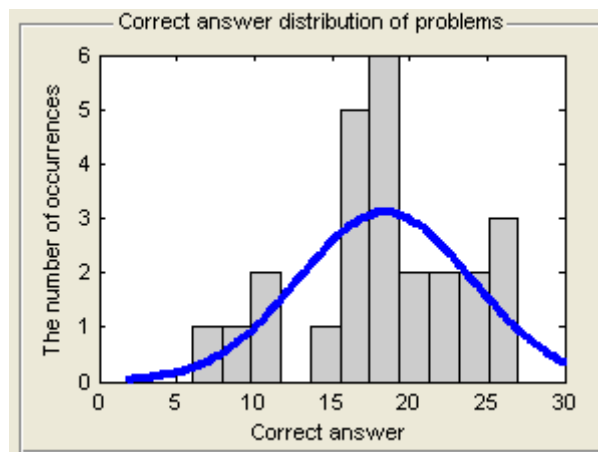


Figure 15. Item correct answer distribution for class B

3. Evaluating Achievement of a Class and Test Difficulty

In practice, GSP chart and Rasch GSP were often combined together as an approach. Calculating LGRG is the most important step that is used in both methods. When calculating LGRG for students, the inspected vector in previous studies is answer response of student and the reference vector is the highest score set which students have actually achieved at each item. This way is only used to evaluate the achievement of students in a class, cannot be used to compare achievement between the different classes. In Rasch GSP toolbox, $y_{S \max}$ and $y_{S \min}$ are added to the set of the inspected vectors, and the reference vector is $y_{S \max}$. For a test, this reference vector is the same for all classes. Thus, all classes are evaluated with the same criteria. The LGRG calculation way that is proposed by this study not only helps to compare the achievement of students in a class, but also helps to compare different classes. Calculating LGRG for items is also performed in similar manner as calculating LGRG for students.

In Rasch GSP method, each class (or each test) is characterized by a Rasch GSP curve. Such a curve can provide two characteristic values for the set that it represents. The meaning of these characteristic values depends on choosing expected goal in grey relational analysis. There are three ways for choosing expected goal: larger-the-better, smaller-the-better and nominal-the-better. Though this program can work with all three ways, there is only the first way is mentioned in this example. The GSP graphs that based on this way have been shown in Fig. 13. Fig. 16 and Fig. 17 describe the Rasch GSP graphs for two classes A and B.

In larger-the-better method, the student whose $\gamma < 0.5$ has low achievement, the student whose $\gamma > 0.5$ has high achievement. Thus, partition b divides set of students into two groups: low achievement group and high achievement group. For a class, the smaller value of b the larger the percentage of high achievement students. The academic result of this class is high. In contrast, the larger value of b the larger the percentage of low achievement students. The academic result of this class is low. Fig. 16 and Fig. 17 show that the achievement of class B (with $b = 64.9\%$) is better than the achievement of class A (with $b = 71.2\%$).

Discrimination a for students is the slope of Rasch GSP curve for students at the point $(b, 0.5)$. This value shows the difference in achievement between high group and low group. Thus, it represents the discrimination of the test done by the class. If a has a larger value, the difference between two groups is high. The discrimination of the test is strong. In contrast, when a has a small value, the difference is low. The discrimination of the test is weak. Comparison between Fig. 16 and Fig. 17 shows the discrimination of the test is stronger for class A (with $a = 1.17$) but weaker for class B (with $a = 0.81$).

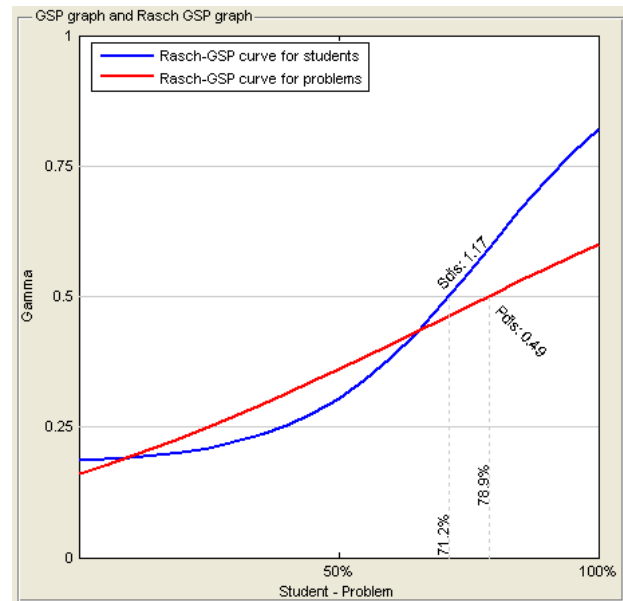


Figure 16. Rasch GSP graphs for class A

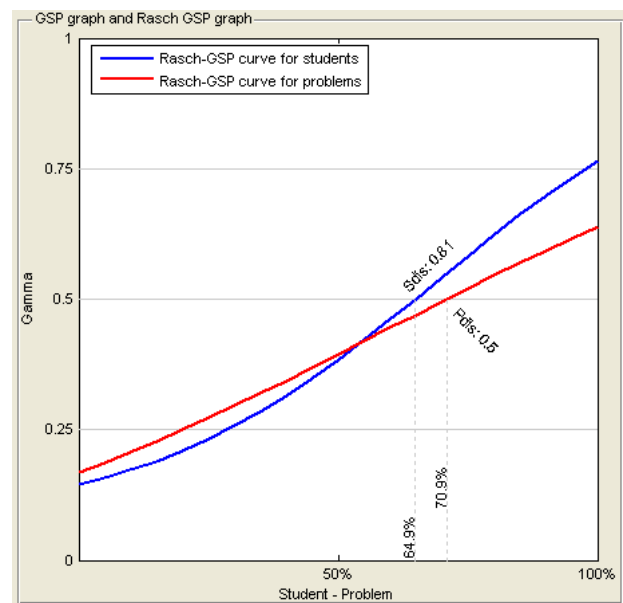


Figure 17. Rasch GSP graphs for class B

In larger-the-better method, the item whose $\gamma > 0.5$ has high difficulty, the item whose $\gamma < 0.5$ has low difficulty. Thus, partition b divides set of items into two groups: high difficulty group and low difficulty group. For a test, the smaller value of b the larger the percentage of low difficulty items. The difficulty of this test is low. In contrast, the larger value of b the larger the percentage of high difficulty items. The difficulty of this test is high. In this example, the difficulty of the test to class A (with $b = 78.9\%$) is higher than class B (with $b = 70.9\%$). Discrimination a for items is the slope of Rasch GSP curve for items at the point $(b, 0.5)$. This value shows the difference in difficulty between items. When a has a larger value, there are the large differences between items in the test.

When a has a small value, the differences are low. For class A, the discrimination a for items is 0.49. For class B, the discrimination a for items is 0.50. Thus, these two classes are almost similar in difference in difficulty of items.

V. CONCLUSION

GSP chart and Rasch GSP are new and effective methods in analyzing and evaluating the student's academic achievement. Based on these approaches and combination with MATLAB software, this study has successfully developed a new assessment toolbox. This toolbox has many advantages such as: easy for use, time-saving, and accurate and clearly visual output.

The main function of this program can be concluded as follows:

1) Based on the students' response result, the program can help to evaluate the academic achievement of students and compare one student to other. Moreover, it not only helps to evaluate academic achievement of each class, but also compares the academic achievements of classes.

2) Based on the result of answering items in test, it can help for evaluating and comparing the difficulty between items become easy and convenience. In addition, it can also help to evaluate the difficulty of the whole test and compare the difficulty of tests with each other easily.

In short, this toolbox is a useful tool for teachers in analyzing and evaluating the learning performance of students, and helps researcher convenience to conduct their educational researches.

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