A Prediction Method for Underwater Acoustic Chaotic Signal Based on RBF Neural Network

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Abstract— **In this paper, the chaotic time series RBF neural network model was designed. A prediction method for underwater acoustic chaotic signal based on RBF neural network is proposed in this paper according to the characteristics of chaotic signal with the short-term prediction. Typical Henon chaotic signal and the actual underwater acoustic chaotic signal are respectively predicted by the RBF neural network. Then the prediction results are analyzed. The results show that the proposed prediction method increases at least two orders of magnitude in the mean square error terms compared with existing prediction method, and that the RBF neural network can be used to predict the chaotic signal effectively.**

Index Terms—**chaotic signal, phase space reconstruction, RBF neural network, prediction**

I. INTRODUCTION

Chaotic behaviors extensively exist in various natural and social systems, such as atmosphere, traffic, acoustics, economics, and biomedicine. They are an evolutionary pattern standing between randomness and certainty [1]. Chaotic signal is generated by deterministic nonlinear dynamical system. The prediction of chaotic signal is of great significance for the analysis and study of nonlinear dynamic system. However, due to the sensitivity of the chaotic system to initial conditions, small changes in the input may cause big differences in the output. At present, the more feasible method is the short-term prediction of chaotic time series [2]. The prediction of chaotic signal can be regarded as inverse problem in the study of dynamics system. Forward problem is to study various properties of the phase space orbit for a given nonlinear dynamic system. Inverse problem is to construct a nonlinear mapping for expressing the original system by a given string of iterative sequence or a group of observation sequence. This mapping can be used as predictive model. Therefore, how to construct predictive model is a key problem in chaotic signal prediction. Prediction of the chaotic signal has become a research focus in the current chaotic signal processing field [3-5].

All traditional methods of time series predicting belong linear approach. Therefore, people put forward many nonlinear prediction methods for chaotic time series based on Takens theory. It can be roughly divided into the global prediction method, local prediction method and adaptive nonlinear filtering prediction method [6-8]. However, these methods require highly appropriate reconstructed phase space, and are more sensitive to noise. In recent years, people put forward the method of complex nonlinear system modeling such as fuzzy system model, support Vector Machine Model, and so on [9-15]. But in order to ensure the prediction accuracy for chaotic time series, these methods also have some problems such as the difficulty of choosing parameters, the complexity of computation time, and so on.

The neural network [16-19] not only has the self-adaptive, parallelism, simple structure, fast training speed, and fault tolerance characteristics, but also has the ability to approximate any nonlinear function. Based on these advantages, the neural network model of the nonlinear system has a very wide range of applications [20-24]. In recent years, particular interest has been put into predicting chaotic time series using neural networks because of their universal approximation capabilities. It is widely used in the prediction of time series [25-27].

This paper is organized as follows. Firstly, chaotic signal is analyzed based on the phase space reconstruction theory. Secondly, Typical Henon chaotic signal and the actual underwater acoustic chaotic signal are respectively predicted by using the RBF neural network. Then the prediction results are analyzed. Finally, meaningful conclusions are obtained.

II. THE PHASE SPACE RECONSTRUCTION OF CHAOTIC TIME SERIES

Phase space reconstruction is the base of chaotic time series analysis by using dynamical system method. Chaotic systems can be usually described by low-order differential equation. Therefore, if all the data of a variable is known, then the system is known. It is clear that any variable of the system is decided by other variables interacting with this variable. The one-dimensional time series is embedded to multi-dimensional phase space through reconstruction, and the new system with the same dynamic characteristics as original system can be obtained by the selection of a suitable embedding dimension D and time delay τ [28]. For the selection of τ, the most commonly used methods are auto-correlation function method, complex auto-correlation function method, multiple correlation function method, average displacement method and mutual information method. For the selection of D, the most

doi:10.4304/jsw.9.6.1581-1586

commonly used methods are GP algorithm, singular value decomposition method, pseudo-nearest-point method, Cao method and C-C method.

Based on the Takens' delay-coordinate phase reconstruct theory, the chaotic time series can be predicted. If the chaotic time series are { $x(n)$ }, then the reconstruct state vector can be represented as follows:

$$\mathbf{x}(n) = (x(n), x(n+\tau), \cdots, x(n+(D-1)\tau)) \quad , \tag{1}$$

where D ($D = 2, 3, \cdots$) and τ are called the embedding dimension ($D = 2d+1$, d is called the freedom of dynamics of the system), and the delay time, respectively. The prediction of chaotic signal can be regarded as inverse problem in the study of dynamics system. Thus, a smooth function defined on the reconstructed manifold in R^m to interpret the dynamics can be got as follows:

$$x(n+T) = F(x(n)) \, , \tag{2}$$

where T ($T > 0$) is forward predictive step length and $F(\cdot)$ is the reconstructed predictive model [28-29].

Evolution equation of nonlinear chaotic system can be represented by nonlinear difference equation:

$$x(n+1) = F(X(n)) \, , \tag{3}$$

where $X(n)$ is d-dimensional state vector at moment *n* in the system. *d* is the system state space dimension. The observing system *h* is assumed. The observed time series is defined as $\{g(n)\}$ by $x(n)$:

$$\mathrm{g}(n) = h(x(n)) + \omega(n) \, , \tag{4}$$

where $h(\cdot)$ is a scalar-valued function, $\omega(n)$ is additive observation noise. Equation (3) and Equation (4) describe the operating state in the state space.

According to Takens embedding theorem, when $\omega(n) = 0$, we select the appropriate delay time τ and dimension D. Then the vector is constructed with the system of a single variable at time *n* observed values $g(n)$.

$$g_r(n) = \left[g(n), g(n-\tau), ..., g(n-(D-1)\tau)\right]. \tag{5}$$

Geometrical structure of the system dynamics characteristics can be expanded in the new state space consisting of vector.

If $D > 2d+1$, the d-dimensional reconstruction vector $g_r(n)$ has the same dynamic characteristics of the original system. The smallest integer dimension D of the original system dynamics characteristics is called the embedding dimension, denoted by D_E. Embedding theorem proves that $g_r(n) \to g_r(n+1)$ in refactoring space evolution is varies with the change of $X(n) \to X(n+1)$ in the original space, and their dynamics characteristics are consistent.

Therefore, $F_\sigma(\cdot)$ vector function exit in the reconstructed phase space and the trajectory of attractor in the reconstruction phase space can be shown as follows:

$$g_r(n+\tau) = F_\tau(g_r(n)) \, . \tag{6}$$

Without loss of generality, we let $\tau = 1$. The Equation (5) can be rewritten as follows:

$$\begin{aligned}&\left[g(n+1), g(n), \cdots g(n-D+2)\right] = \\ &F_\tau\left(\left[g(n), g(n-1), ..., g(n-D+1)\right]\right)\end{aligned} \, . \tag{7}$$

At this point, $F_\sigma(\cdot)$ can be used as a predictive model of the original system. It can be proved that $g(n+1)$ can be got by a step prediction function ψ.

$$\mathrm{g}(n+1) = \psi\left(\left[g(n), g(n-1), \cdots g(n-D+1)\right]\right), \tag{8}$$

where ψ is a nonlinear function and one step prediction function in the chaotic background.

III. A Prediction Method Based on RBF Neural Network

Reconstruction phase space problem is transformed into approximate one step prediction function ψ problem. By using radial basis function (RBF) neural network, the global approximation method is used to determine one step prediction function ψ. After reconstructing the phase space, the RBF neural networks adopt three layers networks which are the hidden unit output function, the network input and output function shown in Figure 1. It consists of an input layer, an output layer and a hidden layer which has K RBF neural unit.

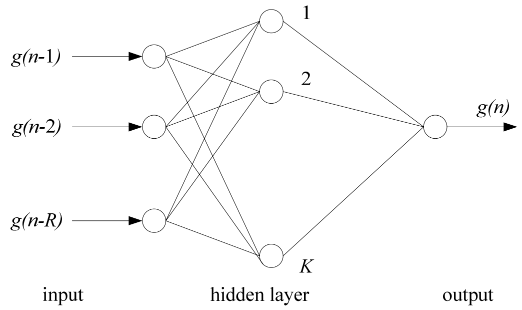


Figure 1. RBF neural network model

Based on Takens embedding theorem on chaotic signal modeling, input unit of designing RBF neural network is an integer number R. When $D = D_E$, Choosing R satisfies the following formula:

$$R \geq D_E \tau \, . \tag{9}$$

So that formula (6) becomes

$$\mathrm{g}(n+1) = \psi\left(\left[g(n), g(n-1), \cdots g(n-R+1)\right]\right). \tag{10}$$

Reconstruction vector provide more information to the training of prediction model in order to obtain more accurate prediction. Gaussian RBF function is selected as implied unit nodes. The output $\hat{y}(n)$ of the prediction model f_K is linear combination of K hidden layer, which can be expressed by

$$\hat{y}(n) = f_K(y_r(n-1)) = \sum_{j=1}^{K} w_j \varphi(\|y_r(n-1) - a_j\|), \quad (11)$$

where $y_r(n)$ is input variable, $\hat{y}(n)$ is the output of the neural network, $\varphi(\cdot)$ is the radial basis function, a_j is RBF center, it represents the Euclidean distance, K is hidden layer unit number.

RBF neural network is trained with training data $\{g(n), n = 1, 2, \cdots N\}$. When K and a_j are certain, weight ω_j can be chosen in the sense of minimum variance, that is

$$\min_{\omega_j} \sum_{n=R+1}^{N} (g(n) - \hat{g}(n)) . \quad (12)$$

Weighting vector matrix solution can be got in the sense of minimum variance.

$$W = \Phi + Q, \quad (13)$$

where the Φ matrix elements are R-dimensional reconstruction vectors of the training data, $Q = [g(R+1), \cdots g(N)]$ is the goal matrix of training data. These matrix elements can be got by the following formula:

$$\varphi_{ji} = \varphi(\|g_r(j) - a_i\|) \quad j = R, \cdots N-1, i = 1, 2, \cdots K . \quad (14)$$

After the training, weights matrix W for radial basis function neural network can be obtained according to Equation (13). Then the RBF neural network f_K can be determined.

RBF neural network is a Gaussian function network. Some parameters can be determined, such as RBF center, the variance and the weights of the output neurons. For the choice of the center, there is no uniform standard. Center is chosen by clustering method. The center of the class is as RBF center. One of the most common is called k-means clustering algorithm.

The basic steps of this algorithm are as follows:

(1) R which is the number of the network input is determined.

The dimension D and the delay time τ are calculated by GP algorithm and the mutual information method, respectively. Based on the Takens' delay-coordinate phase reconstruct theory, a chaotic series demand D variables at least. Thus R can take D, that is $R = D$.

(2) Weight ω_j and the learning rate η are respectively initialized, where ω_j and η take random function between 0 and 1. At the same time, let $k = 1$.

(3) Randomly selected q samples in a given sample. They are different from each other. Select sample X_k, calculate $J = \arg\min_{1 \le i \le q} \|X_k - \mu_i^k\|$, that is got the subscript of the most close to the center. Update the first J center, $\mu_J^{k+1} = \mu_J^k + \eta(X_k - \mu_J^k)$, $k = k+1$. Repeat selecting sample X_k and update the first J center, until the center value no longer update. After determining the RBF center, the various parameters of the Gaussian function should also be determined. The form of Gaussian function is as follows. $\varphi(x) = \exp(-\frac{(x-c)^2}{2\sigma^2})$, $\sigma > 0; x, c \in \mathrm{R}$. General c value is zero. Variance $\sigma = \alpha / \sqrt{2q}$, where $\alpha = \max_{1 \le i, j \le q}(\|\mu_i - \mu_j\|)$ is the maximum distance between any two centers.

(4) The first training network can be done by using the above initialization network and the chaotic signal.

(5) The error is calculated. If the error is in the permitted range, weights matrix W are stored. Otherwise, the second training network will go on.

(6) The prediction error curve, the actual data curve, the predicted data curve and each stored network parameters are output.

IV. Numerical Simulations

A. Henon Chaotic Signal Prediction

Henon chaotic system is as follows:

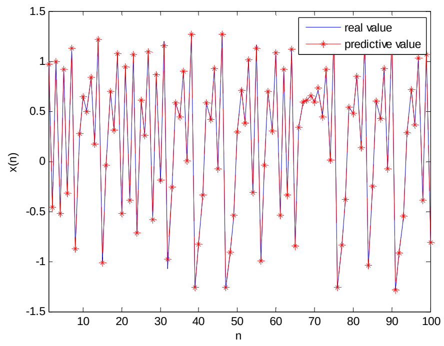
$$x_n = 1 - 1.4x_{n-1}^2 + 0.3x_{n-2} . \quad (15)$$

Prediction error function is defined as follows:

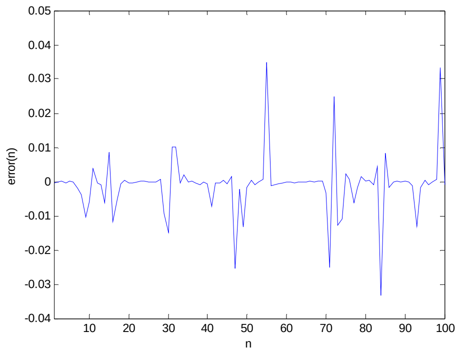
$$error(n) = \frac{1}{N} \sum_{n=1}^{N} [t(n) - y(n)]^2 . \quad (16)$$

where $y(n)$ is predictive value for the first n points, $t(n)$ is actual value for the first n points, N is the number of predicting points.

We reconstruct time series and take the first 500 samples to train the network. Then we take any sample outside of the training sample as input and predict 100 points. The actual data and predicted data are shown in Figure 2(a). The prediction error is shown in Figure 2(b). It can be seen from Figure 2 that the long-term prediction of chaotic time series is difficult, and it can only predict 20-30 points.



(a) Predicted results



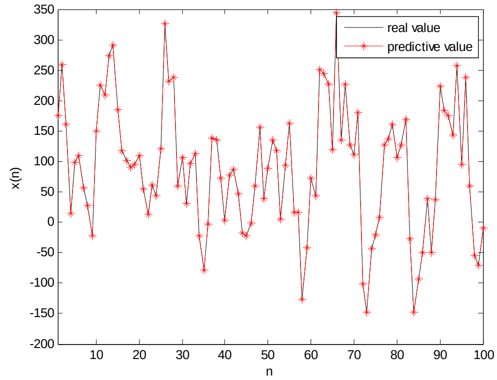
(b) Prediction error

Figure 2. Prediction of Henon chaotic systems by RBF neural network

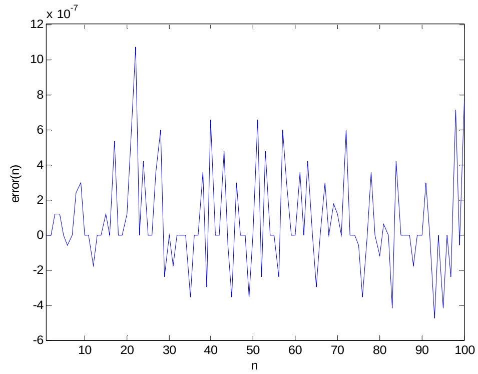
B. The Actual Underwater Acoustic Chaotic Signal Prediction

It has proved that underwater acoustic signal has chaos characteristics. Therefore, it is predictable in the short term. However, the actual ship noise is always mixed with other noise interference. This interference increase the randomness of underwater acoustic signals and make the certainty of underwater acoustic signal become relatively weak. This leads to reduce accuracy of the prediction. Therefore, before predictive model is established, underwater acoustic signal noise reduction processing by local projection filtering algorithm is necessary.

Considering the complexity of underwater acoustic signal, we use 500 data points. The former 400 points are the training sample. The last 100 points are the prediction sample. The actual data and predicted data, the prediction error for underwater acoustic signal 1 are shown in Figure 3(a) and Figure 3(b), respectively. The actual data and predicted data, the prediction error for underwater acoustic signal 2 are shown in Figure 4(a) and Figure 4(b), respectively. The actual data and predicted data, the prediction error for underwater acoustic signal 3 are shown in Figure 5(a) and Figure 5(b), respectively. It is clear from Figure 3, Figure 4 and Figure 5 that predictive model can more accurately predict for the acoustic signal which predictive mean squared error is ten to the negative seven. However, Ref. [3] proposes a prediction method for chaotic signal based on BF neural, and its predictive mean squared error is ten to the negative five. Comparison results show that this proposed prediction method increases at least two orders of magnitude in the mean square error terms.

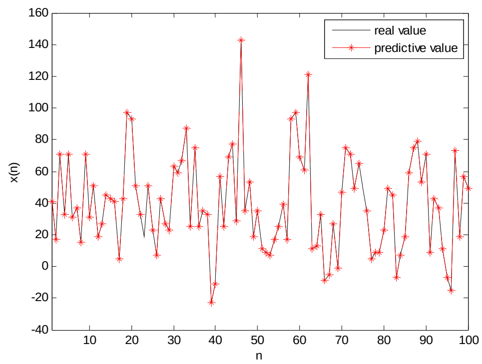


(a) Predicted results

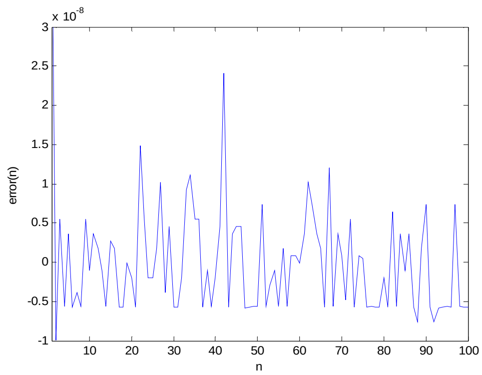


(b) Prediction error

Figure 3. Prediction of the first category of underwater acoustic signal by RBF neural network

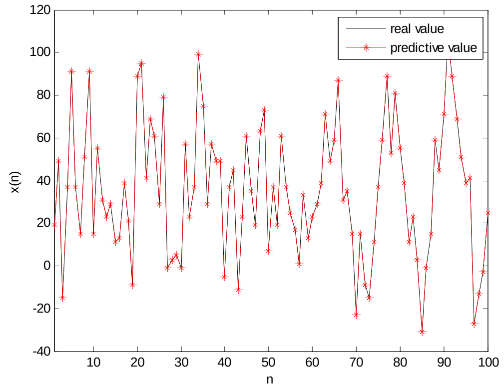


(a) Predicted results

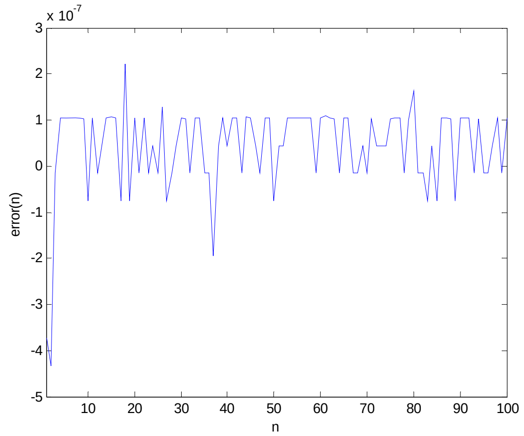


(b) Prediction error

Figure 4. Prediction of the second category of underwater acoustic signal by RBF neural network



(a) Predicted results



(b) Prediction error

Figure 5. Prediction of the third category of underwater acoustic signal by RBF neural network

V. Conclusions

In this paper, the chaotic time series RBF neural network model was designed. A prediction method for underwater acoustic chaotic signal based on RBF neural network is proposed. Typical Henon chaotic signal and the actual underwater acoustic chaotic signal are respectively predicted by the RBF neural network prediction model. The results show that the method can reduce mean squared error, and improve the prediction accuracy, and show better predictive effectiveness and reliability, and prove that it is a more effective method. However, the strength of the underwater acoustic signal randomness directly affects the prediction accuracy. Prediction accuracy rate of randomness strong signal prediction is low.

Acknowledgment

This work was supported in part by a grant from the scientific research project of Shaanxi Provincial education department (No. 2010JK823 and 2012JK0493) and the Scientific Research Foundation for Young Teachers by Xi'an University of Posts and Telecommunications (No. ZL2013-20 and ZL2013-21).

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