A Fractional Order Integral Approach for Reconstructing from Noisy Data

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***Abstract*—Computed tomography (CT) plays an important role in many applications.Recently,total variation(TV) minimization has become a main topic in image reconstruction. This paper focuses on iterative algorithm: SART and EM in both of TV and ordered subset. Iterative reconstruction is an improved algorithm for reconstructing image from noisy projection data. However,image noise will increase after some iterations while the image quality does not meet the requirement. In order to improve the quality of the reconstructed image, for three dimensional cone-beam CT, a new iterative algorithm via fractional order integral is researched. Experimental results show that the proposed method has faster convergence speed and achieve higher PSNR**

***Index Terms*—Cone-beam CT; noise projection data; total variation ;iterative algorithm ; fractional order integral**

I. INTRODUCTION

The iterative reconstruction (IR) algorithm is the key component of computed tomography imaging technology. Iterative algorithms are able to generate higher quality CT images,with the rapid development of computer technology, more and more attention has been given to iterative algorithm [1,2] .

Compared with two-dimensional (2D) CT, in which fan-beam rays are used to scan the object, the three-dimensional (3D) CT, in which cone-beam rays are used to scan the object, has a much shorter scan time because it can make use of the rays more efficiently. So it has attracted increased attention, and is gradually being used in medical diagnosis and engineering [3,4].

For noise projection data, image quality will be worse after the certain number of iteration. There is no method which can completely change this problem, the common method is that we adjust the key factors of iterative algorithm or adopt regularization method enhancement algorithm stability[5]. Total variation ordered subsets iterative algorithm also has this problem, TV-OS-SART algorithm with fractional order integral filtering is researched handle this problem[6], this paper will study two methods TV-OS-SART and TV-OSEM algorithm combined with fractional order integral to enhancement algorithm stability,In order to test those method is possible,Peak Signal to Noise Ratio (PSNR) is adopted.These two methods are addressed as TV-OS-IR-FOI (TV-OS-Iterative reconstruction-fractional order integral).

The rest of the paper is organized as follows.In the next section, we will introduce the cone-beam CT model. In section Ⅲ,TV-OS-IR via fractional order integral will be presented.In section Ⅳ,we present the reconstructed quality evaluation criteria.In section Ⅴ,numerical results will be described to support our method.Finally,we will give a conclusion.

Ⅱ. 3D CONE-BEAM CT

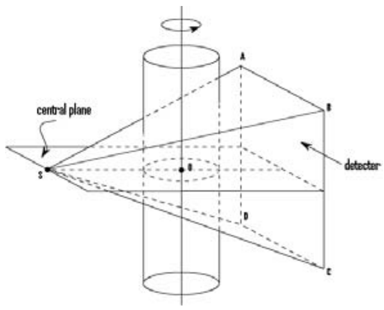

Figure 1. Cone-beam CT

In 3D cone beam scanning (see Fig .1), S is the ray source,the cylinder indicates the object that will be reconstructed,the plane ABCE represents the detector. Source runs around the object on a circle,together with a 2D detector .The scanning process provides us with the line integral of the reconstruct object along each of the lines. From all these integrals we have to reconstruct object.

Ⅲ. ITERATIVE RECONSTRUCTION ALGORITHM

An imaging system can be modeled as follows:

$$Wf = p \quad (1)$$

where $W = (w_{ij})$ denotes an $M \times N$ matrix, projection data is $p = [p_1, p_2, ..., p_M]^T \in R^M$, $f = (f_1, ... f_N) \in R^N$ is the

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image space.The problem is to reconstruct the image space f according to W and p.

3.1 TV-OS-IR

The CT reconstruction problem of the system (1) can be solved by the CS-based reconstruction method to minimize the image TV regularized by the projections. It is equal to solving the following optimization program [7]

$$\arg\min_{f \in H(f)} TV(f), \quad s.t. Wf = p. \tag{2}$$

For the sake of discussion,let's just call this algorithm a TV-IR,this algorithm includes two major steps: in the first step,an iteration algorithm is used to reconstruct a rough image.In this paper,the OS-SART and OSEM are used to reconstruct the image respectively.

Iterative formulas of the OS-SART can be expressed as follows[8]:

$$f_n^{(k+1)} = f_n^{(k)} + \lambda_k \sum_{m \in \phi_l} \frac{w_{mn}}{\sum_{m' \in \phi_l} w_{m'n}} \frac{p_m - \tilde{p}_m}{W_{m+}}, k = 0,1,2..., \tag{3}$$

Iterative formulas of the OSEM can be expressed as follows[9]:

$$f_n^{(k+1)} = f_n^{(k)} \lambda_k \sum_{m \in \phi_l} \frac{w_{mn} p_m}{\tilde{p}_m} \frac{1}{\sum_{m' \in \phi_l} w_{m'n}}, k = 0,1,2..., \tag{4}$$

where k indicates the iteration number, $f_n^{(k)}$ means the (n)th 3D pixels, w_{mn} indicates that the (n)th 3D pixels contribution along the (m)th ray, p_m is the measured projection, $\tilde{p}_m$ is the simulated projection. λ_k is relaxation parameter, ϕ_l represents the set of ray indexes in the (l)th view.In the second step,a searching method is used to minimize the TV of the reconstruction image[10].

3.2 Fractional order integral for TV-OS-IR

The real projection data contain many kinds of noises, so the image quality will be worse after some iterations. We adopted TV-OS-SART and TV-OSEM algorithm to reconstruct image and denoise the image using fractional order integral before the image quality become worse. Those method are addressed as TV-OS-IR-FOI (TV-OS-Iterative reconstruction-fractional order integral) .

The general definition form of factional order calculus follows, let $f(t) \in (a,t)$,it has the $m+1$ order continual derivative, when $a \in R, a > 0$, m is the integer part of a[11]

$$d^a f(t) = \lim_{h \to 0} \frac{1}{h^a} \sum_{m=0}^{n} (-1)^m \binom{a}{m} f(t - mh) = \lim_{h \to 0} \frac{1}{h^a} \sum_{m=0}^{\frac{t-a}{h}} (-1)^m \frac{\Gamma(a+1)}{\Gamma(a-m+1)} f(t - mh) \tag{5}$$

$$\Gamma(a) = \lim_{x \to \infty} \int_0^{\infty} e^{-x} t^{a-1} dt = (a-1)! \tag{6}$$

$d^a f(t)$ differential expression as follows:

$$\frac{d^a f(t)}{dt^a} \approx f(t) + (-a) f(t-1) + \frac{(-a)(-a+1)}{2!} f(t-2) + \frac{(-a)(-a+1)(-a+2)}{3!} f(t-3) + ... + \frac{\Gamma(-a+1)}{n! \Gamma(-a+n+1)} f(t-n) \tag{7}$$

where $n = [t - n]$.

The one-dimensional differential expression is extended to that of two dimensions, which is applied to image reconstruction[12]:

$$\frac{\partial^a f(x,y)}{\partial x^a} \approx f(x,y) + (-a) f(x-1,y) + \frac{(-a)(-a+1)}{2} f(x-2,y) \tag{8}$$

$$\frac{\partial^a f(x,y)}{\partial y^a} \approx f(x,y) + (-a) f(x,y-1) + \frac{(-a)(-a+1)}{2} f(x,y-2) \tag{9}$$

Fractional order of integral template along the x axis and y axis are defined in [13],where $a < 0$.

The TV-OS-IR-FOI can be summarized as the following pseudo-code:

a) Initialization parameters: iteration number n ; relaxation parameter λ_k ;the number of subsets; initial value $f_0 = 1$; fractional order of integral operator;

b) it counts nth iterations of the loop by TV-OS-IR to reconstruct f_n , then f_n is normalized to $0 \le f_n \le 255$ and decomposing f_n along the vertical direction coordinates to get slice sequence; finally, along the x axis and y axis ,convolution operations are performed between fractional order of integral template and f_n to get f_n^x and f_n^y ;

c) Computing $f_n^{TV-OS-IR-FOI}$:

$$f_n^{TV-OS-IR-FOI} = a \times f_n + a_x \times f_n^x + a_y \times f_n^y, \tag{10}$$

where $\{(a, a_x, a_y) | a > 0, a_x > 0, a_y > 0\}$

d)Until the stopping criteria are satisfied; else increment n and return to step b).

IV. Evaluation Criteria

In order to test the method is possible, Mean Squared Error (MSE) and Peak Signal to Noise Ratio (PSNR) are adopted to evaluate the reconstructed image.

Their formula are defined as[14]:

$$MSE = \frac{1}{I \times J \times K} \sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{i=1}^{I} (t_{i,j,k} - r_{i,j,k})^2, \tag{11}$$

$$PSNR = 10 * \log[\frac{I \times J \times K \times 255^2}{\sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{i=1}^{I} (t_{i,j,k} - r_{i,j,k})^2}], \tag{12}$$

where the image size is $I \times J \times K$, $r_{i,j,k}$ is the gray of voxel in original image, $t_{i,j,k}$ is the gray of voxel in reconstructed image.

V. Experimental Results

TABLE Ⅰ.
3D CONE-BEAM SCANNING PARAMETERS

Parameter	value
Model size	128×128×128
Distance between Source and rotation center distance /mm	3000
Detector number	128×128
Projection sampling number	128

A 3D model is adopted to verify the TV-OS-IR-FOI and has much faster convergence speed and can achieve higher PSNR than TV-OS-IR.Update f_n by TV-OS-IR-FOI at 4th and 8th iteration.

The parameters of 3D Cone-beam Scanning are listed in Table I ,8% Poisson noise was added to the simulated projection data; Relaxation parameter λ_k is 1;The numbers of subsets are 8 and 4;Fractional order of integral operator is -0.0001, iteration number *n* is 8.

5.1 The Number of Subsets is 8 for TV-OS-SART-FOI

At the 4th iteration,in which the reconstruction images were shown in Fig.2,we adopt weight coefficient $a_x = a_y = 2$, $a = 0.9$,it can be seen in figure 2

that TV-OS-SART-FIO can improve the brightness of the background and spherical area,while MSE becomes greater, just as shown in Fig .3.

After the 4th iteration .TV-OS-SART-FIO has faster

convergence speed than TV-OS-SART is shown in Fig. 3.

At the 8th iteration,we adopt weight coefficients $a_x = a_y = 0.9$ $a = 2$ .The reconstruction images are shown in Fig. 4,it can be seen in Fig.4 that the reconstructed image using our proposed TV-OS-SART-FOI is in excellent agreement with the original model.

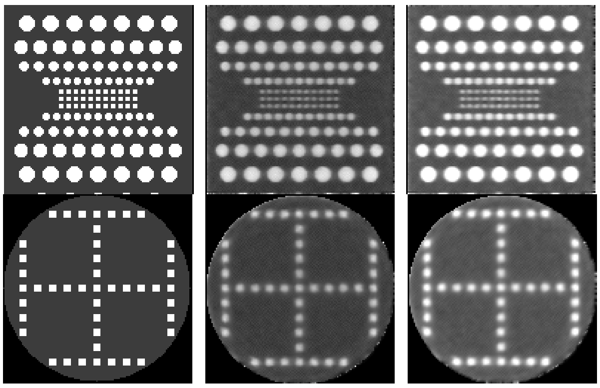

Figure2.Slice of image at x=64 and z=86
Left: original model; middle: reconstruction by TV-OS-SART
Right: reconstruction by TV-OS-SART-FOI

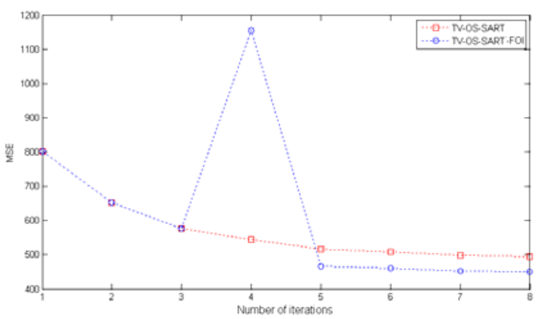

Figure 3. Mean Squared Error

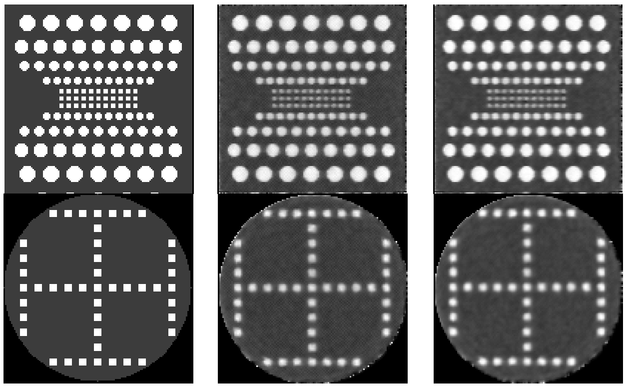

Figure4.Slice of image at x=64and z=86
Left: original model; middle: reconstruction by TV-OS-SART
Right: reconstruction by TV-OS-SART-FOI

TABLE II .
PEAK SIGNAL TO NOISE RATIO

Number of Iterations	1	2	3	4
TV-OS-SART	19.0914	19.9864	20.5224	20.7765
TV- OS-SART-FOI	19.0914	19.9864	20.5224	17.5022
Number of Iterations	5	6	7	8
TV-OS-SART	21.0064	21.0586	21.1584	21.1421
TV- OS-SART-FOI	21.4336	21.4934	21.5768	21.5923

In order to evaluate the reconstruction results objectively,PSNR is adopted to evaluate the reconstructed image,just as in Table II. From Table II, we can see that the TV-OS-SART-FOI performs better than TV-OS-SART in the aspects of PSNR after 4th iteration .

5.2. The number of subsets is 4 for TV-OS-SART-FOI

At the 4th iteration,we adopt weight coefficients $a_x = a_y = 1.8$, $a = 0.9$ .The reconstruction images were shown in Fig. 5,it can be seen in Fig.5 that TV-OS-SART-FOI can improve the brightness of the background and spherical area,while MSE becomes greater, just as shown in Fig.6.

After the 4th iteration,TV-OS-SART-FOI has faster convergence speed than TV-OS-SART is shown in Fig.6.

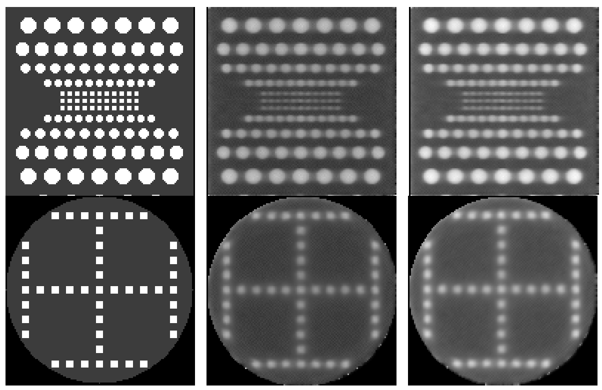

Figure 5. Slice of image at x=64 and z=86
Left: original model; middle: reconstruction by TV-OS-SART
Right: reconstruction by TV-OS-SART-FOI

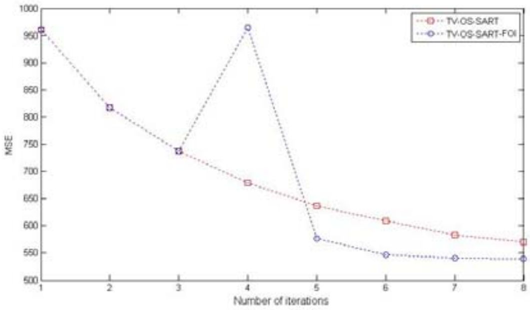

Figure 6 . Mean Squared Error

At the 8th iteration,we adopt weight coefficients $a_x = a_y = 0.7$, $a = 2.4$ .The reconstruction images were shown in Fig.7.It can be seen in Fig.7 that the reconstructed image using our proposed TV-OS-SART-FOI is in excellent agreement with the original model.

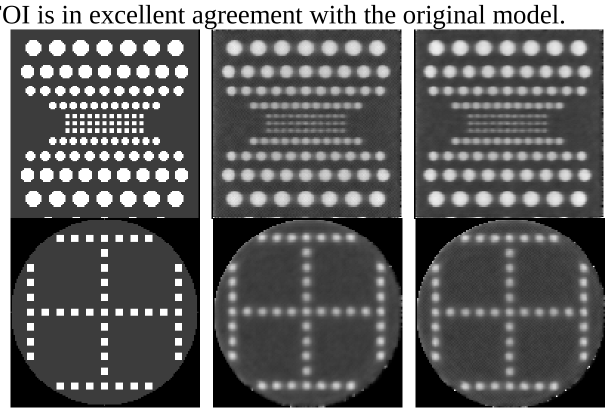

Figure 7. Slice of image at x=64 and z=86
Left: original model; middle: reconstruction by TV-OS-SART
Right: reconstruction by TV-OS-SART-FOI

From Table Ⅲ, after the 4th iteration ,we can see that the TV-OS-SART-FOI performs better than TV-OS-SART in the aspects of PSNR.

TABLE Ⅲ.
PEAK SIGNAL TO NOISE RATIO

Number of Iterations	1	2	3	4
TV-OS-SART	18.3031	19.0062	19.4554	19.8095
TV- OS-SART-FOI	18.3031	19.0062	19.4554	18.2894
Number of Iterations	5	6	7	8
TV-OS-SART	20.0907	20.2795	20.4719	20.5705
TV- OS-SART-FOI	20.5227	20.7569	20.8017	20.8174

5.3 The Number of Subsets is 8 for TV-OSEM-FOI

At the 4th iteration, in which the reconstruction images were shown in Fig.8,we adopt weight coefficient $a_x = a_y = 2$, $a = 0.9$,it can be seen in Fig.8 that TV-OSEM-FOI can still improve the brightness of the background and spherical area, while MSE becomes greater, just as shown in Fig.9.

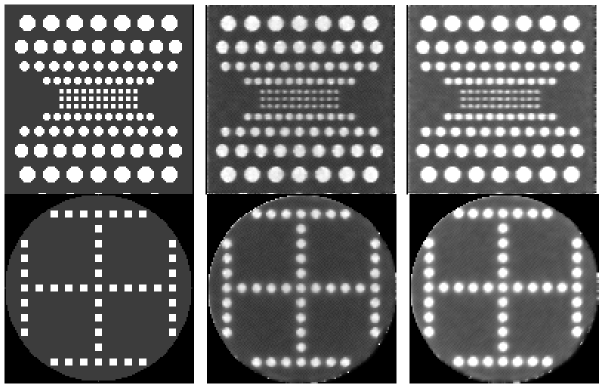

Figure 8. Slice of image at x=64 and z=86
Left: original model; middle: reconstruction by TV-OSEM
Right: reconstruction by TV-OSEM-FOI

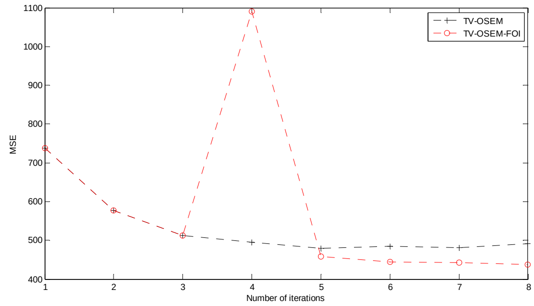

Figure 9 . Mean Squared Error

After the 4th iteration, TV-OSEM-FOI has faster convergence speed than TV-OSEM is shown in Fig.9.

At the 8th iteration, we adopt weight coefficients $a_x = a_y = 0.9$ $a = 2$ .The reconstruction images were shown in Fig.10,It can be seen in Fig.10 that the reconstructed image using our proposed TV-OSEM-FOI is in excellent agreement with the original model.

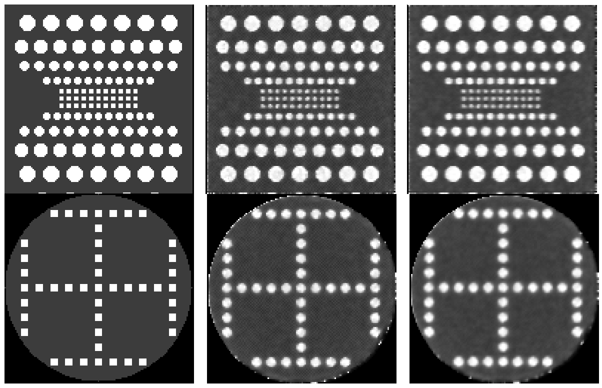

Figure 10 . Slice of image at x=64 and z=86
Left: original model; middle: reconstruction by TV-OSEM
Right: reconstruction by TV-OSEM-FOI

TABLE Ⅳ.
PEAK SIGNAL TO NOISE RATIO

Number of Iterations	1	2	3	4
TV-OSEM	19.4523	20.5147	21.0368	21.1885
TV- OSEM-FOI	19.4523	20.5147	21.0368	17.7510
Number of Iterations	5	6	7	8
TV-OSEM	21.3255	21.2702	21.3001	21.2057
TV- OSEM-FOI	21.5137	21.6533	21.6691	21.7073

From Table Ⅳ, we can see that the TV-OS-EM-FOI performs better than TV-OS-EM in the aspects of PSNR after 4th iteration .

5.4 The Number of Subsets is 4 for TV-OSEM-FOI

At the 4th iteration, we adopt weight coefficients $a_x = a_y = 1.8$, $a = 0.9$ .The reconstruction images were shown in Fig.11,it can be seen in Fig. 11 that TV-OSEM-FOI can improve the brightness of the background and spherical area, while MSE becomes greater, just as shown in Fig.12.

After 4th iteration , TV-OSEM- FIO has faster Convergence speed than TV-OSEM is shown in Fig.12.

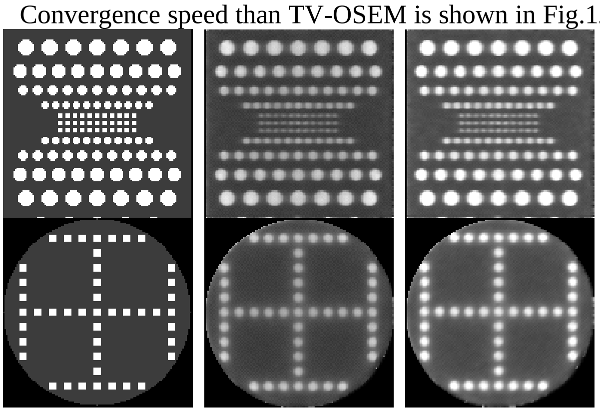

Figure 11.Slice of image at x=64 and z=86
Left: original model; middle: reconstruction by TV-OSEM
Right: reconstruction by TV-OSEM-FOI

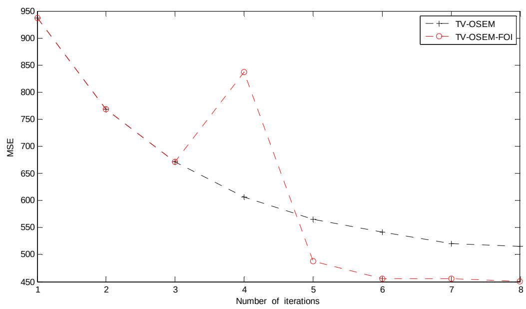

Figure 12 . Mean Squared Error

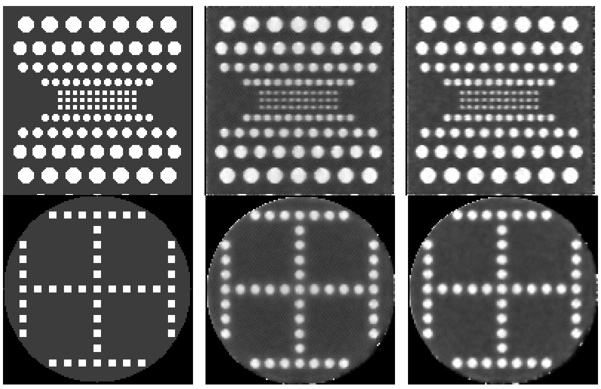

Figure 13. Slice of image at x=64 and z=86
Left: original model; middle: reconstruction by TV-OSEM
Right: reconstruction by TV-OSEM-FOI

At 8th iteration, we adopt weight coefficients $a_x = a_y = 0.7$, $a = 2.4$ .The reconstruction images were shown in Fig.13,it can be seen in Fig.13 that the reconstructed image using our proposed TV-OSEM-FOI is in excellent agreement with the original model.

TABLE Ⅴ.
PEAK SIGNAL TO NOISE RATIO

Number of Iterations	1	2	3	4
TV-OSEM	18.4119	19.2767	19.8612	20.2951
TV- OSEM-FOI	18.4119	19.2767	19.8612	18.8997
Number of Iterations	5	6	7	8
TV-OSEM	20.6070	20.7878	20.9571	20.0097
TV- OSEM-FOI	21.2412	21.5361	21.5387	21.5883

From Table Ⅴ, after 4th iteration, we can see that the TV-OSEM-FOI performs better than TV-OSEM in the aspects of PSNR.

VI. Conclusion

This paper introduces a fractional order integral approach for reconstructing image from noisy data. This method is addressed as TV-OS-IR-FOI. Experimental results show that TV-OS-IR-FOI has faster convergence speed and achieves higher PSNR than TV-OS-IR.

This TV-OS-IR–FOI is evaluated in numerical simulations, in the future, we shall research this approach from theory.

Acknowledgments

The idea of iterative algorithm via fractional order integral was presented at the International Conference on Computational Intelligence and Security (CIS)2012,in this paper, the further computer simulations prove this idea is effective;

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