A New Image Denoising Method Based on Wave Atoms and Cycle Spinning

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***Abstract*—A new method for image denoising was presented，which colligated the strong point of wave atoms transform and Cycle Spinning. Due to lack of translation invariance of wave atoms transform，image denoising by coefficient thresholding would lead to Pseudo-Gibbs phenomena. Cycle Spinning was employed to avoid the artifacts. Experimental results show that the method can remove noisy and remain edges, while Pseudo-Gibbs phenomena are controlled efficiently, and can get better visual effect and PSNR gains compared with the methods like simplex wave atoms or wavelet denoising using Cycle Spinning.And in heavy background noise, this advantage is significant.**

***Index Terms*—image processing, denoising, wavelet transforms, wave atoms, translation invariance, Cycle Spinning**

I. INTRODUCTION

Wavelet theory is widely used in signal processing, but the traditional wavelet transformation showed some limitations in the processing of two-dimensional image[1,2]. The image processing method combined partial differential equations and wavelet theory can better retain the image edge information [3, 4]. In the past two years, Demanet and Ying proposed a variant of wavelet packet-wave atoms[5,6]. Wave atoms transformation is a new type of two-dimensional multi-scale transformation, and still meets the parabolic proportional scaling relation and anisotropic characteristics of curve wave. In the wave atoms, the oscillation function or director texture is sparser than that in the wavelet, Gabor atoms or curve wave [6]. Wave atoms applies to any local direction of the mode and can sparsely spread in the anisotropy mode in the axis direction. Compared with the curve wave, wave atoms can not only capture the vibration mode, but can characterize the pattern through the oscillation. Although wave atoms transformation can sparsely show the two-dimensional image, due to its lack of translation invariance, the artificial visual distortion will be introduced at the same time of being applied to image denoising; especially for the part of image edge, the Pseudo-Gibbs phenomenon is particularly obvious. The Cycle Spinning technology [7] proposed by Coifman and Donoho well avoided this visual distortion. Combined with the effective representation of the wave atoms on the oscillation texture, Cycle Spinning technology was introduced to improve the wave atoms hard threshold denoising, and a denoising algorithm based on the wave atoms transformation was proposed by this paper. The experimental results showed that, compared with traditional denoising method, the algorithm better improved the visual effect of image denoising and obtained a higher PSNR gain, especially had a better effect on the images with rich details and texture. In the strong noise level, this advantage was more apparent.

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II. WAVE ATOMS

We write wave atoms as $\phi_\mu(x)$, with subscript $\mu=(j,m,n)=(j,m_1,m_2,n_1,n_2)$, $j,m_1,m_2,n_1,n_2\in Z$, index a point (x_μ,w_μ) in phase space, as

$$x_\mu=2^{-j}n, w_\mu=2^j m\pi, \quad C_1 2^j\le\max_{i=1,2}|m_i|\le C_2 2^j$$

where $C_1,C_2>0$ are two positive constants. x_μ and w_μ are the centers of $\phi_\mu(x)$ in spatial and frequency domain respectively.

Definition 1. The elements of a frame of wave packets $\{\phi_\mu(x)\}$ are called wave atoms when

$$|\phi_\mu(x)|\le C_M 2^j(1+2^j|x-x_\mu|)^{-M} \quad (1)$$

$$|\hat{\phi}_\mu(w)|\le C_M 2^{-j}(1+2^{-j}|w-w_\mu|)^{-M} + C_M 2^{-j}(1+2^{-j}|w+w_\mu|)^{-M} \quad (2)$$

for all $M>0$.

Definition 1 only presents a qualitative description for wave atoms with spatial frequency location restriction. In practice, Demanet uses the strategy of frequency localization given by Villemose to construct wave atoms from tensor products of adequately chosen 1D wave packets.

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The trick consists in exhibiting adequate symmetric paris of compactly supported bumps in frequency, given by the formula

$$\hat{\psi}_m^0(w) = e^{-iw/2}[e^{i\alpha_m} g(\ \varepsilon_m(\ w-\pi(\ m+\frac{1}{2}\ )))$$
$$+e^{-i\alpha_m} g(\ \varepsilon_{m+1}(\ w+\pi(\ m+\frac{1}{2}\ )))]$$

where $\varepsilon_m = (-1)^m$ and $\alpha_m = \frac{\pi}{2}(m+\frac{1}{2})$. The function g is an appropriate real-valued, C^∞ bump function, compactly supported on an interval of length 2π, and chosen such that $\sum_m \left|\hat{\psi}_m^0(w)\right|^2 = 1$. Let g supported on $[-7\pi/6, 5\pi/6]$,and such that for $|w| \le \pi/3$, $g(\pi/2-w)^2 + g(\pi/2+w)^2 = 1$ and $g(-2w-\pi/2) = g(\pi/2+w)$. Then the translates $\{\psi_m(x-n)\}$ form an orthonormal basis of $L^2(R)$ .This construction provides a uniform, or Gabor, tiling of the frequency axis. We need to introduce the subscript j to index scale, and write our basis functions as

$$\psi_{m,n}^j(x) = \psi_m^j(x-2^{-j}n) = 2^{j/2}\psi_m^0(2^j x - n)$$

Then the resulting basis of wavelet packets $\psi_{m,n}^j(x)$ form an orthonormal basis of $L^2(R)$. We emphasize here that these constructed basis functions have a good property, namely the uniformly bounded location in both time and frequency, which is the most important difference with wavelet packets from a standard multi-resolution analysis and plays a key role in designing wave atoms. For all $f(x) \in L^2(R)$, the coefficients can be seen as a decimated convolution at scale 2^{-j},

$$C_{j,m,n} = \int \psi_m^j(x-2^{-j}n) f(x)dx = \psi_m^j(x-2^{-j}n) * f(x)$$

By Plancherel,

$$C_{j,m,n} = \frac{1}{2\pi}\int e^{i2^{-j}nw}\, \overline{\hat{\psi}_m^j(w)}\, \hat{u}(w)dw$$

In two dimension, let us abbreviate $\mu = (j,m,n)$, where $m=(m_1,m_2)$ and $n=(n_1,n_2)$. H be Hilbert Transform. We define an orthonormal basis

$$\phi_\mu^+(x_1,x_2) = \psi_{m_1}^j(x_1-2^{-j}n_1)\psi_{m_2}^j(x_2-2^{-j}n_2)$$

A dual orthonormal basis can be defined from the "Hilbert-transformed" wavelet packets,

$$\phi_\mu^-(x_1,x_2) = H\psi_{m_1}^j(x_1-2^{-j}n_1)H\psi_{m_2}^j(x_2-2^{-j}n_2)$$

We denote $\phi_\mu^{(1)} = \frac{\phi_\mu^+ + \phi_\mu^-}{2}$ and $\phi_\mu^{(2)} = \frac{\phi_\mu^+ - \phi_\mu^-}{2}$, then $\{\phi_\mu\} = \{\phi_\mu^{(1)}, \phi_\mu^{(2)}\}$ form the wave atoms frame in two dimension, and satisfy

$$\sum_\mu \left|\left\langle \varphi_\mu^{(1)}, f\right\rangle\right|^2 + \sum_\mu \left|\left\langle \varphi_\mu^{(2)}, f\right\rangle\right|^2 = \|f\|^2$$

The coefficients of two-dimension wave atoms transform can be obtained as follows:

$$WA_\mu(f) = \left\langle f, \varphi_\mu^{(1)}\right\rangle + \left\langle f, \varphi_\mu^{(2)}\right\rangle$$

Figure 1 and figure 2 show the space-frequency domain forms of one-dimensional wave atoms at increasingly scales.

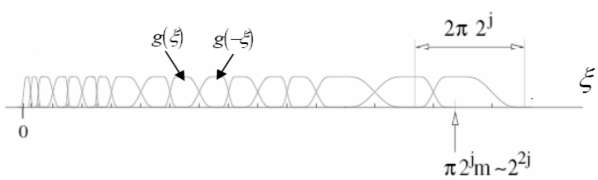
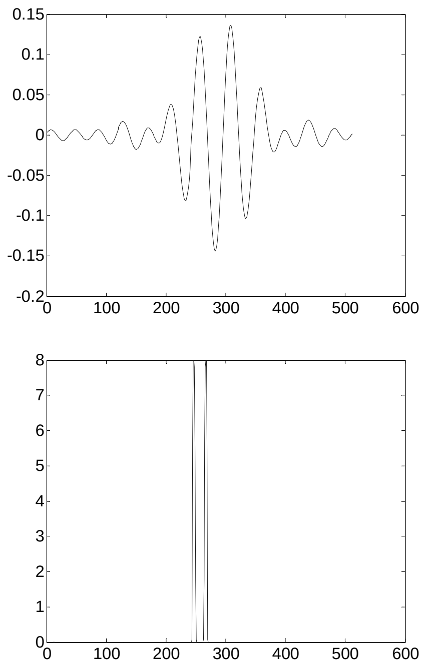
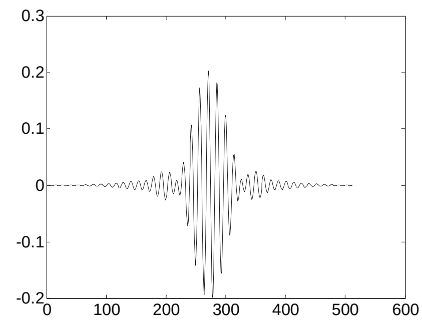
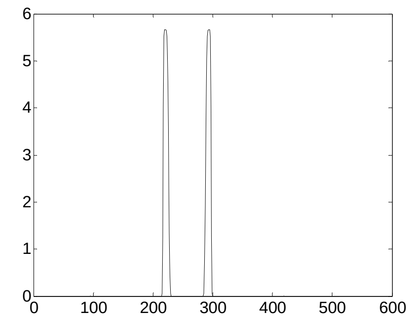


Figure 1. The frequency bands divide of one-dimensional wave atomes.

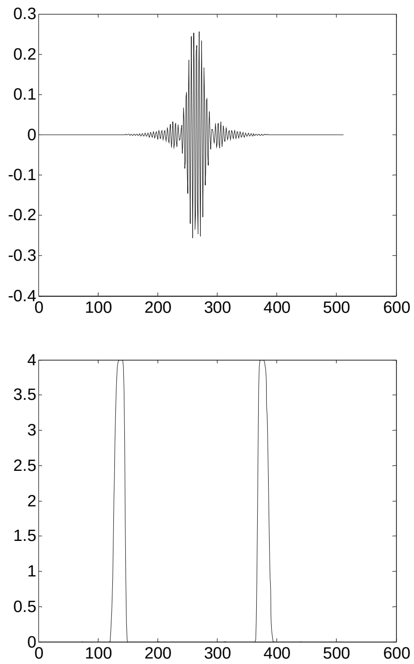


(a) j=3,m=3





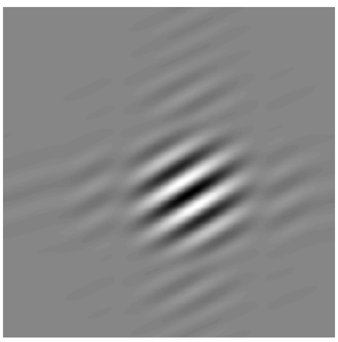
(b) j=4,m=5



(c) j=5,m=8

Figure 2. One-dimensional wave atoms in space-frequency domain at increasingly scales.

Figure 3 shows two-dimensional wave atoms at increasingly scales. The upper panels represent wave atoms in the spatial domain and the nether panels show wave atoms in the frequency domain.



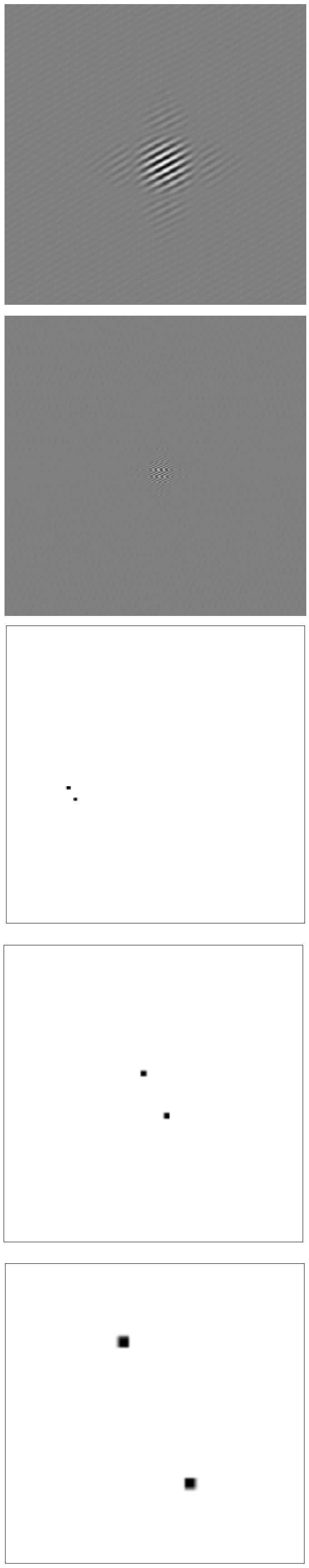


Figure 3. Two-dimensional wave atoms at increasingly scales.

III. Algorithm Description

A. Wave Atoms Hard Threshold Denoising

The basic idea of hard threshold denoising based on wave atoms transformation is consistent with the wavelet based denoising method. Assume the noisy image u can be expressed as $u = u_0 + \eta$, where u_0 is the noise clean image, η is the Gaussian noise of zero mean and variance σ^2; the purpose of image denoising is to recover a clear image from u_0. Soft threshold function because of its continuity makes the edge of the image denoising too vague, and too much detail is lost. However, hard threshold method can better retain the local characteristics of image edge, so this paper uses the method of wave atoms hard threshold denoising.

B. Cycle Spinning

In the threshold denoising process, if the transformation is lack of translation invariance, pseudo-Gibbs phenomenon will be produced in the image discontinuous point neighborhood area (edges and textures), leading to image distortion; this distortion is closely related to the location of image discontinuous points. For example, for Haar wavelet, the pseudo-Gibbs phenomenon will not be produced in the discontinuous point neighborhood area of $n/2$, but obvious pseudo-Gibbs phenomenon will occur in the discontinuous point neighborhood in other location (such as $n/3$). A method to prevent this phenomenon is to change the location of image discontinuous points via image translation, conduct threshold denoising on the translated image and then reversely translated the denoised image to avoid the pseudo-Gibbs phenomenon. If, however, the image to be analyzed contains a plurality of discontinuous points, the optimal translation of a certain discontinuous point may result in pseudo-Gibbs phenomenon in the neighborhood area of another discontinuous point. So it is difficult to find a translation amount that can satisfy the requirements of all discontinuous points. To inhibit the pseudo-Gibbs phenomenon occurred due to the lack of translation invariance in threshold denoising process, Coifman and Donoho proposed Cycle Spinning technology, that is, to carry out "cycle spinning-threshold denoising-reverse cycle spinning". As the threshold denoising on the image after each translation will make the occurrence of pseudo-Gibbs phenomenon in different places; therefore, single translation is not used, but a different denoising result $\hat{s}_{i,j}$ will be obtained from each translation in image rows and columns, and the denoising result $\hat{s}$ inhibiting pseudo-Gibbs phenomenon by linear average on all the denoising results, that is:

$$\hat{s}_{i,j} = S_{-i,-j}(T^{-1}(\Lambda[T(S_{i,j}(x))]),$$

$$\hat{s} = \frac{1}{K_1 K_2}\sum_{i=0}^{K_1}\sum_{j=0}^{K_2}\hat{s}_{i,j}.$$

K_1, K_2 is the maximum translation amount in the row and column direction, S is the cycle spinning operator, the subscript is the translation amount in the i, j row and column directions, T, T^{-1} is the transformation operator and its inverse operator respectively, and Λ is the threshold operator.

C. Cycle Spinning Based Wave Atoms Denoising Algorithm

Although the hard threshold can well preserve the image details, the processed image will have the vision distortions such as ringing, pseudo-Gibbs phenomenon; to inhibit pseudo-Gibbs phenomenon in the process of hard threshold denoising, the wave atoms based image denoising new algorithm is proposed by combining with Cycle Spinning technology; the specific algorithm steps are as follows:

1) Conduct cycle spinning on the noisy image u by the use of cycle spinning operator S, and obtain image $S(u)$;

2) Conduct wave atoms transformation T on the image $S(u)$ after cycle spinning and get the transformation coefficient $TS(u)$;

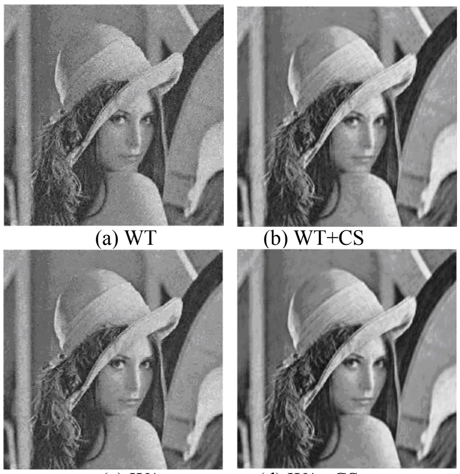
3) Process these coefficients by hard threshold operator Λ_h and obtain the transformation coefficient $\Lambda_h(TS(u))$ after denoising;

4) Carry out inverse wave atoms transformation on the wave atoms coefficient $\Lambda_h(TS(u))$ after hard threshold process and get the denoised image $T^{-1}(\Lambda_h(TS(u)))$;

5) The restored image $\tilde{u} = S^{-1}T^{-1}(\Lambda_h(TS(u)))$ can be obtained by conducting reverse cycle spinning on the denoised image $T^{-1}(\Lambda_h(TS(u)))$, where S^{-1} represents the reverse cycle spinning operator, and the final denoising results can be derived by averaging on all results.

IV. Experimental Results and Analysis

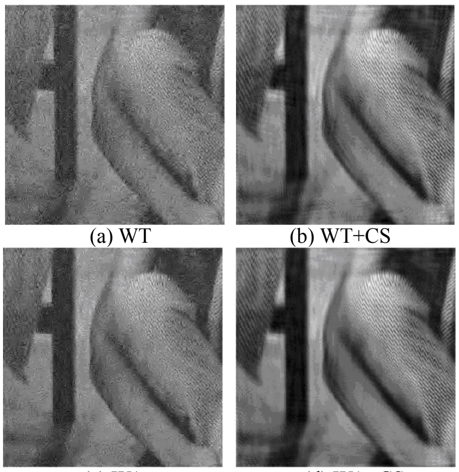
In order to verify the correctness and validity of the proposed algorithm, select some images with the size of about 512×512 and the white Gaussian noise with the mean of zero for experiments, such as the seismic profile with rich texture information, fingerprint image and Lena figure (figure 1) with rich edge details, and Barbara figure (figure 2), etc. Select 8 as the maximum translation amount in the image row and column direction. In the experiment, the comparison of denoising effects has been made of the wavelet hard threshold denoising (WT), cycle spinning wavelet hard threshold denoising (WT+ CS), wave atoms hard threshold denoising (WA) and the proposed method in this paper (WA + CS).



(a) WT (b) WT+CS

(c) WA (d) WA+CS

Figure 4. The comparison of denoising effects of Lena（σ=0.1，PSNR=19.99dB）.



(a) WT (b) WT+CS

(c) WA (d) WA+CS

Figure 5. The comparison of denoising effects of Barbara（σ=0.15，PSNR=16.50dB）.

From the visual effects, wave atoms denoising method can better retain the edge (the brim of Lena figure) and the texture information (hair of Lena figure and the pants stripes of Barbara figure). It indicates wave atoms can well retain the curved edge contour of the image, and is superior to the other two methods in terms of PSNR gain and characterization texture. The texture details in the figures such as the hair and pants stripes, figure (c) and figure (d) are much clearer than figure (a) and figure (b). In the edge area such as the hat brim, the effect of wave atoms method is better than that of the wavelet method. Moreover, the visual effect of the wave atoms denoising by the use of Cycle Spinning is significantly better than the traditional wave atoms denoising, and the peak signal to noise ratio has been improved by more than 1dB. Figure (d) has the highest peak signal to noise ratio of the image by the use of Cycle Spinning wave atoms denoising, which has more effectively inhibited the pseudo-Gibbs phenomenon caused by the lack of translation invariance in the process of threshold denoising, and significantly improved the visual quality of the image. The wave atoms anisotropy and its efficient representation of oscillation texture determine that the wave atoms is superior to traditional wavelet.

TABLE I.
THE COMPARISON ON THE PSNRS OF THE DENOISED IMAGES WITH DIFFERENT NOISE VARIANCES（DB）

Varian-ces	Lena				Barbara			
	WT	WT+CS	WA	WA+CS	WT	WT+CS	WA	WA+CS
0.10	24.75	27.58	28.41	29.53	23.00	25.45	27.24	28.27
0.15	21.81	24.75	26.43	27.57	20.30	22.86	25.36	26.43
0.20	19.58	22.58	24.94	26.17	18.58	21.24	24.03	25.14
0.25	17.84	20.91	23.78	25.04	17.05	19.80	22.86	24.12

To illustrate the algorithm proposed by this paper is significantly better than other methods in the objective performance, comparison has been made on the PSNRs of the denoised images with different noise variances (see Table 1). Whether the traditional wave atoms threshold denoising algorithm or the Cycle Spinning based wave atoms threshold denoising algorithm, the effect is better than the traditional wavelet threshold denoising, and even better than the cycle spinning wavelet denoising. Furthermore, in the strong noise level, this advantage is more obvious. The comparison of the CPU time consumed by the denoising of images with different noise variances has been made, and the results showed the wavelet method consumed less time than the wave atoms method. After the use of Cycle Spinning technology, the speeds of the algorithms are slower and the new algorithm is the most time consuming. This is due to the complexity of the wave atoms transformation computing; the repeated translation invariance will also increase the workload, so the cost of the new algorithm computing is great.

V. CONCLUSIONS

Wave atoms are an emerging new direction multi -scale transformation used for image processing and numerical analysis, with the oscillation cycle and support size satisfying the parabolic scaling relation. Its notable feature is the multi-scale and anisotropy, which can sparsely spread the smooth oscillation function (such as texture). In the processing of traditional image, pseudo-Gibbs phenomenon often arises from the lack of translation invariance and thus leading to image distortion, Cycle Spinning is an effective way to eliminate this distortion. On the basis of literature [6], this paper proposed an image denoising algorithm on the basis of wave atoms and Cycle Spinning. At the same time of effective removal of the image noise, this algorithm better retained the image edge and texture, and could effectively inhibit the pseudo-Gibbs phenomenon, making the denoised image look more realistic and natural, with better visual effects. The texture sharpness and contrast of the processed image were superior to the traditional wavelet threshold, cycle spinning wavelet threshold and wave atoms threshold methods. For images with rich texture information, particularly the texture images, it has ideal denoising effect, and this advantage is more obvious in the context of strong noise. Due to the costly new algorithm computing, the fast algorithm of the given wave atoms transformation and the improvement of the computing speed of cycle spinning method will be the focus of research in the future.

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