Balanced Growth Solutions and Related Problems of Hua's Macroeconomic Model

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***Abstract*—In this paper, the balanced growth solution and relative stability of solution on Hua's macroeconomic model is studied. Firstly, by deriving the greatest eigenvalue and nonnegative eigenvector, and analyzing the range of the eigenvalue of nonnegative matrix, the existence of the balanced growth solution on a sort of Hua's macroeconomic model is proved, no matter whether direct consumption coefficient matrix is irreducible or reducible. Furthermore, the concept of balanced solution's relative stability is introduced to Hua's macroeconomic model and the necessary and sufficient condition for the existence of these solutions is obtained. Finally, based on the price equation proposed by Prof. Hua, the dynamic price system which is inclusive of the interest rate is proposed and the relationship between price and output on the basis of the relative stability of price system is illuminated.**

***Index Terms*—Hua's macroeconomic model, balanced growth solution, relative stability, price system**

I. Introduction

In the 1980s,Prof. Luogeng Hua established the well-known theory which called "mathematical theory of large-scale optimization in planned economy" in several papers. In these papers, he proposed the so-called Hua's macroeconomic model. Under the condition of enough large productivity elasticity, the model depicts the relationship between input and output. Compared with Leontief's macroeconomic model, Hua's model fits better in the current Chinese economy. Hence the model can be used to analyze and forecast effectively in China

Due to the causal indeterminacy, it is difficult to apply Hua's macroeconomic model into practice. Therefore it is necessary to study the model's output feature. Prof. Hua also introduced the balanced solution which is based on the nonnegative irreducible square matrix [1]. Recently, many researchers put forward to generalize Hua's macroeconomic model involving consumption and investment and gave the balanced solutions of these generalized models [8-11]. Furthermore, the positive eigenvector method was researched [13, 14]. Since all the generalized models can be simplified to Hua's macroeconomic model, so it is important to study the balanced growth solution on Hua's macroeconomic model.

II. Preliminaries

According to the input-output analysis method, national economy is divided into n production sectors. Let $A=(a_{ij})_{n\times n}$ stands for the direct consumption coefficient matrix. Here a_{ij} is the product quantity of sector i which sector j needs when sector j produces one unit product. obviously, $A=(a_{ij})_{n\times n}\ge 0$, that is $a_{ij}>0$ for some i, j. $X^{(t)}$ is an n-dimensional column vector, which stands for the output in period t. Then Hua's macroeconomic model is

$$X^{(t)} = AX^{(t+1)}, t\in T=\{0,1,2,\cdots\} \qquad (1)$$

Let A be an invertible matrix, then Hua's macroeconomic model can be defined as follow

$$X^{(t+1)} = A^{-1}X^{(t)}, t\in T=\{0,1,2,\cdots\} \qquad (2)$$

The balanced solution of the model is $X^{*(t)}=\left(1/\lambda_*^{\,t}\right)X_*$. Here, A is a nonnegative irreducible matrix, λ_* is the largest eigenvalue of A and X_* is the nonnegative eigenvector of λ_*.

Price equation is

$$(q_1 \quad \cdots \quad q_n)\lambda_* = (q_1 \quad \cdots \quad q_n)A \qquad (3)$$

Where, $q_i(i=1,2,\cdots,n)$ stand for the price of per unit product of sector i, λ_* is the price change rate which is also the largest eigenvalue of A.

Lemma 1 [2]. If A is a nonnegative irreducible square matrix, for any positive vector $X^{(0)}$ which is not an eigenvector of A, then there is a positive integer $l_0>0$, when a positive integer $l\ge l_0$, $X^{(l)}=A^{-l}X^{(0)}$ must be a variable vector, that is some entries of $X^{(l)}$ are positive, others are negative.

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Lemma 1 shows that if initial input isn't a positive eigenvector of A, then some sectors' output will be negative in several years. In consequence, the economic system will collapse.

III. Balanced Growth Solution on Hua's Macroeconomic Model

Definition 1. On Hua's macroeconomic model, if there is $X^{(t)} = \alpha X^{(t-1)}$, then $X^{(t)}$ is called balanced solution. Here, $\forall t \in N$, $\alpha > 0$ is a constant and called growth coefficient.

By lemma 1, we know the balanced solution is the right positive eigenvector of A. Economic growth rate is $1/\lambda_*$, when the direct consumption coefficient matrix A is a nonnegative irreducible square matrix. However, the range of $1/\lambda_*$ was not given, which is an important index to show if the economic system will healthily develop. According to the definition 1, when $1/\lambda_* > 1$, economic gross increases and when $0 < 1/\lambda_* < 1$, economic gross decreases. If A is a reducible matrix, it is also possible that a balanced solution on the model may exist. For the two problems mentioned above, we are trying to get the balanced growth solution on a sort of Hua's macroeconomic model. So we suppose $I - A$ is a diagonal strictly dominant matrix.

Definition 2 [15]. If the relative structure of initial input does not meet any balanced solutions in an economic system, then for any solutions of the input-output economic model, it is possible at least one section's product output is negative. If it happens, the model has causal indeterminacy.

According to lemma 1, we know there is causal indeterminacy on Hua's macroeconomic model.

Lemma 2 ([16] Perron-Frobenius theorem on general nonnegative matrix). Let A be a $n \times n$ matrix with nonnegative real entries. Then,

(i) A has a nonnegative real eigenvalue $\lambda \ge 0$, which dominates the absolute values of all other eigenvalues λ_i of A, that is $\lambda \ge |\lambda_i|$.

(ii)Exist a positive eigenvector $X \ge 0$ of λ. Here, $X \ge 0$ means that $\forall i, x_i \ge 0$ and $\exists j, x_j > 0$.

Lemma 3 [16]. Let A be a $n \times n$ matrix with nonnegative real entries, A has a nonnegative real eigenvalue $\lambda \ge 0$, which dominates the absolute values of other eigenvalues λ_i of A, that is $\lambda \ge |\lambda_i|$, then for any positive vector $x = (x_1, \cdots, x_n)^T > 0$, we have

$$\min_{1 \le i \le n}\left(\frac{1}{x_i}\sum_{j=1}^{n} a_{ij}x_j\right) \le \lambda \le \max_{1 \le i \le n}\left(\frac{1}{x_i}\sum_{j=1}^{n} a_{ij}x_j\right).$$

Theorem 1. Let A be a direct consumption coefficient matrix. Suppose $I - A$ is a diagonal strictly dominant matrix. Then there is a positive real $\hat{\lambda}$, $0 < \hat{\lambda} < 1$ and a positive vector $\hat{X} \ge 0$, which satisfy $A\hat{X} = \hat{\lambda}\hat{X}$.

Proof: For Hua's macroeconomic model $X^{(t)} = AX^{(t+1)}$, since $A \ge 0$, we know existing $\hat{\lambda} \ge 0$ and $\hat{X} \ge 0$, which satisfy $A\hat{X} = \hat{\lambda}\hat{X}$ by lemma 2. Since $I - A$ is a diagonal strictly dominant matrix, then $1 - a_{ii} > \sum_{j \ne i} a_{ij}$.

By Gerschgorin theorem, all the eigenvalues of A belong to the set

$$\bigcup_i \Omega_i = \left\{\lambda \,\middle|\, |\lambda - a_{ii}| \le \sum_{j \ne i} a_{ij}\right\}.$$

Since $1 - a_{ii} > \sum_{j \ne i} a_{ij}$, so $|\lambda - a_{ii}| \le \sum_{j \ne i} a_{ij} \le 1 - a_{ii} \Rightarrow$ $2a_{ii} - 1 < \lambda < 1$. Hence $\hat{\lambda} < 1$. According to lemma 3, let $x = (x_1, \cdots, x_n)^T = (1, \cdots, 1)^T$, then

$$\min_{1 \le i \le n}\left(\sum_{j=1}^{n} a_{ij}\right) \le \hat{\lambda} \le \max_{1 \le i \le n}\left(\sum_{j=1}^{n} a_{ij}\right).$$

Because A is a nonnegative matrix, therefore there exists $\sum_{j=1}^{n} a_{ij} \ge 0$. According to the economic means of A, it is impossible that one sector does not offer its product to others to produce, so $\sum_{j=1}^{n} a_{ij} > 0$, hence $0 < \hat{\lambda}$. Whence we obtain $0 < \hat{\lambda} < 1$ and $\hat{X} \ge 0$, which satisfy $A\hat{X} = \hat{\lambda}\hat{X}$.

For Hua's macroeconomic model

$$X^{(0)} = AX^{(1)}. \tag{4}$$

Suppose there is a balanced solution in the economic system, the growth rate is $1/\lambda$, that is

$$X^{(1)} = \frac{1}{\lambda} X^{(0)} \tag{5}$$

From (5) and (4), we obtain

$$\lambda X^{(0)} = AX^{(0)} \tag{6}$$

By inductive method, we obtain $X^{(t)} = (1/\lambda)^t X^{(0)}$. Let $X^{(0)}$ be $\hat{X}$, then the growth rate is $1/\hat{\lambda}$. Whence we have $X^{(t)} = (1/\hat{\lambda})^t \hat{X}$, that is if initial input is $\hat{X}$, the economic system could have a balanced solution. Since $0 < \hat{\lambda} < 1$, hence the balanced solution also is a balanced growth solution. By the analysis above, we obtain the following theorem.

Theorem 2. Let A be a direct consumption coefficient matrix, and satisfy $1 - a_{ii} > \sum_{j \ne i} a_{ij}$, then $X^{(t)} = (1/\hat{\lambda})^t \hat{X}$

is a balanced growth solution on Hua's macroeconomic model, the growth rate is $1/\hat{\lambda}$, here $0<\hat{\lambda}<1$ is the eigenvalue of A, $\hat{X}\geq 0$ is the eigenvector of $\hat{\lambda}$.

There is no restriction that the consumption coefficient matrix is irreducible and the balanced growth solution on a sort of Hua's macroeconomic model have been proposed in the theorem 2. It is illuminate when all sectors consume their own product less, it is impossible that the total amount of products that one sector consumes will be more than the total amount of products that it produces, which would result in a negative output in an economic system. Therefore this kind of model has no causal indeterminacy.

IV. Relative Stability of Balanced Solution

Definition 3. $A\geq 0$ is a direct consumption coefficient matrix and is invertible. For Hua's macroeconomic model, suppose $X^{*(t)}=(1/\lambda^t)X$ is a balanced solution, $\hat{X}^{(t)}$ is a general solution which is determined by any initial input $\hat{X}^{(0)}\geq 0$. If $\lim_{t\to\infty}(\hat{X}_i^{(t)}/X_i^{*(t)})=\sigma$, here, $\hat{X}_i^{(t)}$, $X_i^{*(t)}$ respectively stand for the i^{th} entry of $\hat{X}^{(t)}, X^{*(t)}$, $0<\sigma<\infty$ and there is no relation between σ and i, then the balanced solution is relatively stable.

Fig.1 shows the concept of relative stability in the two-dimensional situation. It means that $\hat{X}^{(t)}$ asymptotic approximation to a balanced solution $X^{*(t)}$ when $\lim_{t\to\infty}(\hat{X}_i^{(t)}/X_i^{*(t)})=\sigma\ i\in\{1,2\cdots\}$. Since $\hat{X}^{(t)}>0$, exist T, when $t>T$, we have $\hat{X}^{(t)}>0$. So when there is a relatively stable balanced solution on an economic model, the model's solution which is determined by any initial input is always greater than zero, so that there is no causal indeterminacy in the economic system.

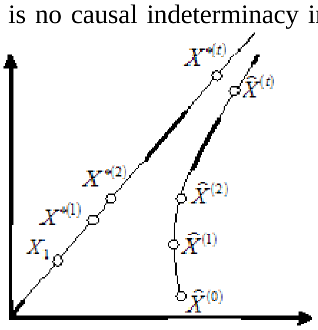

Figure 1. Diagram of two-dimensional relative stability.

Theorem 3. $A\geq 0$ is a direct consumption coefficient matrix, and is invertible, then exist a positive integer t, which satisfy $(A^{-1})^t>0$ if and only if $|\lambda_i|>\lambda_1>0$, $X_1>0$, here $\lambda_1,\lambda_i(i=2,3,\cdots,n)$ are the eigenvalue of A, X_1 is the eigenvector of λ_1.

Proof: ($\Rightarrow$) If the eigenvalue of A is λ_j, then the eigenvalue of A^{-1} is $1/\lambda_j$, and the eigenvalue of A^{-t} is $1/\lambda_j^t$. When there is a positive integer t, which satisfies $(A^{-1})^t>0$, then $(A^{-1})^t$ is irreducible matrix. According to Frobenius' theorem [16], there exists an eigenvalue $\mu_1>0$ and an eigenvector $Y_1>0$, which satisfies

$$\mu_1>|\mu_i|(i=2,\cdots,n),\ (A^{-1})^tY_1=\mu_1Y_1,$$

Here $\mu_i(i=2,\cdots,n)$ also are eigenvalue of $(A^{-1})^t$. Let λ_i be the eigenvalue of A and X_1 be the eigenvector of λ_1. Due to the relationship between A and $(A^{-1})^t$, we have

$$\mu_i=\frac{1}{\lambda_i^t}(i=1\cdots n),\ Y_1=X_1.$$

Then $(1/\lambda_1^t)>(1/|\lambda_i^t|)\ (i=2,\cdots n)$, therefore we obtain

$$|\lambda_i|>\lambda_1>0(i=2,\cdots n),\ X_1>0.$$

($\Leftarrow$) The general solution of difference equation (2) can be written as

$$\hat{X}^{(t)}=h_1\frac{1}{\lambda_1^t}X_1+h_2\frac{1}{\lambda_2^t}X_2+\cdots+h_n\frac{1}{\lambda_n^t}X_n \quad (7)$$

Here $\lambda_1,\cdots,\lambda_n$ are the eigenvalue of A, $X_1,\cdots,X_n$ are the eigenvector, $h_1,\cdots,h_n$ are determined by initial input vector.

Let e^i be an n-dimensional vector, its the i^{th} entry is 1, others are 0. Now let e^i be the initial input, we obtain

$$\hat{X}^{(0)}=e^i=h_1X_1+h_2X_2+\cdots+h_nX_n$$

$$(X_1\cdot e^i)=(X_1\cdot\hat{X}^{(0)})=\left(X_1\cdot\sum_{i=1}^n h_iX_i\right)$$

$$=\sum_{i=1}^n h_i(X_1\cdot X_i).$$

Since $AX_1=\lambda_1X_1$, we get $A^{-1}X_1=(1/\lambda_1)X_1$. So

$$\frac{1}{\lambda_1}(X_1\cdot X_i)=\left(\frac{1}{\lambda_i}X_1\cdot X_i\right)=(A^{-1}X_1\cdot X_i)$$

$$=(A^{-1}X_1)^TX_i=X_1^T(A^T)^{-1}X_i$$

$$=X_1^T\frac{1}{\lambda_i}X_i=\frac{1}{\lambda_i}X_1^TX_i$$

$$=\frac{1}{\lambda_i}(X_1\cdot X_i)$$

When $i\neq 1$, $\lambda_1\neq\lambda_i$, $(X_1\cdot X_i)=0$. Therefore $(X_1\cdot e^i)=h_1(X_1\cdot X_1)$, that is $h_1=X_1e^i/(X_1\cdot X_1)$. Since $(X_1\cdot X_1)>0$, $(X_1\cdot X_1)>0$, so $h_1>0$.

Since $|\lambda_i| > \lambda_1$, $(1/\lambda_1^t) > (1/|\lambda_i^t|)$, according to the following equation

$$\widehat{X}^{(t)} = \left(A^{-1}\right)^t e^i = h_1 \frac{1}{\lambda_1^{\ t}} X_1 + h_2 \frac{1}{\lambda_2^{\ t}} X_2 + \cdots + h_n \frac{1}{\lambda_n^{\ t}} X_n,$$

we know there is a positive integer t , which satisfy $\left(A^{-1}\right)^t e^i > 0$. Because i is randomized, therefore $\left(A^{-1}\right)^t > 0$.

According to definition 3, if $X^{*(t)} = \left(1/\lambda_1^t\right)X_1$ is a relatively stable balanced solution, the general solution $\widehat{X}^{(t)}$ which is determined by any initial input vector $\widehat{X}^{(0)} \geq 0$ satisfies

$$\lim_{t\to\infty}\frac{\widehat{X}_i^{(t)}}{X_i^{*(t)}} = \lim_{t\to\infty}\frac{h_1 \frac{1}{\lambda_1^{\ t}} X_{1i} + h_2 \frac{1}{\lambda_2^{\ t}} X_{2i} + \cdots + h_n \frac{1}{\lambda_n^{\ t}} X_{ni}}{\frac{1}{\lambda_1^{\ t}} X_{1i}}$$

$$= \lim_{t\to\infty}(h_1 + h_2 \frac{\lambda_1^{\ t}}{\lambda_2^{\ t}} \frac{X_{2i}}{X_{1i}} + \cdots + h_n \frac{\lambda_1^{\ t}}{\lambda_n^{\ t}} \frac{X_{ni}}{X_{1i}}) = \sigma$$

Since there isn't relation between σ and i , and $\sigma > 0$, we have $|\lambda_i| > \lambda_1 > 0$, that is the balanced solution is relatively stable if and only if $|\lambda_i| > \lambda_1 > 0$. We get the following theorem by the analyzing above.

Theorem 4. Suppose $A \geq 0$ is a direct consumption coefficient matrix and is invertible, then the balanced solution $X^{*(t)} = 1/\left(\lambda_1^t\right)X_1$ is relatively stable if and only if $|\lambda_i| > \lambda_1 > 0$, here λ_1, λ_i ($i = 2 \cdots n$) are the eigenvalue of A , X_1 is the eigenvector of λ_1 .

Corollary. Suppose $A \geq 0$ is a direct consumption coefficient matrix and is invertible, then exist a positive integer t , which satisfy $\left(A^{-1}\right)^t > 0$ if and only if balanced solution $X^{*(t)} = \left(1/\lambda_1^t\right)X_1$ is a relatively stable solution.

By the theorem 4, we know , for Hua's macroeconomic model , the balanced solution $X^{*(t)} = (1/\lambda_1^t)X_1$ is relatively stable $\Leftrightarrow$ the eigenvalue and the eigenvector of A satisfy $|\lambda_i| > \lambda_1 > 0$ and $X_1 > 0$ $\Leftrightarrow$ exist a positive integer t , which satisfies $A^{-t} > 0$ $\Leftrightarrow$ there isn't the causal indeterminacy on Hua's macroeconomic model $\Leftrightarrow$ all sectors' output is not negative in several years.

V. Dynamic Price System

The price equation (3) doesn't include interest rate and was proposed under the presumption that products price changes proportionally in each period, that is $P^{(t+1)} = \lambda_* P^{(t)}$. Since interest rate has a direct impact on the product's sales and price, so it can't be disregarded. The product's price may not change proportionally. thus we suppose the price of i sector's per unit product is $P_i^{(t)}$ in t period, then $P^{(t)} = \left(P_1^{(t)} \ \cdots \ P_n^{(t)}\right)$ is the price vector in t period, and the price vector is a constant vector during each period, any product prices will not be affected by human factors, and the costs of raw material will be paid at the beginning of each period. The costs that j sector produce per unit j product in t period is

$$v_j^{(t)} = \sum_{i=1}^{n} P_j^{(t)} a_{ij} = P^{(t)} \ a_j \tag{8}$$

Here, a_j is the j^{th} column of the direct consumption coefficient matrix A , a_{ij} is the i^{th} entry of a_j . Then the net profit that j sector produce per unit j product in t period is

$$\pi_j^{(t)} = P_j^{(t+1)} - P^{(t)} a_j \tag{9}$$

Now there is an individual, who has an amount of money $v_j^{(t)}$ at the beginning of t period, we suppose this money can be lent out at an interest rate $r^{(t)}$, and then the interest who will get at the beginning of $t+1$ period is

$$R_j^{(t)} = r^{(t)} v_j^{(t)} = r^{(t)} P^{(t)} a_j \tag{10}$$

According to the competition arbitrage principle, the interest and the profit are the same in equilibrium, so we have

$$\pi_j^{(t)} = R_j^{(t)}$$

$$P_j^{(t+1)} - P^{(t)} a_j = r^{(t)} P^{(t)} a_j$$

$$P_j^{(t+1)} = (1 + r^{(t)}) P^{(t)} a_j \tag{11}$$

Obviously, for all $j = 1, \cdots, n$, (8) is right. Hence we obtain

$$P^{(t+1)} = (1 + r^{(t)}) P^{(t)} A \tag{12}$$

Let $(1 + r^{(t)})A = M$, (12) can be written as

$$P^{(t+1)} = P^{(t)} M \tag{13}$$

(13) is called dynamic price equation on Hua's macroeconomic model.

When the interest rate $r^{(t)}$ is 0, and the price changes proportionally in every period, (13) and (3) are the same. When the interest rate $r^{(t)}$ and the price vector $P^{(t)}$ are known in t period, we can get the price vector $P^{(t+1)}$ by (13), therefore (13) can also be called expected price equation.

VI. Relative Stability of Price System

In order to simplify, we suppose the interest rate r is a constant when we study the relative stability of the price system, thus $M = (1 + r)A$. The general solution of difference equation (13) can be written as

$$\widehat{P}^{(t)} = \alpha_1 \zeta_1^t p_1 + \alpha_2 \zeta_2^t p_2 + \cdots + \alpha_n \zeta_n^t p_n \qquad (14)$$

Here, $\zeta_1, \zeta_2, \cdots, \zeta_n$ are eigenvalue of M , $p_1, p_2, \cdots p_n$ are eigenvector of M , that is $p_i M = \zeta_i p_i$. $\alpha_1, \alpha_2, \cdots, \alpha_n$ are determined by initial price vector. Since $(1+r)A = M$, whence by the output balanced solution $X^{*(t)} = \left(1/\lambda_1^t\right) X_1$, we can get price balanced solution, that is

$$P^{*(t)} = \zeta_1^t p_1 \qquad (15)$$

Here, $\zeta_1 = (1+r)\lambda_1$.

Definition 4. For the dynamic price equation on Hua's macroeconomic model, suppose $P^{*(t)} = \zeta_1^t p_1$ is a balanced solution, $\widehat{P}^{(t)}$ is the price vector which is determined by any initial price vectors $\widehat{P}^{(0)} \geq 0$ by (13). If $\lim_{t\to\infty}\left(\widehat{P}_i^{(t)} / P_i^{*(t)}\right) = \sigma$, here $\widehat{P}_i^{(t)}, P_i^{*(t)}$ stand for the i^{th} entry of $\widehat{P}^{(t)}, P^{*(t)}$ respectively, $0 < \sigma < \infty$ and there is no relation between σ and i , then the price balanced solution is a relatively stable balanced solution.

Definition 4 illuminates when there is a relatively stable price balanced solution $P^{*(t)}$, the price vector $\widehat{P}^{(t)}$ which is determined by any initial price vectors $\widehat{P}^{(0)} \geq 0$ asymptotic approximation to the price balanced solution which is fit for the economic growth. According to definition 4, if $P^{*(t)} = \zeta_1^t p_1$ is relatively stable, then $\widehat{P}^{(t)}$ satisfies

$$\begin{aligned} &\lim_{t\to\infty} \frac{\widehat{P}_i^{(t)}}{P_i^{*(t)}} \\ &= \lim_{t\to\infty} \left(\alpha_1 + \alpha_2 \left(\frac{\zeta_2}{\zeta_1}\right)^t \frac{p_{2i}}{p_{1i}} + \cdots + \alpha_n \left(\frac{\zeta_n}{\zeta_1}\right)^t \frac{p_{ni}}{p_{1i}}\right) \\ &= \sigma \end{aligned}$$

Because there is no relation between σ and i , and $\sigma > 0$, so $\zeta_1 > |\zeta_i|$, that is the price balanced solution is relatively stable if and only if $\zeta_1 > |\zeta_i|$. Hence we get the following theorem.

Theorem 5. Suppose $A \geq 0$ is a direct consumption coefficient matrix, r stands for the interest rate, let $M = (1+r)A$, then the price balanced solution $P^{*(t)} = \zeta_1^t p_1$ is relatively stable if and only if $\zeta_1 > |\zeta_i|$, here $\zeta_1, \zeta_2, \cdots, \zeta_n$ are n different eigenvalue of M , p_1 is the eigenvector of ζ_1.

According to $(1+r)A = M$, we know $\zeta_i = (1+r)\lambda_i$, when the price balanced solution is relatively stable, that is $\zeta_1 > |\zeta_i|$, we obtain $\lambda_1 > |\lambda_i|$, and then the output balanced solution is not relatively stable. On the contrary, when the output balanced solution is relatively stable, that is $|\lambda_i| > \lambda_1$, we obtain $|\zeta_i| > \zeta_1$, and then the price balanced solution isn't relatively stable.

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