Color Image Quantization Algorithm Based on Differential Evolution

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Abstract—Some stochastic optimization methods, such as Particle Swarm Optimization Algorithms (PSO) and Genetic Algorithms (GA), have been used to solve the color image quantization. Differential Evolution Algorithm (DE) is one of the powerful stochastic optimization methods. Few researches have been done for using DE to solve the color image quantization. This paper proposes a DE-based color image quantization algorithm. In the proposed algorithm, a better colormap is designed by using DE to update some randomly initialized candidate colormaps. Numerical experiments are conducted on a set of commonly used test images. The experimental results show that the proposed algorithm is practicable, and it has better performance than the color image quantization algorithm using PSO.

Index Terms—Color Image Quantization, Differential Evolution, Particle Swarm Optimization

I. INTRODUCTION

Color image quantization, one of the common image processing techniques, is the process of reducing the number of colors presented in a color image with less distortion [1]. The main purpose of color quantization is reducing the use of storage media and accelerating image sending time [2]. Color image quantization consists of two essential phases. The first one is to design a colormap with a set of colors (typically 8-256 colors [3, 4]), which is smaller than the set of a color image. The second one is to map each pixel of the color image to one color in the colormap. Most of color quantization methods focus on creating a colormap. To address this problem, researchers have applied several stochastic optimization methods, such as GA and PSO. Especially, literature [5]-[8] have compared the color image quantization algorithm using PSO (PSO-CIQ) and several other well-known color image quantization methods. The experimental results show that PSO-CIQ has higher performance.

Differential evolution (DE) [9-11] is a population-based heuristic search approach. DE has been applied to the classification for gray images [12]-[14]. In literature [12]-[14], DE and PSO show similar performance. However, due to simple operation, litter parameters and fast convergence, DE is the better choice to use than PSO [12]. However, few researches have been done for using DE to solve the color image quantization. This paper proposes a DE-based color image quantization algorithm. The proposed algorithm starts with an initialized population, in which each individual represents a candidate colormap. Then each individual (candidate colormap) is repeatedly updated by mutation, crossover and selection operations of DE until a given stopping condition is satisfied. The solution obtained by the above operations is the last colormap, by which the quantized image is generated. By some commonly used color images, the performance of the proposed algorithm is compared with that of the color image quantization approach using PSO.

This paper is organized as follows. Section 2 introduces the classical DE algorithm. In section 3, the DE-based color image quantization algorithm is proposed. In section 4, numerical experiments are given to compare the quantization quality of the proposed algorithm and the color image quantization algorithm using PSO. Section 5 concludes this paper.

II. CLASSICAL DIFFERENTIAL EVOLUTION

The classical DE is a powerful stochastic global optimization algorithm. It regenerates a population through executing some simple arithmetic operations such as mutation, crossover and selection. Here the classical DE will be introduced. For the convenience of introduction, some symbols used throughout this paper are defined.
Consider the following optimization problem:

\[
\min g(x), \quad x = (x_1, x_2, \ldots, x_D) \in \prod_{i=1}^{D} [L_i, U_i], \quad i = 1, 2, \ldots, D,
\]

where each variable \( x_i \) has a lower bound \( L_i \) and an upper bound of \( U_i \), and \( \prod_{i=1}^{D} [L_i, U_i] \) is the feasible domain of this problem.

For solving the optimization problem, an initial population is sampled in the feasible domain randomly. Set

\[
X = \{x_1, x_2, \ldots, x_N\}
\]

is a population with \( N \) individuals. Each individual is \( D \)-dimensional, and we denote the \( j \)-th one as

\[
x^j = (x^j_1, x^j_2, \ldots, x^j_D), \quad j = 1, 2, \ldots, N.
\]

To obtain the optimal solution of the problem, the classical DE works through a simple cycle of operators including the following mutation, crossover, and selection operations.

a) Mutation

For each individual \( x^j \), DE creates a donor vector \( y^j \) by using the following mutation operator.

\[
\begin{align*}
\{u^j_i, v^j_i, \ldots, u^j_D\} & = x^j + F \cdot (x^j - x^j_i), \quad j = 1, 2, \ldots, N, \\
\{u^j_i\} & = L_i, \quad \text{if} \left( u^j_i < L_i \right),
\end{align*}
\]

\[
\begin{align*}
y^j & = U_i, \quad \text{if} \left( u^j_i > U_i \right), \quad i = 1, 2, \ldots, D, \\
\{u^j_i\} & = u^j_i, \quad \text{otherwise}
\end{align*}
\]

\[
y^j = (y^j_1, y^j_2, \ldots, y^j_D), \quad j = 1, 2, \ldots, N.
\]

(1)

Here \( r_1, r_2, r_3 \) are three different integers randomly obtained on \([1, N]\). The scaling factor, \( F \), is an empirical parameter belong on \([0, 1]\).

b) Crossover

For each individual \( x^j \), do

\[
\begin{align*}
z^j & = (z^j_1, z^j_2, \ldots, z^j_D), \quad j = 1, 2, \ldots, N, \\
z^j_i & = x^j_i, \quad \text{if} \left( \text{rand} \leq CR \text{ or } i = \text{rnbr} \right), \quad i = 1, 2, \ldots, D, \\
z^j_i & = x^j_i, \quad \text{if} \left( \text{rand} > CR \text{ and } i \neq \text{rnbr} \right), \quad i = 1, 2, \ldots, D.
\end{align*}
\]

(2)

Here a trial vector \( z^j \) is generated for the individual \( x^j \) by a random value \( \text{rand} \) on \([0, 1] \). \( CR \), a crossover rate, is a empirical parameter on \([0, 1] \). \( \text{rnbr} \), a random integer on \([1, D]\), is used to assure that at least one component of \( z^j \) is taken from the donor vector.

c) Selection

Finally, for each individual \( x^j \), the following selection operator is employed to maintain the most promising individual in the next generation.

\[
x'^j = \begin{cases} x^j, & \text{if} \left( g(x^j) \leq g(z^j_i) \right), \quad j = 1, 2, \ldots, N, \\
z^j_i, & \text{if} \left( g(x^j) > g(z^j_i) \right).
\end{cases}
\]

(3)

Here \( x' \) represents the updated individual according to the fitness values of \( g(x) \) for the next generation population \( X' = \{x'_1, x'_2, \ldots, x'_N\} \).

As stated above, for obtaining the best solution of the fitness function \( g(x) \), DE starts with a randomly generated initial population and then repeatedly updates the population by using the mutation, crossover and selection operations until the stopping condition is satisfied.

III. THE DE-BASED COLOR IMAGE QUANTIZATION ALGORITHM (DE-CIQ)

In the RGB color space \([0, 255]^3 \), a color image is a set of some color pixels consisting of red, green and blue values, which determine the color of these pixels. Set \( I \) is a color image with \( N \) different colors, and \( S \) is the collection of the \( N \) different colors. For the problem of color quantization, the first phase is to determine \( K \) different colors in \([0, 255]^3 \), where \( K < N \). The collection \( S \) of this \( K \) colors is called a colormap. The second phase is to create a map \( f : S \rightarrow S' \), by which each color pixel in \( S \) is replaced by one of the colors in \( S' \). Thus a new color image \( I' \), called the quantized image of \( I \), is constructed. In the image \( S' \), there are \( K \) colors. Commonly, the corresponding rule in the map \( f \) is the minimum Euclidean distance between two colors.

The objective to quantize the color image \( I \) is to minimize the color error between the color image \( I \) and its quantized image \( I' \). The mean square error (MSE) is the most general measure of quality of a quantized image [4]. It is defined as follows:

\[
\text{MSE} = \frac{1}{N_p} \sum_{r=1}^{k} \left[ \min_{p \in S} \{ d(p, c_r) \} \right],
\]

(4)

where the symbols used in Eq.(4) are explained as follows.

- \( N_p \) : the number of image pixels
- \( K \) : the color number in the colormap
- \( p_r = (p_{r,x}, p_{r,y}, p_{r,z}) \) : the \( r \)-th pixel of the color image \( I \), \( r = 1, 2, \ldots, N_p \)
- \( c_k \) : the \( k \)-th color triple in the colormap, \( k = 1, 2, \ldots, K \)

Shown in the above two phases of color quantization problem, a better colormap could improve the quality of a quantized image. Some heuristic techniques, including some pre-clustering approaches and some post-clustering
approaches, have been applied to design colormaps. Among these techniques, the post-clustering approaches are superior to the pre-clustering approaches in the quantization quality. Post-clustering approaches perform clustering of the color space [15]. Post-clustering algorithms start with a randomly given colormap including $K$ colors. Then each color pixel of the image $I$ is mapped to a color in the colormap according to their color similarity. Thus all the color pixels in image $I$ are clustered into $K$ clusters which centers are separately the color triples in the RGB color space. Iteratively, the colormap and the $K$ clusters are adjusted. Post-clustering algorithms are different in their iterative process. This section presents a new post-clustering color image quantization approach using DE, called the DE-based color image quantization algorithm (DE-CIQ), in which the classical DE introduced in section 2 is used.

![Input parameters](image)

**Input color image $I = \{z\}$**

Set parameters $NP$, $F$, $CR$, $t_{\text{max}}$.

$x_{ij}^{0} = \text{rand}(0,1) \times 255$, $i=1,2,\ldots,D$

$x_{ij} = (x_{ij}^{0}, x_{ij}^{1}, \ldots, x_{ij}^{D})$, $j=1,2,\ldots, NP$

//population initialization

For $t = 0$ to $t_{\text{max}}$

For $j=1,2,\ldots, NP$

$r_{nhr} = \text{rand}(0,1)$

$u_{ij}^{t} = x_{ij}^{t} + F \times (x_{ij}^{t} - x_{ij}^{t-1})$

For $i=1,2,\ldots,D$

If $u_{ij}^{t} < 0$ then $y_{ij}^{t} = 0$

Else if $u_{ij}^{t} > 255$ then $y_{ij}^{t} = 255$

Else $y_{ij}^{t} = u_{ij}^{t}$

End if

End for

End for

//mutation

$rand_{t} = \text{rand}(0,1)$

If $rand_{t} \leq CR$ or $i = r_{nhr}$ then $z_{ij}^{t} = y_{ij}^{t}$

Else $z_{ij}^{t} = x_{ij}^{t}$

End if

End for

//crossover

$z_{ij}^{t} = (z_{ij}^{t}, z_{ij}^{t}, \ldots, z_{ij}^{t})$

Calculate $g(x^{t})$ and $g(z^{t})$

If $g(x^{t}) > g(z^{t})$ then $x_{ij}^{t+1} = z_{ij}^{t}$

Else $x_{ij}^{t+1} = x_{ij}^{t}$

End if

Find the optimal solution $x_{\text{opt}} = (x_{1,\text{opt}}, x_{2,\text{opt}}, \ldots, x_{D,\text{opt}})$

Output the optimal colormap \(c_{1}, c_{2}, \ldots, c_{K}\)

where $c_{k} = (x_{1,\text{opt}}, x_{2,\text{opt}}, x_{D,\text{opt}})$, $k = 1,2,\ldots,K$

Construct the quantized image $I'$. 

![Figure 1](image)

**Figure 1.** The pseudocode of the DE-CIQ algorithm

In the DE-CIQ algorithm, the fitness function is as follows:

$$
g(x) = \text{MSE}(x) = \frac{1}{N_{p}} \sum_{i=1}^{N_{p}} \left[ \min_{c_{i}} d(p_{i}, c_{i}) \right]
$$

$$
= \frac{1}{N_{p}} \sum_{i=1}^{N_{p}} \left[ \sum_{k=1}^{K} \left( p_{i,k} - x_{k(i)} \right)^{2} \right]^{1/2}
$$

Where $x = (x_{1}, x_{2}, \ldots, x_{N}) \in [0, 255]^D$.

A population $X = \{x^{1}, x^{2}, \ldots, x^{NP}\}$ represents a set of candidate colormaps. Each individual represents a candidate colormap with $K$ color triples in the RGB color space $[0, 255]^3$, that is, the dimension of each individual is $D = 3 \times K$. The $j$th individual is denoted by

$$
x^{j} = (c_{j,1}^{1}, c_{j,1}^{2}, \ldots, c_{j,1}^{K}) = (x_{1,j}, x_{2,j}, x_{3,j}, x_{4,j}, \ldots, x_{8,5}, x_{9,6})
$$

In the performance of the DE-CIQ algorithm, a population including $NP$ candidate colormaps are randomly initialized in the color space $[0, 255]^3$. Then the population is updated by the mutation, crossover and selection operations in DE. The mutation operation, some better colormaps are determined by the values of the fitness function in Eq.(5). The mutation, crossover and selection operations are repeated until a specified maximal number of iteration $t_{\text{max}}$. The optimal solution obtained by DE is the optimal colormap. Finally, according to the minimal color distance rule, the color of each pixel in the image $I$ is replaced with its corresponding color in the optimal colormap. And the quantized image of $I$ is reconstructed.

The pseudocode of the DE-CIQ algorithm is shown in Figure 1.

IV. NUMERICAL EXPERIMENTS

In this section, the DE-CIQ algorithm is tested on a set of four commonly used test images in the quantization literature. In addition, the performance of the DE-CIQ algorithm is compared with that of the color image quantization algorithm using PSO (PSO-CIQ) presented in literature [5].

A. Images and Parameters Set

The set of test images include Lena, Peppers, Baboon and Airplane, which have the same size $512 \times 512$ pixels. They are shown in Figure 2.

The parameters in the DE-CIQ algorithm are set as the population size $NP = 100$, the scaling factor $F = 0.4$, the crossover rate $CR = 0.7$ and the maximal number of iteration $t_{\text{max}} = 200$. The values of $F$ and $CR$ are suggested by literature [13], in which the classical DE is applied to solve the image binarization.

The PSO-CIQ algorithm has more parameters than the DE-CIQ algorithm. Suppose the swarm size $NP = 100$, with $CR = 0.7$, and the maximal number of iteration $t_{\text{max}} = 200$. The values of $F$ and $CR$ are suggested by literature [13], in which the classical DE is applied to solve the image binarization.
Figure 2. The quantized images obtained by DE-CIQ and PSO-CIQ ($K=16$)
<table>
<thead>
<tr>
<th>Alg.</th>
<th>$K=16$</th>
<th>$K=32$</th>
<th>$K=64$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DE-CIQ</td>
<td>PSO-CIQ</td>
<td>diff</td>
</tr>
<tr>
<td>Peppers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>min</td>
<td>29.3859</td>
<td>36.3436</td>
<td></td>
</tr>
<tr>
<td>max</td>
<td>32.4772</td>
<td>40.9532</td>
<td></td>
</tr>
<tr>
<td>Baboon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>min</td>
<td>33.0359</td>
<td>35.8892</td>
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</tr>
<tr>
<td>max</td>
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<td>41.9940</td>
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</tr>
<tr>
<td>Lena</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>max</td>
<td>27.0157</td>
<td>34.5876</td>
<td></td>
</tr>
<tr>
<td>Airplane</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*diff is the difference between the average MSEs of DE-CIQ and PSO-CIQ.

The iteration-num is the number of iteration. The ave-MSE is the average MSE over 10 simulations.

Figure 3. The average MSE variations with the number of iteration of DE-CIQ and PSO-CIQ.
the inertia weight $\omega = 0.72$, the acceleration constants $c_1 = c_2 = 1.49$, the maximum velocity $V_{\text{max}} = 0.4$, the maximal number of iteration $t_{\text{max}} = 200$. These parameters except for the last one are as same as those in the literature [5].

B. Experimental Results

For each algorithm, the test images are quantized into 16, 32, 64 colors. The 16 colors quantized images with the smallest MSEs over 10 simulations are shown in Figure 2. The smallest, the largest and the average MSEs over 10 simulations are listed in TABLE I. The Figure 3 shows the evolution landscapes of the average MSE.

C. Analysis of Experimental Results

As shown in Figure 2, the DE-CIQ algorithm outperforms the PSO-CIQ algorithm in the visual quality of the quantized images for all test images. The quantized image a-2, b-2, c-2 and d-2 have richer layers and more details than the quantized image a-3, b-3, c-3 and d-3.

As illustrated in TABLE I, the DE-CIQ algorithm generates a smaller MSEs than the PSO-CIQ algorithm for each test image. Moreover, the smaller $\kappa$ is, the larger the differences between the MSEs of the two algorithms are. Shown in Figure 3, the DE-CIQ algorithm has a smaller average MSE than the PSO-CIQ algorithm at each same number of iteration. Moreover, the average MSE resulting from the DE-CIQ algorithm decreases more quickly than that resulting from the PSO-CIQ algorithm with the increasing number of iteration.

The above experimental results can be summarized as follows:

- The DE-CIQ algorithm is an effective color image quantization method;
- The DE-CIQ algorithm has the better quantization quality than the PSO-CIQ algorithm on the test images set;
- The smaller the color number in a colormap is, the better the DE-CIQ algorithm performs than the PSO-CIQ algorithm on the test images set;
- The DE-CIQ algorithm converges more quickly than the PSO-CIQ algorithm on the test images set.

V. CONCLUSIONS

This paper presents a novel color image quantization, called the DE-based color image quantization algorithm (DE-CIQ). Numerical experiments are implemented to investigate the performance of the DE-CIQ algorithm, and to compare it against the color image quantization algorithm using PSO (PSO-CIQ) presented in literature [5]. For a set of commonly used test images, the experimental results demonstrate the feasibility of the DE-CIQ algorithm and its superiority to the PSO-CIQ algorithm in the quantization quality. In addition, the DE-CIQ algorithm has simpler operation, litter parameters and faster convergence than the PSO-CIQ algorithm.

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