Relative to

issues. The attribute reduction is specific criteria, deleting

knowledge discovery, rough set theory is one of the core

systematic approach for classification of objects through

insufficient and incomplete information. It provides a

the study of intelligent systems characterized by

Pawlak(1982)(1991)[1, 2], is an extension of set theory for

with the uncertainty knowledge which proposed by

various equivalence classes combine to describe the

approximation operator, the knowledge base of the

constitute the domain of the under and upper

equivalence relation disjoint equivalence classes of points,

is called the similarity about the attribute

defining the relations sets valued information system, for any

attribute values the problem of attribute reduction and the

information system based on similarity grade of attribute

relevance or unnecessary attributes\(^{9,10}\). However, this

property is necessary to entirely rely on information

systems knowledge or relationship definition, for a

variety of practical problems, you can define different

knowledge or relationship, when the property is

necessary change. Paper\(^{10}\) include relationship defines a

reflexive advantage to passing relationship by means of a

set of attributes values. On this basis, we define a

reflexive relationship which has only advantages

relationship and set the value of information systems in

this relationship attribute reduction judgment, given the

attribute reduction of specific methods of operation.

Rough sets theory is a new mathematical tool to deal

with the uncertainty classes of points, constitute the domain of the under and upper approximation operator, the knowledge base of the various equivalence classes combine to describe the uncertainty of knowledge, to further study the corresponding knowledge reduction and knowledge acquisition. But in practical problems, due to the complexity of the objective world, people's access to the lack of data or data measurement error, for various reasons, so that the information system is incomplete, coupled with essentially equivalent between objects or price relationship is difficult to construct, which will limit rough set model application of classic rough set model for the promotion of various forms of expansion equivalence relation compatible relationship of dominance relations\(^{4,8}\).

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I. INTRODUCTION

Rough sets theory is a new mathematical tool to deal

with the uncertainty classes of points, constitute the domain of the under and upper approximation operator, the knowledge base of the various equivalence classes combine to describe the uncertainty of knowledge, to further study the corresponding knowledge reduction and knowledge acquisition. But in practical problems, due to the complexity of the objective world, people's access to the lack of data or data measurement error, for various reasons, so that the information system is incomplete, coupled with essentially equivalent between objects or price relationship is difficult to construct, which will limit rough set model application of classic rough set model for the promotion of various forms of expansion equivalence relation compatible relationship of dominance relations\(^{4,8}\).

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II. THE ADVANTAGES RELATIONSHIP OF SET-VALUED INFORMATION SYSTEM

Definition 2.1\(^{10}\). Let \((U, A, F)\) be a Set-valued Information System, \(U = \{x_1, x_2, \cdots, x_l\}\) be objects Set, \(A = \{a_1, a_2, \cdots, a_n\}\) be attributes set for any set-valued function \(F = \{f_i: i \leq n\}\), where

\[
f_i : U \longrightarrow P_\alpha(V_i)(U \leq m), a_i \in V_i, P(V_i) \in 2^{V_i}.
\]

Definition 2.2. Let \((U, A, F)\) be a Set-valued Information System, for any \(a_i \in A, x_i \in U\),

\[
\alpha \in [0, 1], \exists \alpha \in (0, 1], \text{define the relations } R^\alpha_a \text{ and } [x_i]_b^\alpha \text{ as follow:}
\]

\[
R^\alpha_a = \{(x_i, x_j) \in U \times U : \forall \alpha \in B, f_i(x_i) \subseteq f_j(x_j), c_j^\alpha \geq \alpha\}
\]

\[
[x_i]_b^\alpha = \{x_j \in U : (x_i, x_j) \in R^\alpha_a\}
\]

Where \(\alpha\) is called Similar levels, Greater about the \(c_j^\alpha\),

the higher similarity degree about the attribute

\(a_i \in A\) for the object \(x_j\) relative to \(x_i, R^\alpha_a\) is called \(\alpha\) -

dominance Relation and \([x_i]_b^\alpha\) is called \(\alpha\) - dominance
class.

Theorem 2.1 Let \((U, A, F)\) be a Set-valued Information System, \(R^\alpha_a\) is a binary relation, then
(1) \( R^a_\alpha \) is reflexive
(2) when \( B_i \subseteq B_j \subseteq A \),
\( \forall \alpha \in (0,1], \ R^a_\alpha \subseteq R^a_\beta \subseteq R^a_\gamma \);
(3) when \( B_i \subseteq B_j \subseteq A \),
\( \forall \alpha \in (0,1], [x_i]_{J_i}^a \subseteq [x_j]_{J_\beta}^a \subseteq [x_j]_{J_\gamma}^a \);
(4) \( J = [x_i]_{J_i}^a : x_i \in U \) is a covering of \( U \).

III. ATTRIBUTE REDUCTION METHOD

Definition 3.1. \( B \subseteq A \) is called \( \alpha \) horizontal coordination Set of Set-valued Information System \((U,A,F)\) , if \( R^a_\alpha = R^a_\beta \). \( B \) is called \( \alpha \) horizontal reduction If \( R^a_\alpha \neq R^a_\beta \), for any \( b \in B \).

Definition 3.2. \( D_\alpha^a = \{ a_i \in A : (x_i, x_j) \notin R^a_\alpha \} \) is called \( \alpha \) distinguish attribute set of Set-valued Information System \((U,A,F)\),
\[ D_\alpha^a = (D_\alpha^a : i, j \leq n) \]
is called \( \alpha \) distinguish matrix of Set-valued Information System \((U,A,F)\), for any \( x_i, x_j \in U \) and \( \alpha \), \( D_\alpha^a = \emptyset \), Which express that all elements on the main diagonal of the \( \alpha \) distinguish matrix is \( \emptyset \).

Theorem 3.1 Let \((U,A,F)\) be a Set-valued Information System, \( B \subseteq A \), \( \forall \alpha \in (0,1], \)
\[ D_\alpha^a = \{ D_\alpha^a : x_i, x_j \in U \} \]
Then the following are equivalent:
(1) \( B \) is \( \alpha \) horizontal coordination Set of Set-valued Information System \((U,A,F)\),
(2) \( B \cap D_\alpha^a \neq \emptyset \) for any \( D_\alpha^a \in D_\alpha^a \);
(3) for any \( B' \subseteq A \), if \( B' \cap B = \emptyset \), then \( B' \notin D_\alpha^a \).

Proof. \( B \) is \( \alpha \) horizontal coordination Set
\( \Leftrightarrow R^a_\alpha \subseteq R^a_\beta \)
\( \Leftrightarrow (x_i, x_j) \notin R^a_\alpha, (x_i, x_j) \notin R^a_\beta \)
\( \Leftrightarrow \) there exist \( a_i \in A \), when \( (x_i, x_j) \notin R^a_\alpha \), there must exist \( a_i \in B \), such that
\( (x_i, x_j) \notin R^a_\alpha \Leftrightarrow a_i \in D_\alpha^a \),
there have \( a_i \in B \) and \( a_i \in D_\alpha^a \Leftrightarrow D_\alpha^a \in D_0 \), \( B \cap D_\alpha^a \neq \emptyset \), i.e.(1) and(2) are equivalent.
(2) and(3) are equivalent clearly established, so the proposition holds.

From the Theorem 3.1, to get the \( \alpha \) horizontal reduction. In fact in seeking minimal set \( B \) to meet the \( B \cap D_\alpha^a \neq \emptyset \), which can be get by The conjunctive type in the minimal disjunctive normal to definition to the discernibility function on matrix\(^{[10]}\).

Example 3.1 Set-valued Information System

<table>
<thead>
<tr>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
<th>( a_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>{1}</td>
<td>{1}</td>
<td>{2}</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>{1}</td>
<td>{1,2,3}</td>
<td>{1,2}</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>{1,2}</td>
<td>{1,2}</td>
<td>{2}</td>
</tr>
<tr>
<td>( x_4 )</td>
<td>{1}</td>
<td>{1,2}</td>
<td>{1,2,3}</td>
</tr>
<tr>
<td>( x_5 )</td>
<td>{1,2}</td>
<td>{1,2}</td>
<td>{1}</td>
</tr>
<tr>
<td>( x_6 )</td>
<td>{2}</td>
<td>{1}</td>
<td>{1}</td>
</tr>
</tbody>
</table>

Let \( \alpha = 0.6 \), then \( \alpha \) distinguish matrix is:
\[ \begin{bmatrix}
\emptyset & \{a_2,a_3\} & \{a_1,a_2,a_3\} & \{a_1,a_3\} & \{a_1,a_2,a_3\} \\
\{a_2,a_3\} & \emptyset & \{a_1,a_2,a_3\} & \{a_3\} & \{a_1,a_2,a_3\} \\
\{a_1,a_3\} & \{a_1,a_2,a_3\} & \emptyset & \{a_1\} & \{a_1,a_2,a_3\} \\
\{a_1,a_2,a_3\} & \{a_3\} & \{a_1\} & \emptyset & \{a_1,a_2,a_3\} \\
\{a_1,a_2,a_3\} & \{a_1,a_2,a_3\} & \{a_1,a_3\} & \{a_1\} & \emptyset \\
\{a_1,a_2,a_3\} & \{a_1,a_2,a_3\} & \{a_1,a_3\} & \{a_1\} & \emptyset \\
\end{bmatrix} \]

discernibility function:
\[ \Delta = a_1 \land (a_2 \lor a_3) = (a_2 \land a_3) \lor (a_1 \land a_3) \]
so there are two \( \alpha \) horizontal reduction
\[ \{a_1,a_3\} \] and \( \{a_1,a_4\} \).

Definition 3.3. Let \( \{B_i : i \leq I\} \) be the \( \alpha \) horizontal reduction of \((U,A,F)\), note
\[ C = \bigcap_{i \in I} B_i, K = \bigcup_{i \in I} B_i - \bigcap_{i \in I} B_i, I = A - \bigcup_{i \in I} B_i, \]
then \( C \cap K \cap I \) are called \( \alpha \) horizontal core attributes set, \( \alpha \) horizontal relative necessary attribute set, \( \alpha \) horizontal absolutely unnecessary attribute set.

From the Definition 3.3, \( \alpha \) horizontal core attributes set belong to the any \( \alpha \) horizontal reduction, while \( \alpha \) horizontally absolutely unnecessary attribute set does not belong to any \( \alpha \) horizontal reduction. In example3.1, \( \alpha \) horizontal core attributes set is \( \{a_1\} \). \( \alpha \) horizontal relative necessary attribute set is \( \{a_1,a_3\} \). \( \alpha \) horizontal absolutely unnecessary attribute set is \( \{a_1\} \).

Theorem 3.3 Let \((U,A,F)\) be a Set-valued Information System, \( \alpha \in (0,1], \) then the following are equivalent:
(1) \( a_i \in A \) is \( \alpha \) horizontal core attributes;
(2) there exist \( x_i, x_j \in U \), such that \( D_\alpha^a = \{a_i\} \);
(3) \( R^a_{\alpha \land \neg a_i} \subseteq R^a_\alpha \)

Proof. (1) \( \Rightarrow \) (2): if(2) not established, then there are at least two elements in \( \alpha \) distinguish attribute set which includes \( a_i \). Let
\[ B = \bigcup_{i \in I} D_\alpha^a - \{a_i\}, D_\alpha^a \in D_\alpha^a, \]
for any \( D_\alpha^a \in D_\alpha^a \), \( B \cap D_\alpha^a \neq \emptyset \). Then \( B \) is \( \alpha \) horizontal coordination set from Theorem 3.1, therefore there exist \( B' \subseteq B \) and \( a_i \notin B' \), which contradiction with \( a_i \) \( \alpha \) horizontal core attribute.
(2) ⇒ (3): from (2), if \( x_i, x_j \in U \), then \((x_i, x_j) \notin R_{\alpha}^a\) and for any \( a_k \in A - \{a_i\} \), there is \((x_i, x_j) \in R_{\alpha}^{a_k}\) i.e.
\n\[ (x_i, x_j) \in R_{\alpha}^{a_k}, (x_i, x_j) \in R_{\alpha} \]

therefore \( R_{\alpha}^{a_k} \subset R_{\alpha}^a \).

(3) ⇒ (1): if \( a_i \) is not the \( \alpha \) horizontal core attributes set, then there is \( B \subset A \), \( a_i \notin B \).

\[ B \subset A - \{a_i\}, R_{\alpha}^{a_i} \subset R_{\alpha}^B \]. But we have \( R_{\alpha}^{a_k} \subset R_{\alpha}^a \) from \( B \) is \( \alpha \) horizontal reduction, \( R_{\alpha}^{a_i} \subset R_{\alpha}^a \) which contradicts with (3).

IV. THE AFFECT OF SIMILAR LEVEL TO SET-VALUED INFORMATION SYSTEM

**Theorem 4.1** Let \((U, A, F)\) be a Set-valued Information System, for any \( \alpha, \beta \in (0,1) \), if \( \alpha \leq \beta \), then
\n\[ R_{\alpha}^a \subseteq R_{\beta}^a, [x_i]_{\alpha}^a \subseteq [x_i]_{\beta}^a \]

Proof. For any \((x_i, x_j) \in R_{\alpha}^a\), \( a_i \in B \), then
\n\[ f_i(x_i) \subseteq f_j(x_j), c_{ij}^\alpha \geq \beta \]

but \( \alpha \leq \beta \), therefore
\n\[ c_{ij}^\beta \geq \alpha, (x_i, x_j) \in R_{\beta}^a \]

So
\n\[ R_{\beta}^a \subseteq R_{\alpha}^a \]

from the relation of \( R_{\alpha}^a \) and \([x_i]_{\alpha}^a, [x_i]_{\beta}^a \subseteq [x_i]_{\alpha}^a \).

**Theorem 4.2** Let \((U, A, F)\) be a Set-valued Information System, for any \( x_i, x_j \in U \), \( \alpha, \beta \in (0,1) \), if \( \alpha \leq \beta \), then

(1) \[ D_{\alpha}^a \subseteq D_{\beta}^a \]

(2) \[ D_{\alpha}^a \] is more meticulous than \( D_{\beta}^a \), i.e. for any \( D_{\alpha}^a \in D_{\beta}^a \), if \( D_{\alpha}^a \in D_{\beta}^a \), then \( D_{\alpha}^a \subseteq D_{\beta}^a \).

**Example 4.1** Taking Example 3.1 again, let \( \alpha = 0.5 \) then \( \alpha \) distinguish matrix can be got as following:

\[
\begin{pmatrix}
\emptyset & {} [a_i] & [a_i, a_j] & [a_i, a_k] & [a_i, a_l] \\
{[a_i, a_j]} & \emptyset & [a_i, a_k] & [a_i, a_l] & [a_i, a_l] \\
{[a_i, a_k]} & [a_i, a_j] & \emptyset & [a_i, a_l] & [a_i, a_l] \\
{[a_i, a_l]} & [a_i, a_j] & [a_i, a_k] & \emptyset & [a_i, a_l] \\
{[a_i, a_l]} & [a_i, a_j] & [a_i, a_k] & [a_i, a_l] & \emptyset \\
\end{pmatrix}
\]

From Theorem 3.1, \( B = \{a_i, a_j, a_k\} \) is the only \( \alpha \) horizontal reduction, \( \{a_i, a_j, a_k\} \) is \( \alpha \) horizontal core attributes set. \( \alpha \) horizontal relative necessary attribute set is \( \emptyset \) and \( \alpha \) horizontal absolutely unnecessary attribute set is \( \{a_i\} \).

V. CONCLUSIONS

the set-value of information systems can be used to deal with the data incomplete information system, the advantage of a new relationship between the degree of similarity in this paper by means of the value of the

attribute set defined on the set-valued information system, given the dominance relations set attribute reduction of the value of information systems with the judgment, seeking a reduction of the valued information systems operation, and at the same time similar level of valued information systems attribute reduction.

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