# Attribute Reduction of Set-valued Information System Based on Dominance Relation

Chunzhen Zhong
School of Mathematics and Information Science , Neijiang Normal University, Neijiang , China
Email: nj_zhongcz@126.com

***Abstract*—A new dominance relation is defined in set-valued information system based on similarity grade of attribute value sets the problem of attribute reduction and the judgment in set-valued information system are studied, from which a new approach to attribute reductions is provided in set-valued information system, and the influence of similarity level on attribute reduction are discussed..**

***Index Terms*—Set-valued information system, minance relation, similarity level, attribute reduction**

## I. Introduction

Rough sets theory is a new mathematical tool to deal with the uncertainty knowledge which proposed by Pawlak(1982)(1991)[1, 2], is an extension of set theory for the study of intelligent systems characterized by insufficient and incomplete information. It provides a systematic approach for classification of objects through an in discern ability relation[2,3]. Many examples of applications of the rough sets method to process control, economics, medical diagnosis, biochemistry, environmental science, biology, chemistry psychology, conflict analysis and other fields.

Classic rough set model is an assumption based on incomplete information system, all attribute values for each object are known, plays a very important role in On the equivalence relation on the domain, the domain of equivalence relation disjoint equivalence classes of points, constitute the domain of the under and upper approximation operator, the knowledge base of the various equivalence classes combine to describe the uncertainty of knowledge, to further study the corresponding knowledge reduction and knowledge acquisition. But in practical problems, due to the complexity of the objective world, people's access to the lack of data or data measurement error, for various reasons, so that the information system is incomplete, coupled with essentially equivalent between objects or price relationship is difficult to construct, which will limit rough set model application of classic rough set model for the promotion of various forms of expansion equivalence relation compatible relationship of dominance relations[4,8].

Attribute reduction is an important research topic of knowledge discovery, rough set theory is one of the core issues. The attribute reduction is specific criteria, deleting irrelevant or unnecessary attributes[9,10]. However, this property is necessary to entirely rely on information systems knowledge or relationship definition, for a variety of practical problems, you can define different knowledge or relationship, when the property is necessary change. Paper[10] include relationship defines a reflexive advantage to passing relationship by means of a set of attributes values. On this basis, we define a reflexive relationship which has only advantages relationship and set the value of information systems in this relationship attribute reduction judgment, given the attribute reduction of specific methods of operation.

## II. The Advantages Relationship of Set-valued Information System

**Definition**2.1[10]. Let $(U, A, F)$ be a Set-valued Information System, $U=\{x_1, x_2, \cdots, x_n\}$ be objects Set, $A=\{a_1, a_2, \cdots, a_m\}$ be attributes set for any set-valued function $F=\{f_l : l \le m\}$ ,where

$$f_l : U \longrightarrow P_0(V_l)(l \le m), a_l \in V_l, P_0(V_l) \in 2^{V_l} .$$

**Definition2.2.** Let $(U, A, F)$ be a Set-valued Information System, for any $a_l \in A, x_i, x_j \in U$ ,

$$c_{ij}^l = \frac{|f_l(x_i)|}{|f_l(x_j)|}$$

is called the similarity about the attribute $a_l \in A$ for the object $x_i$ relative to $x_j$ .

For $B \subseteq A, \alpha \in (0,1]$ ,define the relations $R_B^\alpha$ and $[x_i]_B^\alpha$ as follow:

$$R_B^\alpha = \left\{(x_i, x_j) \in U \times U : \forall \alpha_i \in B, f_i(x_i) \subseteq f_i(x_j), c_{ij}^i \ge \alpha\right\}$$

$$[x_i]_B^\alpha = \{x_j \in U : (x_i, x_j) \in R_B^\alpha\}$$

Where $\alpha$ is called Similar levels, Greater about the $c_{ij}^l$ , the higher similarity degree about the attribute $a_l \in A$ for the object $x_i$ relative to $x_j$ . $R_B^\alpha$ is called $\alpha$ - dominance Relation and $[x_i]_B^\alpha$ is called $\alpha$ - dominance class.

**Theorem 2.1** Let $(U, A, F)$ be a Set-valued Information System, $R_B^\alpha$ is a binary relation, then


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(1) $R_B^\alpha$ is reflexive

(2) when $B_1 \subseteq B_2 \subseteq A$,

$$\forall \alpha \in (0,1],\ R_A^\alpha \subseteq R_{B_2}^\alpha \subseteq R_{B_1}^\alpha ;$$

(3) when $B_1 \subseteq B_2 \subseteq A$,

$$\forall \alpha \in (0,1],\ [x_i]_A^\alpha \subseteq [x_i]_{B_2}^\alpha \subseteq [x_i]_{B_1}^\alpha ;$$

(4) $\{J = [x_i]_B^\alpha : x_i \in U\}$ is a covering of $U$ .

## III. Attribute Reduction Method

**Definition3.1.** $B \subseteq A$ is called $\alpha$ horizontal coordination Set of Set-valued Information System $(U, A, F)$ , if $R_B^\alpha = R_A^\alpha$ . $B$ is called $\alpha$ horizontal reduction If $R_{B-\{b\}}^\alpha \neq R_A^\alpha$, for any $b \in B$.

**Definition3.2.** $D_{ij}^\alpha = \{a_l \in A : (x_i, x_j) \notin R_{\{a_l\}}^\alpha\}$ is called $\alpha$ distinguish attribute set of Set-valued Information System $(U, A, F)$ ,

$$D^\alpha = (D_{ij}^\alpha : i, j \leq n)$$

is called $\alpha$ distinguish matrix of Set-valued Information System $(U, A, F)$ . for any $x_i, x_j \in U$ and $\alpha$ , $D_{ii}^\alpha = \varnothing$ ,Which express that all elements on the main diagonal of the $\alpha$ distinguish matrix is $\varnothing$.

**Theorem 3.1** Let $(U, A, F)$ be a Set-valued Information System, $B \subseteq A, \forall \alpha \in (0,1]$,

$$D_0^\alpha = \left\{ D_{ij}^\alpha \neq \varnothing : x_i, x_j \in U \right\}$$

Then the following are equivalent:

(1) $B$ is $\alpha$ horizontal coordination Set of Set-valued Information System $(U, A, F)$

(2) $B \cap D_{ij}^\alpha \neq \varnothing$ for any $D_{ij}^\alpha \in D_0^\alpha$;

(3)for any $B' \subseteq A$ ,if $B' \cap B = \varnothing$ , then $B' \notin D_0^\alpha$

Proof. $B$ is $\alpha$ horizontal coordination Set

$\Leftrightarrow$ $R_B^\alpha \subseteq R_A^\alpha$

$\Leftrightarrow$ $(x_i, x_j) \notin R_A^\alpha, (x_i, x_j) \notin R_B^\alpha$

$\Leftrightarrow$ there exist $a_l \in A$ ,when $(x_i, x_j) \notin R_{\{a_l\}}^\alpha$ ,There must exist $a_k \in B$ ,such that

$$(x_i, x_j) \notin R_{\{a_k\}}^\alpha \Leftrightarrow a_l \in D_{ij}^\alpha ,$$

there have $a_k \in B$ and $a_k \in D_{ij}^\alpha \Leftrightarrow D_{ij}^\alpha \in D_0$ , $B \cap D_{ij}^\alpha \neq \varnothing$ ,i.e.(1) and(2)are equivalent.

(2)and(3)are equivalent clearly established, so the proposition holds.

From the Theorem 3.1, to get the $\alpha$ horizontal reduction, In fact in seeking minimal set $B$ to meet the $B \cap D_{ij}^\alpha \neq \varnothing$ ,which can be get by The conjunctive type in the minimal disjunctive normal to definition to the discernibility function on matrix[10] .

Example 3.1 Set-valued Information System

| | $a_1$ | $a_2$ | $a_3$ | $a_4$ |
|---|---|---|---|---|
| $x_1$ | {1} | {1} | {2} | {2} |
| $x_2$ | {1} | {1,2,3} | {1,2} | {2} |
| $x_3$ | {1,2} | {1,2} | {2} | {1} |
| $x_4$ | {1} | {1,2} | {1,2,3} | {1} |
| $x_5$ | {1,2} | {1,2} | {1} | {1} |
| $x_6$ | {2} | {1} | {1} | {1,2} |

Let $\alpha$ =0.6, then $\alpha$ distinguish matrix is :

$$\begin{pmatrix} \varnothing & \{a_2,a_3\} & \{a_1,a_2,a_4\} & \{a_2,a_3,a_4\} & \{a_1,a_2,a_3,a_4\} & \{a_1,a_3,a_4\} \\ \{a_2,a_3\} & \varnothing & \{a_1,a_2,a_3,a_4\} & \{a_2,a_4\} & \{a_1,a_2,a_3,a_4\} & \{a_1,a_2,a_3,a_4\} \\ \{a_1,a_2,a_4\} & \{a_1,a_3,a_4\} & \varnothing & \{a_1,a_3\} & \{a_3\} & \{a_1,a_2,a_3,a_4\} \\ \{a_2,a_3,a_4\} & \{a_3,a_4\} & \{a_1,a_3\} & \varnothing & \{a_1,a_3\} & \{a_1,a_2,a_3,a_4\} \\ \{a_1,a_2,a_3,a_4\} & \{a_1,a_3,a_4\} & \{a_3\} & \{a_1,a_3\} & \varnothing & \{a_1,a_2,a_4\} \\ \{a_1,a_3,a_4\} & \{a_1,a_2,a_3,a_4\} & \{a_1,a_2,a_3,a_4\} & \{a_1,a_2,a_3,a_4\} & \{a_1,a_2,a_4\} & \varnothing \end{pmatrix}$$

discernibility function:

$$\Delta = a_3 \wedge (a_2 \vee a_4) = (a_2 \wedge a_3) \vee (a_3 \wedge a_4)$$

so there are two $\alpha$ horizontal reduction

$$\{a_2, a_3\} \text{ and } \{a_3, a_4\} .$$

**Definition 3.3**. Let $\{B_i : i \leq l)\}$ be the $\alpha$ horizontal reduction of $(U, A, F)$ , note

$$C = \bigcap_{i \leq l} B_i, K = \bigcup_{i \leq l} B_i - \bigcap_{i \leq l} B_i, I = A - \bigcup_{i \leq l} B_i ,$$

then $C$ 、 $K$ 、 $I$ are called $\alpha$ horizontal core attributes set, $\alpha$ horizontal relative necessary attribute set, $\alpha$ horizontal absolutely unnecessary attribute set.

From the Definition 3.3, $\alpha$ horizontal core attributes set belong to the any $\alpha$ horizontal reduction, while $\alpha$ horizontal absolutely unnecessary attribute set does not belong to any $\alpha$ horizontal reduction. In example3.1, $\alpha$ horizontal core attributes set is $\{a_3\}$ . $\alpha$ horizontal relative necessary attribute set is $\{a_2, a_4\}$ . $\alpha$ horizontal absolutely unnecessary attribute set is $\{a_1\}$ .

**Theorem 3.3** Let $(U, A, F)$ be a Set-valued Information System, $\alpha \in (0,1]$ ,then the following are equivalent:

(1) $a_l \in A$ is $\alpha$ horizontal core attributes;

(2)there exist $x_i, x_j \in U$ ,such that $D_{ij}^\alpha = \{a_l\}$ ;

(3) $R_{A-\{a_l\}}^\alpha \not\subset R_A^\alpha$

Proof. (1) $\Rightarrow$ (2): if(2)not established, then there are at least two elements in $\alpha$ distinguish attribute set which includes $a_l$ . Let

$$B = \bigcup \{D_{ij}^\alpha - \{a_l\} : D_{ij}^\alpha \in D_0^\alpha\} ,$$

for any $D_{ij}^\alpha \in D_0^\alpha$ , $B \cap D_{ij}^\alpha \neq \varnothing$ . Then $B$ is $\alpha$ horizontal coordination set from Theorem 3.1. therefore there exist $B' \subseteq B$ and $a_l \notin B'$ ,which contradiction with $a_l$ $\alpha$ horizontal core attribute.

(2) $\Rightarrow$ (3):from (2), if $x_i, x_j \in U$ , then $(x_i, x_j) \notin R^{\alpha}_{\{a_l\}}$ and for any $a_k \in A-\{a_l\}$ ,

there is $(x_i, x_j) \in R^{\alpha}_{\{a_k\}}$ ,i.e.

$$(x_i, x_j) \notin R_A, (x_i, x_j) \in R_{A-\{a_l\}}.$$

therefore $R^{\alpha}_{A-\{a_l\}} \not\subset R^{\alpha}_A$ .

(3) $\Rightarrow$ (1):if $a_l$ is not the $\alpha$ horizontal core attributes set ,then there is $B \subseteq A$ , $a_l \notin B$ . $B \subseteq A-\{a_l\}$ , $R^{\alpha}_{A-\{a_l\}} \subseteq R^{\alpha}_B$ . But we have $R^{\alpha}_B \subseteq R^{\alpha}_A$ from $B$ is $\alpha$ horizontal reduction, $R^{\alpha}_{A-\{a_l\}} \subseteq R^{\alpha}_A$ which contradicts with(3).

## IV. The Affect of Similar Level to Set-valued Information System

**Theorem4.1** Let $(U, A, F)$ be a Set-valued Information System, for any $\alpha, \beta \in (0,1]$ ,if $\alpha \le \beta$ ,then

$$R^{\beta}_B \subseteq R^{\alpha}_B, [x_i]^{\beta}_B \subseteq [x_i]^{\alpha}_B.$$

Proof. for any $(x_i, x_j) \in R^{\beta}_B$, $a_l \in B$ , then

$$f_l(x_i) \subseteq f_l(x_j), c^l_{ij} \ge \beta,$$

but $\alpha \le \beta$ ,therefore

$$c^l_{ij} \ge \alpha, (x_i, x_j) \in R^{\alpha}_B.$$

So

$$R^{\beta}_B \subseteq R^{\alpha}_B,$$

from the relation of $R^{\alpha}_B$ and $[x_i]^{\alpha}_B$, $[x_i]^{\beta}_B \subseteq [x_i]^{\alpha}_B$ .

**Theorem4.2** Let $(U, A, F)$ be a Set-valued Information System, for any $x_i, x_j \in U$ , $\alpha, \beta \in (0,1]$ ,if $\alpha \le \beta$ ,then

(1) $D^{\alpha}_{ij} \subseteq D^{\beta}_{ij}$ ;

(2) $D^{\alpha}_0$ is more meticulous than $D^{\beta}_0$ ,i.e. for any $D^{\alpha}_{ij} \in D^{\alpha}_0$ ,if $D^{\beta}_{ij} \in D^{\beta}_0$ , then $D^{\alpha}_{ij} \subseteq D^{\beta}_{ij}$ .

**Example 4.1** Taking Example 3.1 again, let $\alpha$ =0.5 then $\alpha$ distinguish matrix can be got as following:

$$\begin{pmatrix} \varnothing & \{a_2\} & \{a_4\} & \{a_3,a_4\} & \{a_3,a_4\} & \{a_1,a_3\} \\ \{a_2,a_3\} & \varnothing & \{a_2,a_3,a_4\} & \{a_2,a_4\} & \{a_2,a_3,a_4\} & \{a_1,a_2,a_3\} \\ \{a_1,a_2,a_4\} & \{a_1,a_4\} & \varnothing & \{a_1,a_3\} & \{a_3\} & \{a_1,a_2,a_3\} \\ \{a_2,a_3,a_4\} & \{a_3,a_4\} & \{a_3\} & \varnothing & \{a_3\} & \{a_1,a_2,a_3\} \\ \{a_1,a_2,a_3,a_4\} & \{a_1,a_4\} & \{a_3\} & \{a_1,a_3\} & \varnothing & \{a_1,a_2\} \\ \{a_1,a_3,a_4\} & \{a_1,a_2,a_4\} & \{a_3,a_4\} & \{a_1,a_3,a_4\} & \{a_4\} & \varnothing \end{pmatrix}$$

From Theorem3.1, $B = \{a_2, a_3, a_4\}$ is the only $\alpha$ horizontal reduction,, $\{a_2, a_3, a_4\}$ is $\alpha$ horizontal core attributes set. $\alpha$ horizontal relative necessary attribute set is $\varnothing$ and $\alpha$ horizontal absolutely unnecessary attribute set is $\{a_1\}$ .

## V. Conclusions

the set -value of information systems can be used to deal with the data incomplete information system, the advantage of a new relationship between the degree of similarity in this paper by means of the value of the attribute set defined on the set-valued information system, given the dominance relations set attribute reduction of the value of information systems with the judgment, seeking a reduction of the valued information systems operation, and at the same time similar level of valued information systems attribute reduction.


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**Chunzhen Zhong** She received Master in1993, in Sichuan Normal University. Now, she is an Associate Professor in Neijiang Normal University. Her research interest including: rough set, information system.