Approach for Optimizing 3D Highway Alignments Based on Two-stage Dynamic Programming

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Abstract—**Optimizing highway alignments is a very complex engineering problem. None of existing methods has totally solved the problem In this paper we build an optimization model for highway 3D alignment based on two-stage dynamic programming, in which the comprehensive cost and design constraints are embedded. In the first stage: we formulate the optimization of alignment as a network problem by establishing a three-dimensional grid representation for the study area. Then dynamic programming is used to find the best corridor alignment. In order to obtain a group of scenarios, we propose a bidirectional searching strategy in dynamic programming. In the second stage: we put forward an improved direction acceleration method considering constraints to refine the corridor alignments for resulting the alignment deviate from the grid points and reaching the global optimum. Experimental results show that the global optimal alignment with good diversity was ultimately obtained.**

Index Terms— **highway alignment, alignment optimization, dynamic programming, direction acceleration method**

I. Introduction

Highway alignment is a very complex problem. Various factors such as design specifications, costs, geographic features and environment should be considered [1]. The problem is so complex that the designer can only afford a few candidate alignments by manual design. In fact the number of alternatives is unlimited. So many good possible alternatives may be overlooked and the last alignment is merely a satisfactory solution.

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Alignments optimization mainly includes horizontal alignments optimization, vertical alignments optimization and three dimensional alignments optimization. The mainly used methods to optimize horizontal alignments are calculus of variations [2], network optimization [3], dynamic programming [4] and genetic algorithms [5]. Existing approaches for optimizing vertical alignments are enumeration [6], linear programming [7], numerical search [8], dynamic programming [9, 10], while existing approaches for 3D alignments optimization (i.e. simultaneously optimizing horizontal and vertical alignments are distance transform [11], neighborhood search [12] neural network [13], genetic algorithms [14-17] and dynamic programming [18].

Among these existing methods, dynamic programming (DP) has been widely applied and can yield good results especially in the optimization of vertical alignments [10]. However, there are still some problems. Firstly, previous research about dynamic programming generally focuses on vertical alignments or horizontal alignments solely, while dynamic programming for simultaneously optimizing horizontal and vertical alignments can rarely be seen from literatures. Secondly, traditional dynamic programming can only obtain discrete solutions that confined to certain coarse grid of points. Such solutions can by no means be recognized as global optimum or nearly global optimum because the search area is only a subset of the whole search space. Thirdly, the final alignment obtained by dynamic programming is generally composed of piecewise linear segments, which is not good enough for real applications as a typical alignment consists of geometric curves and tangent lines. Finally, dynamic programming can only obtain one single solution, which obviously cannot satisfy the scenario-diversity demand of alignment selection process, because a group of scenarios tend to be more desirable and valuable for highway planners than merely one so-called best solution.

Due to the rigorous geometric specifications of alignments and the complexity of the problem, none of

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existing methods has totally solved the problem of alignments optimization. To sort out the problems mentioned above, we developed a two-stage approach for optimizing 3D highway alignments. The preliminary alignments are generated by using DP in the first stage. Then, direction acceleration method which is desirable for multiple function optimizations was introduced to refine the preliminary alignments so that the resulting alignment could deviate from the grid points to reach the global optimum. Moreover, we proposed a bidirectional searching strategy in dynamic programming to obtain a group of scenarios ranking by their corresponding comprehensive cost.

II. Optimization Model of Alignment

In the process of alignment optimization, generally the starting point and the ending point will not be included in the points of intersections (PIs) because they are known and constant as well. So, optimization of three dimensional alignments can be regarded as the following non-linear programming issue:

$$\min \quad \mathrm{f}(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{R}, \boldsymbol{L}, \boldsymbol{Z}), \tag{1}$$

$$s.t. \quad \mathrm{g}_i(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{R}) \le 0 \quad i = 1, 2, \cdots, n, \tag{1a}$$

$$\mathrm{h}_j(\boldsymbol{L}, \boldsymbol{Z}) \le 0 \quad j = 1, 2, \cdots, m. \tag{1b}$$

A. Design Variables

$\boldsymbol{X} = [x_1, x_2, ..., x_n]^{\mathrm{T}}$ and $\boldsymbol{Y} = [y_1, y_2, ..., y_n]^{\mathrm{T}}$ are the coordinates for horizontal points of intersection (HPIs), $\boldsymbol{R} = [r_1, r_2, ..., r_n]^{\mathrm{T}}$ are the horizontal curve radius. Where, n is the number of HPIs.

$\boldsymbol{L} = [l_1, l_2, ..., l_m]^{\mathrm{T}}$ are the mileages for vertical points of intersection (VPIs), $\boldsymbol{Z} = [z_1, z_2, ..., z_m]^{\mathrm{T}}$ are the corresponding heights of VPIs. Where, m is the number of VPIs.

B. Objective Function

$$\mathrm{f}(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{R}, \boldsymbol{L}, \boldsymbol{Z}) = f_1 + f_2 + f_3 + f_4 + f_5 + f_6. \tag{2}$$

Where $f_1, ..., f_6$ are respectively the earth work costs, land acquisition costs, bridges costs, tunnels costs, retaining structure costs and length-related costs (i.e. pavement, guardrail et al.).

C. Constraint Conditions

$\mathrm{g_i}(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{R}) \le 0$ are horizontal constraints, including minimum curvature radius constraints, curvature length constraints and general area constraints.

$\mathrm{h_j}(\boldsymbol{L}, \boldsymbol{Z}) \le 0$ are vertical constraints, including gradient constraints, vertical curvature constraints and general area constraints.

Based on this optimization model, we develop a two-stage approach to optimize the high alignment.

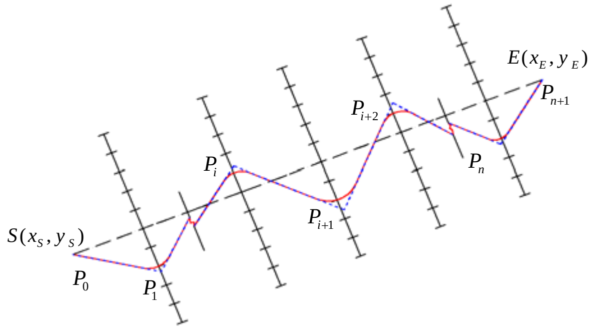

Figure 1. Plane of the Search Grid

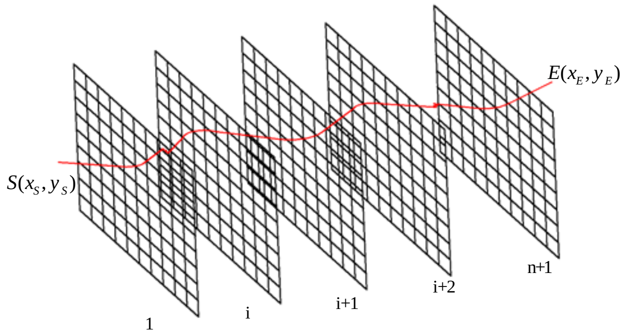

Figure 2. A 3D Search Grid

III. Stage 1: Dynamic Programming

A. Three-dimensional Network

The study areas can be divided into a three dimensional lattice (see Fig. 1 and Fig. 2). Let S and E be the starting point and ending point respectively. SE is the direct line linking the two points, which can be divided by n cutting faces $A_1 \cdots A_n$, crossing at n points $P_1 \cdots P_n$.

B. Generating Horizontal & Vertical Alignments

1) Horizontal Alignments

Here each optional point on the cutting face is regarded as the midpoints of the circular curve of horizontal alignments rather than the intersection points of piecewise lines. If the numbers of optional points in the network are $m_1 \cdots m_n$ (see Fig.1), then each midpoint of the circular curve should be selected from these network points. Curve radius will be designated with reference to the curve radius suggested by design standard. When the suggested curve radius cannot satisfy constraint conditions, curve radius to be assigned will be reduced until they are eligible and meanwhile can ensure that the curve radius should not be smaller than the minimum curve radius.

2) Vertical Alignments

Generally, the number of HPIs and those of VPIs should not be necessarily coincident, and the relevant vertical alignment in one single segment between two midpoints of horizontal circular curves not necessarily has merely one vertical slope, as the slopes should be adjusted to the specific topography. We developed a special method to automatically generate the optimal vertical alignments by integrating smoothing the terrain and least square fit, which has been interpreted elaborately in another paper [18].

C. Dynamic Programming Model

1) Stages

Alignments are divided into n+1 parts by n cutting face (see Fig.2). These n+1 parts can be considered as the n+1 stages of dynamic programming.

2) State and Decision

Take the k stage for example, where k=0,1,…,n, If there are m_k points (i.e. $P_{k,1}, P_{k,2} \cdots P_{k,m_k}$) in the k cutting face, then the advisable point in the k stage can be selected from the following set

$$S_k \in \{P_{k,1}, P_{k,2} \cdots P_{k,m_k}\} . \quad (3)$$

Let $D_k(S_k)$ denote the decision set, and then the points that will be selected in the next stage should be

$$U_k \in D_k(S_k) = \{P_{k+1,1}, P_{k+1,2} \cdots P_{k+1,m_{k+1}}\} . \quad (4)$$

3) Policy

Let P_0 and P_{n+1} denote the starting point and ending point respectively, one possible alignment can be represented by one policy:

$$Q_{0,n} = \{U_0(P_0), U_1(P_1), \cdots, U_n(P_n)\} . \quad (5)$$

The k^{th} process policy is:

$$Q_{k,n} = \{U_k(P_k), U_{k+1}(P_{k+1}), \cdots, U_n(P_n)\} . \quad (6)$$

4) Transfer Equation

$$S_{k+1} = T_k(S_k, U_k) . \quad (7)$$

4) Objective Function

Let $\mathrm{f}_k(P_k, P_{k+1})$ represent the comprehensive cost of the k part of the alignments, then the stage function is:

$$V_k(S_k, U_k) = \mathrm{f}_k(P_k, P_{k+1}) . \quad (8)$$

The process function is:

$$\begin{aligned} V_{k,n} &= V_{k,n}(S_k, U_k, S_{k+1}, U_{k+1}, ..., S_n, U_n) \\ &= \sum_{j=k}^{n} V_j(S_j, U_j) = \sum_{j=k}^{n} \mathrm{f}_j(P_j, P_{j+1}) \end{aligned} \quad (9)$$

The optimal state $Q_{k,n}$ can be achieved by optimizing $V_{k,n}$. Meanwhile the optimal function S_k, the minimum comprehensive cost for the k^{th} stage P_{n+1} (denoted by $\mathrm{f}_k(S_k)$) can be achieved.

$$\mathrm{f}_k(S_k) = \min_{U_k,...,U_n} V_{k,n}(S_k, U_k, S_{k+1}, U_{k+1}, ..., S_n, U_n) . \quad (10)$$

5) Basic Equation

Thus, the basic equation of dynamic programming can be presented as:

Recurrence equation

$$\mathrm{f}_k(S_k) = \min_{U_k \in D_k(S_k)} \{V_k(S_k, U_k) + \mathrm{f}_{k+1}(S_{k+1})\} , \quad (11)$$

Boundary condition

$$\mathrm{f}_{n+1}(S_{n+1}) = 0 . \quad (12)$$

The recurrence equation can be deduced from k=n, then the optimal decisions and best objective function can be achieved. When $k = 0$, the $\mathrm{f}_0(S_0)$ can be acquired, which is also the solution of the entire optimization problem.

D. Calculation of the k^th^ Stage Function

In the k^{th} stage, presume points selected from cutting face A_k and A_{k+1} are respectively $P_k(x_k, y_k, z_k)$ and $P_{k+1}(x_{k+1}, y_{k+1}, z_{k+1})$. Taking P_k as the midpoint of the circular curve, horizontal alignment between P_k and P_{k+1} can be inversely calculated.

If the horizontal alignment segment does not satisfy horizontal constraints, let: $V_k(S_k, U_k) = \mathrm{f}_k(P_k, P_{k+1}) = \infty$.

Else if the horizontal alignment segment satisfies horizontal constraints, then if the slope connecting z_k and z_{k+1} is larger than the acceptable maximum slope or shorter than the acceptable minimum slope, which means by no means the vertical alignments can satisfy the vertical constraint, similarly the corresponding function of this alignment function should be regarded as infinitely large. Let: $V_k(S_k, U_k) = \mathrm{f}_k(P_k, P_{k+1}) = \infty$.

Otherwise, the vertical alignments between $P_k(x_k, y_k, z_k)$ and $P_{k+1}(x_{k+1}, y_{k+1}, z_{k+1})$ should be automatically designed according to the specific terrain, and function value can be calculated by (2), namely the value of $V_k(S_k, U_k)$.

E. Bidirectional Search Strategy

As dynamic programming can only obtain one single global optimum, which cannot satisfy the scenario-diversity demand for highway alignment selection, a bidirectional searching strategy in dynamic programming was proposed to obtain a group of scenarios ranking by their corresponding comprehensive cost.

Dynamic programming is essentially memorized searching. For the search process from the starting point to the ending point, in the k^{th} stage, each optional point $P_{k,i}$ in the state S_k records the minimum cost function from the current optional point to the ending point,

$$\mathrm{f}_k(P_{k,j}) = \min_{U_k \in D_k(S_k)} \{V_{k,n}(P_{k,j}, U_k(P_{k,j})) + \mathrm{f}_k(S_k)\} \quad (13)$$

And the route from the current optional point to the ending point

$$Q_{k,n}(P_{k,j}) = \{U_k(P_{k,j}), U_{k+1}, \cdots, U_n\} . \quad (14)$$

For the search process from the ending point to the starting point, in the k^{th} stage, each optional point $P_{k,j}$ records the minimum cost function from the current optional point to the starting point,

$$\mathrm{f}_k^{'}(P_{k,j}) = \min_{U_k \in D_k(S_k)} \{V_{k,0}(P_{k,j}, U_k(P_{k,j})) + \mathrm{f}_{k-1}(S_{k-1})\} . \quad (15)$$

And the route from the current optional point to the starting point

$$Q'_{k,0}(P_{k,j}) = \{U_k(P_{k,j}), U_{k-1}, \cdots, U_0\}\,. \quad (16)$$

Denote:

$$\mathrm{f}(P_{k,j}) = \mathrm{f}_k(P_{k,j}) + \mathrm{f}'_k(P_{k,j})\,, \quad (17)$$

$$Q(P_{k,j}) = Q_{k,n}(P_{k,j}) + Q'_{k,0}(P_{k,j})\,. \quad (18)$$

We get the following two theorems:

Theorem.1:

$\mathrm{f}(P_{k,j})$ should be the minimum cost function linking the starting point and the ending point while passing through the control point $P_{k,j}$.

Theorem.2:

$Q(P_{k,j})$ should be the corresponding best route linking the starting point and the ending point while passing through the control point $P_{k,j}$.

Proof:

We suppose there exists another route passing through the control point $P_{k,j}$ with a much smaller cost, the corresponding cost functions from $P_{k,j}$ to the starting point and from $P_{k,j}$ to the ending point being denoted by $f_{QD}(P_{k,j})$ and $f_{ZD}(P_{k,j})$, so the hypothesis can be denoted by $f_{QD}(P_{k,j}) + f_{ZD}(P_{k,j}) < \mathrm{f}(P_{k,j})$.

Considering the optimal substructure characteristic of DP, we have:

$$f_{QD}(P_{k,j}) \ge \mathrm{f}'_k(P_{k,j})\,, \quad (19)$$

$$f_{ZD}(P_{k,j}) \ge \mathrm{f}_k(P_{k,j})\,, \quad (20)$$

By adding (19) to (20), we get

$$f_{QD}(P_{k,j}) + f_{ZD}(P_{k,j}) \ge \mathrm{f}_k(P_{k,j}) + \mathrm{f}'_k(P_{k,j}) = \mathrm{f}(P_{k,j})\,. \quad (21)$$

Which is contradictory to our previous hypothesis: $f_{QD}(P_{k,j}) + f_{ZD}(P_{k,j}) < \mathrm{f}(P_{k,j})$.

In this way, by adopting bidirectional search strategy in dynamic programming, the minimum cost function $\mathrm{f}(P_{k,j})$ passing through each control point $P_{k,j}$ while linking the starting point and ending point can be obtained. If we rank the routes according to the value of $\mathrm{f}(P_{k,j})$, we can obtain a group of optimized scenarios, then according to the transfer equation $S_{k+1} = T_k(S_k, U_k)$, corresponding alignments can be available.

The DP can only obtain discrete solutions that confined to certain relatively coarse grid of points. In order to result the alignment deviate from the grid points to reach the global optimum, direction acceleration method was introduced to refine the preliminary alignment in the second stage.

IV. Stage 2: Direction Acceleration Method

Direction acceleration method, which is also called Powell method, has a fast convergence speed and is so far the most effective direct method. In the calculation of direction acceleration method, the selection of initial point is of significant importance and theoretically it should be selected near the optimal point. By taking the preliminary alignments yielded from dynamic programming as the initial scenarios for Powell method, the resulting alignment can deviate from the grid points to reach the global optimum.

However, it is an unconstrained problem solving tool which is not suitable for dealing with constrained problems. In this paper constrained problem was converted to unconstrained problem by adding penalties to the objective function to avoid constraint violations. The basic idea is to formulate an assistant function, which has its accurate value in viable points while adding a considerable large number at ineligible points.

Define an assistant function:

$$\begin{aligned}&\mathrm{F}(\boldsymbol{X},\boldsymbol{Y},\boldsymbol{R},\boldsymbol{L},\boldsymbol{Z},\sigma,\eta)\\ &= \mathrm{f}(\boldsymbol{X},\boldsymbol{Y},\boldsymbol{R},\boldsymbol{L},\boldsymbol{Z}) + \sigma\sum_{i=1}^{n}\left[\max\left\{0, \mathrm{g_i}(\boldsymbol{X},\boldsymbol{Y},\boldsymbol{R})\right\}\right]^2 \quad (22)\\ &+\eta\sum_{j=1}^{m}\left[\max\left\{0, \mathrm{h_j}(\boldsymbol{L},\boldsymbol{Z})\right\}\right]^2\end{aligned}$$

Where, penalty indicators σ and η are very large numbers.

The main steps are as follows.

Step (1): Choose Initial Point

Take the scenario achieved by dynamic programming as the initial point:

$$\boldsymbol{x}_0 = [\boldsymbol{X}_0^{\mathrm{T}}, \boldsymbol{Y}_0^{\mathrm{T}}, \boldsymbol{R}_0^{\mathrm{T}}, \boldsymbol{L}_0^{\mathrm{T}}, \boldsymbol{Z}_0^{\mathrm{T}}]^{\mathrm{T}}\,. \quad (23)$$

Searching matrix is:

$$\boldsymbol{E_O} = \begin{bmatrix} e_{11} & \cdots & e_{1(3n+2m)} \\ e_{21} & \cdots & e_{2(3n+2m)} \\ \vdots & \ddots & \vdots \\ e_{3n+2m} & \cdots & e_{(3n+2m)(3n+2m)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{e_0} \\ \boldsymbol{e_1} \\ \vdots \\ \boldsymbol{e_{3n+2m-1}} \end{bmatrix}. \quad (24)$$

Where, $\boldsymbol{e_0}, e_1, \ldots e_{3n+2m-1}$ are $3n+2m$ searching linear irrelevant direction, $\boldsymbol{e_0} = [1,0,...,0]$, $\boldsymbol{e}_1 = [0,1,...,0]$, … $\boldsymbol{e}_{3n+2m-1} = [0,0,...,1]$.

Set the control error $\varepsilon > 0$ and let $k = 0$.

Step (2): Conduct Basic Search

Let $\boldsymbol{p_0} = \boldsymbol{e_k}$, conduct linear searching alternately according to all the directions $\boldsymbol{e_0}, \boldsymbol{e_1}, \cdots, \boldsymbol{e_{3n+2m-1}}$. For all $i = 0,1,\cdots,3n+2m-1$, we denote:

$$F\left(\boldsymbol{p_i} + \alpha_i \boldsymbol{e_i}\right) = \min_{\alpha} F\left(\boldsymbol{p_i} + \alpha \boldsymbol{e_i}\right), \quad (25)$$

$$\boldsymbol{p_{i+1}} = \boldsymbol{p_i} + \alpha_i \boldsymbol{e_i}\,. \quad (26)$$

Step (3): Check Stop Condition

Let $\boldsymbol{e}_{3n+2m} = \boldsymbol{p}_{3n+2m} - \boldsymbol{p}_0$. If $\|\boldsymbol{e}_{3n+2m}\| < \varepsilon$, then stop, and $\boldsymbol{p}_{3n+2m}$ is the best option. Otherwise, turn to step (4).

Step (4): Determine Searching Direction

Determine t according (27):

$$F(\boldsymbol{p}_t) - F(\boldsymbol{p}_{t+1}) = \max_{0 \le j \le 3n+2m-1} \left\{ F(\boldsymbol{p}_j) - F(\boldsymbol{p}_{j+1}) \right\} \quad (27)$$

If (28) is confirmed, turn to step (5). Otherwise, turn to step (6)

$$\begin{aligned} &F(\boldsymbol{p}_0) - 2F(\boldsymbol{p}_{3n+2m}) + F(2\boldsymbol{p}_{3n+2m} - \boldsymbol{p}_0) \\ &< 2\left[F(\boldsymbol{p}_t) - F(\boldsymbol{p}_{t+1})\right] \end{aligned} \quad (28)$$

Step (5): Adjust Search Direction

Conduct linear search from the point $\boldsymbol{p}_{3n+2m}$ and along with the direction to get the best α_{3n+2m} .

$$F(\boldsymbol{p}_{3n+2m} + \alpha_{3n+2m}\boldsymbol{e}_{3n+2m}) = \min_{\alpha} F(\boldsymbol{p}_{3n+2m} + \alpha\boldsymbol{e}_{3n+2m}) \quad (29)$$

Let : $\boldsymbol{x}_{k+1} = \boldsymbol{p}_{3n+2m} + \alpha_{3n+2m}\boldsymbol{e}_{3n+2m}$, $\boldsymbol{e}_i := \boldsymbol{e}_{i+1}$, $i = t, t+1, \cdots, 3n+2m-1$, $k := k+1$, turn to step (2).

Step (6): Maintain Search Direction

Let $\boldsymbol{x}_{k+1} = \boldsymbol{p}_{3n+2m}$, $k := k+1$, turn to step (2).

Finally, according to $\boldsymbol{p}_{3n+2m} = (\boldsymbol{X}^*, \boldsymbol{Y}^*, \boldsymbol{R}^*, \boldsymbol{L}^*, \boldsymbol{Z}^*)$, the best parameters representing the optimal scenario can be achieved.

V. APPLICATION

Based on the theory and method presented in this paper, a 3D intelligent alignment system has been developed. The formulation has been applied to various highway alignments. The horizontal and vertical alignments for a typical example are shown in Fig.3 and Fig.4.

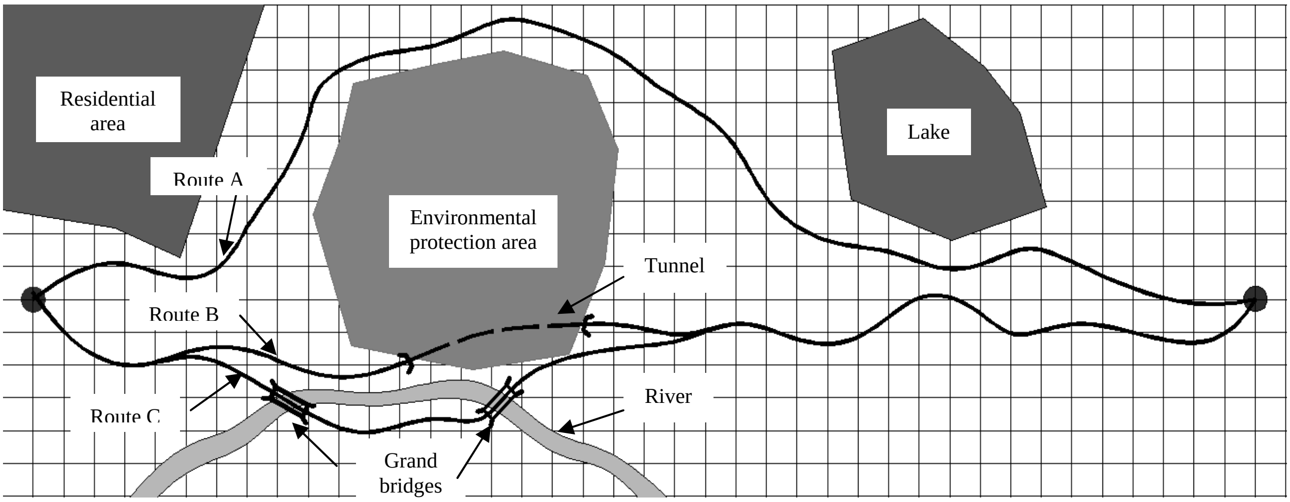

Figure 3. Horizontal alignments in a typical example

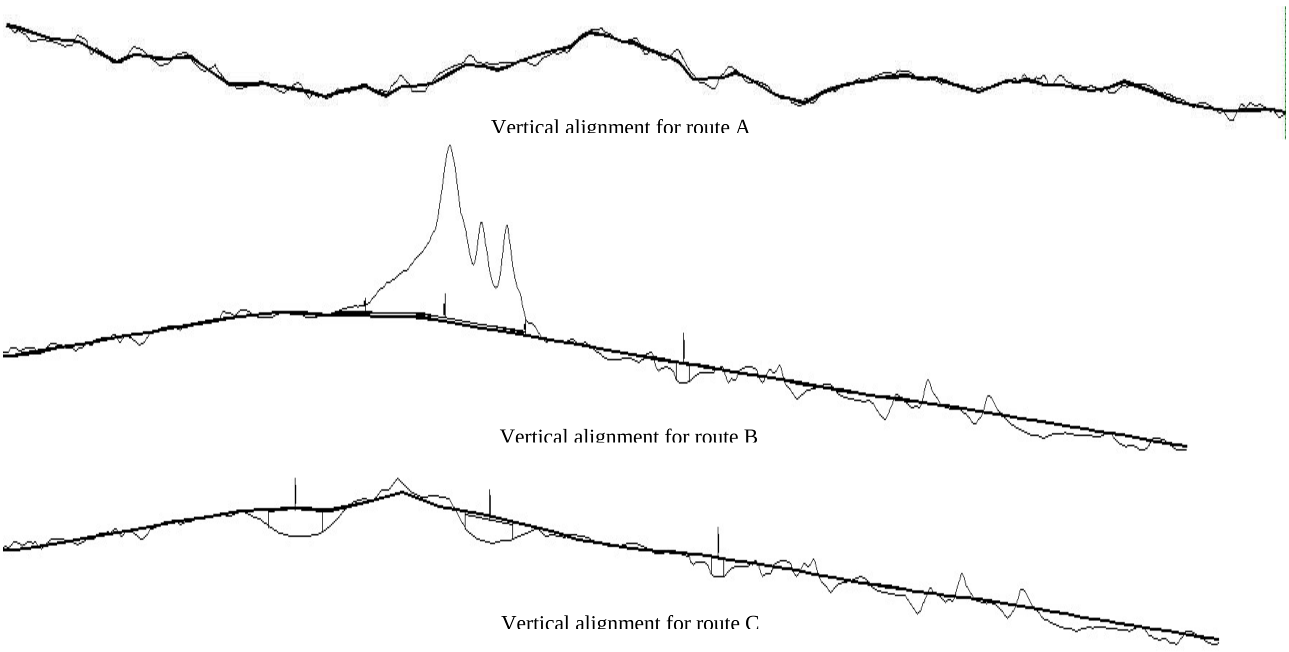

Figure 4. Vertical alignments in a typical example

We select 3 representative alignments from various alternatives generate by this formulation. The maximum fill and cut height is 9.2m. The average of fill and cut height is 2.8m.

The result indicates that the formulation has some merits which can be summarized as follows: (1) The global optimal alignment can be created; (2) The alignment can jump over barriers such as residential area, lake and so on; (3) Various of alternatives can be generated to avoid omitting valuable alignments.

VI. Conclusions

A two-stage approach for solving three-dimensional alignment optimization problems which integrates dynamic programming with Powell method was developed. Stage 1: we formulated the optimization of alignment as a network problem by establishing a three-dimensional search grid in the study area. Stage 2: dynamic programming as one of the most well-developed network optimization techniques for solving shortest path problems was used to find the best corridor alignment within the vast study area. Finally, the corridor alignment was then refined with Powell method to ultimately obtain the globally optimal route location.

Compared with other studies, the approach proposed in this has such advancement solving alignment optimization problem:

(1)The bidirectional searching strategy in dynamic programming can generate a group of scenarios ranking by their corresponding comprehensive cost to avoid omitting valuable alignments.

(2)Taking the preliminary alignments yielded from dynamic programming as the initial scenarios for the Powell method, the resulting alignment could deviate from the grid points to reach the global optimum.

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