Production Planning with Postponement Strategy Based on Classification of Product Differentiations

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Abstract—This paper was motivated by the practices of some Chinese automobile manufacturers who had changed their production planning processes in order to obtain more demand information. We regard their methods as an example of postponement implementation in production planning and name it as production planning postponement (PPP). Firstly, by treating PPP as a two-level hierarchical structure problem which is characterized by families and items we develop an analytical model for evaluating the value of this kind of postponement. Secondly, we propose an algorithm based on generalized linear programming for solving the model. Then we illustrate the effectiveness of PPP by using a numerical example and discuss the impact of prediction accuracy on the performance of PPP. The results of a numerical example show that if the demand is overestimated at initial time PPP can perform quite well. Finally, we set future research directions on PPP suggested by our results.

***Index Terms*—production planning, postponement strategy, mass customization**

I. Introduction

Our research was motivated by the practices of some Chinese and Japanese auto manufacturers. In 2006 Shanghai General Motors (SGM) in China announced they had introduced a new business method. Under this method the order cycle time of SGM was shortened to 6 weeks. In the first week of each cycle, denoted by Week 1, SGM received the orders from their dealers and balanced these orders against its own monthly sales target which had been set beforehand so as to form a general production plan for each car-line, such as plans for Chevrolet and Buick. At the end of Week 1, SGM allocated the planned units of these car-lines to their dealers and let each dealer know, for all of the vehicles the dealer had ordered at the cycle, which one had been scheduled to be built on which day. In the following weeks, SGM offered their dealers two opportunities to modify their orders based on the dealers' expectations and plans of sales. In Week 2 could the dealers revised the quantities of the models in each car-line, such as the quantities of Cruze and Malibu within car-line Chevrolet. In Week 3 the dealers were allowed to revise the colors of these models. Therefore until the end of Week 3 all the details of the production plan were completely determined. In Week 4 and Week 5, SGM prepared the components and assembled them into vehicles. In Week 6 SGM fulfilled their dealers' orders. Guangqi Honda (GH) in China had adopted an N+3 order processing method since 2005. At the beginning of each year GH set a target amount of sales and then allocated it into each month to form a basic monthly production plan. This plan was not fixed. In the four weeks of each month, GH could adjust the planned vehicles' models in Week 1, colors in Week 2 and quantities in Week 3. However, to keep the production stability this adjustment was strictly confined in 30%~50% in relation to the basic monthly production plan. In week 4, GH completed monthly production plan and shipped the finished vehicles to their dealers and customers. The objective of collaborative forecasting and production planning with their dealers in SGM and GH is an increased satisfaction and enthusiasm of customers which could lead to higher sales, greater profitability and an improved market [1]. These two Chinese examples above are quite similar to the operations of some Japanese auto manufacturers. Toyota and Nissan in Japan had adopted a production method in which they could allow their dealers to modify the orders daily, so that these two manufactures achieved a great progress in production flexibility [2]. Lee and Tang regarded this kind of method as an important application of postponement strategy in auto industry [3].

The concept of postponement is not new. It was firstly introduced by Alderson in 1950s [4]. It means to delay the commitment to a product's characteristics until more information is available so that the decisions about the product can be made more accurately [5]. Postponement

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strategy can save inventory, reduce delivery lead times and provide various products to satisfy various customer demands. As a result, it has been regarded as a powerful way to enable cost-effective mass customization. Automobile, apparel and consumer electronics industries are the prime fields that postponement has been successfully implemented [5]- [7].

A number of papers on postponement strategy in the literature focus on process standardization, process re-sequencing and component standardization which are the main methods to achieve postponement [8]. Recently a lot of scholars have extended their research interests to some special forms of postponement strategy, such as partial postponement, price postponement, intelligent system for mass customization, product substitution and operational postponement [9]-[12].

However to our knowledge there is very limited research that studies the application of postponement in production planning. Fisher and Raman managed to reduce the high cost of demand uncertainty of fashion products through accurate response to early sales [13]. Leung and Ng developed a goal programming model for production planning of perishable products with postponement [14]. Trentin *et al.* studied the form postponement from a decision-making perspective[15]. Deng *et al.* constructed a fuzzy bi-level programming model to describe the relationship between producing and dealing [16]. However, our study differs from their papers in some vital aspects. First, in [13] product differentiations are not taken into consideration, whereas the existence of product differentiations is a critical assumption in our study. Second, the model in [14] is deterministic since the market demand data is known whereas in our problem the demand is stochastic. Third, Ref. [15] focuses on the timings of decision processes, whereas our research interest is the value of planning postponement strategy rather than planning process itself. Last, Ref. [16] investigates the optimal amount of production under customer demand uncertainty by means of fuzzy programming while in our research we adopt nonlinear programming.

The rest of this paper is organized as follows. In Section II we describe the sequence of the operation of production planning postponement and formulate the expected cost. Section III proposes an algorithm to find the approximate optimal solutions to the model. The results of numerical examples are presented in Section IV. Section V discusses the impact of prediction on the performance of production planning postponement and provides some managerial insights. Finally, Section VI presents summary comments and discusses promising areas for future research.

II. Model Formulation

We consider a problem where the product differentiations are classified into two levels: *basic specification* and *secondary specificatio*n. For convenience we apply the concept of *family* to represent a set of products that have the same basic specification components. In a certain family, there are multiple *items* which are distinguished from each other by the second specification components. The production plan of families is made with the lead time L_M measured backward from the first day of the production period. The plans of items are made multiple times with the same lead time L_T measured backward from the first day of each sub-production periods. Fig. 1 illustrates the planning process. In this method a monthly production plan can be made with different levels of product differentiations step by step as time proceeds. We name it as *production planning postponement* (PPP).

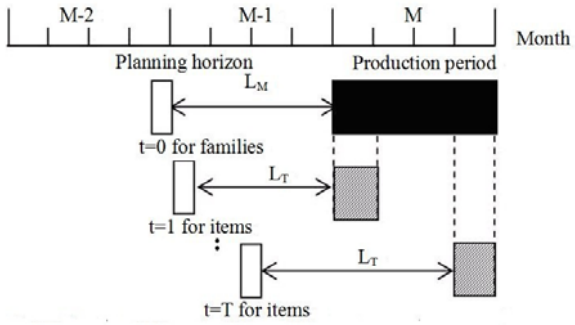

Figure 1. The process of plan postponement strategy.

Compared with the traditional planning method, PPP can determine the planned units of items based on the demand forecasting which are made at much later dates in calendar. For instance, in Fig. 1, the items planned to be built in the last week of the month M are based on the most accurate demand forecasting. Consequently PPP can substantially reduce the risk of the manufacturers and their dealers to make misjudgment as to which specifications of products are easier to sell than others. Furthermore, since the plan is made multiple times PPP can be adapted to the change in demand. As a result the errors in preceding planning can be compensated by the succeeding planning.

A. Notations

n : the number of families.

N : the number of items.

i : index for families($i = 1, \cdots, n$).

j : index for items($j = 1, \cdots, N$).

$J(i)$: set of indexes of items in family i .

N_i : number of items in family i .

T : number of sub-periods in production period(or number of time points in planning horizon).

t : index for time points in planning horizon($t = 0,1, \cdots, T$).

t' : index for sub-periods in production period($t' = 0,1, \cdots, T$).

D_{jt} : time t estimate of the demand of item j .

m_{jt} : time t estimate of the expected demand of item j .

$X_{it,t'}$: time t planned production quantity of family i in production period t' .

$X_{jt,t'}$: time t planned production quantity of item j in production period t'.

M_i: target sale quantity of family i.

α: maximum changeable ratio of planned production quantity of family to target sale quantity.

r_i: resource consumption for producing one unit of an item in family i.

R: resource available in production period.

U_j: underproduction cost for product j, the per unit cost of producing less than is demanded.

O_j: overproduction cost for product j, the per unit cost of producing more than is demanded.

v_t: the expected total cost with the planned quantity of families at time t in planning horizon.

a^+: $\max(0,a)$.

$z_j(X_{jt,t'})$: $O_j(X_{jt,t'}-D_{jt})^+ + U_j(D_{jt}-X_{jt,t'})^+$.

B. Assumptions

The assumptions of our model are as follows:

(1) There is no setup cost associated with a changeover from one item to another within the same family, so does from one family to another.

(2) The production period is divided into multiple sub-periods. Correspondingly the planning horizon is separated into several stages by time points and the quantity of time points is the same as that of sub-production periods.

(3) The planned production quantity of each family is limited within a range of the target quantity which is fixed in accordance with the monthly sales target.

(4) The demand estimates of items in a family follow a joint normal distribution. The negative tail is typically negligible. Note that the actual demand occurs later than the last sub-production-period T.

(5) The demand estimates of items in family i have the covariance matrix $\sigma_{it}^2\sum_i$ at time t for $i=1,\cdots,n$ and $t=0,\cdots,T$ where $\sum_i$ is a $N_i \times N_i$ correlation coefficient matrix. The demand estimates of items D_{jt} are independent of t.

(6) Since the forecast accuracy increases as t increases, we have $\sigma_{i0} \ge \sigma_{i1} \ge \cdots \ge \sigma_{iT}$ for $i=1,\cdots,n$ [17].

C. Mathematical Model

Our model includes two stages. In the first stage, the decision process involves at time $t=0$ determining the production quantity of each family $X_{i0,0}$ for $i=1,\cdots,n$ to minimize the expected total cost of over and under production. Here $t=0$ means the beginning of planning horizon and $t'=0$ means the whole production period. It is a family-level problem where $X_{i0,0}$ is conditioned by D_{j0} which has the largest variance. We name it as Aggregate Problem(AP). In the second stage, allocate $X_{i0,0}$ into sub-production periods (i.e. $X_{i0,t'}, t'=1,2,\ldots,T$) and then with the constraint of each family's $X_{i0,t'}$ for $i=1,\cdots,n$, determine $X_{jt,t'}$ for $j \in J(i)$ and $t=t'=1,\cdots,T$ based on the updated demand estimates D_{jt} to minimize the expected cost. This is an item-level problem, named as Disaggregate Problem (DP). Therefore we have

PROBLEM (AP):

$$v_0 = \min_{X_{i0,0}} \sum_{i=1}^{n} z_i(X_{i0,0}) \tag{1}$$

subject to

$$\sum_{i=1}^{n} r_i X_{i0,0} \le R \tag{2}$$

$$(1-\alpha)M_i \le X_{i0,0} \le (1+\alpha)M_i \tag{3}$$

$$X_{i0,0} \ge 0, \quad i=1,\ldots,n$$

where

$$z_i(X_{i0,0}) = \min_{X_{j0,0}} E_{D_{j0}} \sum_{j\in J(i)} z_j(X_{j0,0}) \tag{4}$$

subject to

$$\sum_{j\in J(i)} X_{j0,0} = X_{i0,0} \tag{5}$$

$$X_{j0,0} \ge 0,\ j \in J(i)$$

PROBLEM(DP):

For $t=1,2,\ldots,T$

$$v_t = \min_{X_{jt,t'=t}} E_{D_{jt}} \sum_i \sum_{j\in J(i)} z_j(X_{jt,t'=t} + \sum_{k=1}^{t-1} X_{jk,t'=k}) \tag{6}$$

subject to

$$\sum_{j\in J(i)} X_{jt,t'=t} \le X_{i0,t'=t},\ i=1,\ldots,n \tag{7}$$

$$X_{i0,t'=t} = \frac{1}{T} X_{i0,0} \tag{8}$$

$$X_{jt,t'} \ge 0,\ j=1,2,\ldots,N$$

In the above formulations, $z_j(X_{jt,t'})$ is valid regardless of whether the items' demands are dependent. Thus we only need to consider the items' individual marginal demand distributions. Constraint (2) represents the capacity constraint. Constraint (3) ensures that the production quantities of families are confined within the allowable range of the target sale quantity. Constraint (5) is a balance constraint which assures that the total production quantity of the items in a family equals the

production quantity of that family. Constraint (7) assures that in a certain family the total production quantity of the items in a family produced in sub-production periods does not exceed the allocated production quantity of corresponding family which is determined at time $t=0$. Constraint (8) shows that the planned quantities of families determined at time $t=0$ are averagely allocated into the sub-production periods.

III. Solution Procedure for (AP) and (DP)

A. Solution Procedure for (LP)

Before giving the algorithms for solving (AP) and (DP), we define a multi-item newsvendor problem with one capacity constraint as Problem (LP), shown as follows:

PROBLEM (LP):

$$v = \min_{X_{j0,0}} E_{D_{j0}}\left\{\sum_{j\in J(i)} z_j(X_{j0,0}) + \lambda^K \sum_{j\in J(i)} r_i X_{j0,0}\right\}$$
$$= \min \sum_{j\in J(i)} \left\{O_j \int_0^{X_{j0,0}} (X_{j0,0}-D) f_j(D)dD + U_j \int_{X_{j0,0}}^{\infty} (D-X_{j0,0}) f_j(D)dD + \lambda^K r_i X_{j0,0}\right\} \tag{9}$$

Subject to

$$\sum_{j\in J(i)} w_j X_{j0,0} \le C \tag{10}$$

$$X_{j0,0} \ge 0 \ , \ j\in J(i) \tag{11}$$

where $f_j(\cdot)$ is Product j's demand density function, C is the capacity of the resource, λ^K is a known value and w_j is the amount of resource consumed by one unit of Product j. Note that the marginal distribution of item j in family i follows the normal distribution with mean m_{jt} and standard deviation σ_{it}. The optimal (unconstrained) order quantity $X^u_{j0,0}$ for item j without capacity constraint is such that

$$X^u_{j0,0} = m_{j0} + \sigma_{i0}\Phi^{-1}\left(\frac{U_j - \lambda^K r_i}{O_j + U_j}\right) \tag{12}$$

where $\Phi(\cdot)$ is the cumulative normal density function. By using Lagrangian multiplier method, we can obtain the optimal order quantity for item j such that

$$X_{j0,0} = m_{j0} + \sigma_{i0}\Phi^{-1}\left(\frac{U_j - \lambda^K r_i - \lambda w_j}{O_j + U_j}\right) \tag{13}$$

where λ is the Lagrange multiplier. Therefore, the solution procedure can be summarized as follows [18]:

Step 1: For each item $j\in J(i)$, determine the optimal unconstrained order quantity $X^u_{j0,0}$ via (12). If $\sum_{j\in J(i)} w_j X^u_{j0,0} \le C$, then stop. The solution is optimal. Let $X_{j0,0} = X^u_{j0,0}$.

Step 2: Choose an initial value of $\lambda > 0$.

Step 3: Determine $X_{j0,0}$ for all $j\in J(i)$ via (13).

Step 4: If

(a) $\sum_{j\in J(i)} w_j X_{j0,0} = C$,then stop. The $X_{j0,0}$ are optimal.

(b) $\sum_{j\in J(i)} w_j X_{j0,0} < C$, then go to step 3 with a smaller value of λ.

(c) $\sum_{j\in J(i)} w_j X_{j0,0} > C$, then go to step 3 with a larger value of λ.

Once $X_{j0,0}$ for all $j\in J(i)$ are calculated then the expected cost can be obtained from

$$v = \lambda^K \sum_{j\in J(i)} r_i X_{j0,0} + \sum_{j\in J(i)} \left\{O_j\left(X_{j0,0} - m_{j0}\right) + \left(O_j + U_j\right)\sigma_{i0} G\left(\frac{X_{j0,0} - m_{j0}}{\sigma_{i0}}\right)\right\} \tag{14}$$

where $G(\cdot)$ is the loss function and defined as $G(x) = \phi(x) - x(1-\Phi(x))$. Here $\phi(x)$ is the unit normal probability density function.

B. Solution Procedure for (AP)

Different from (LP), (AP) is a multi-item newsvendor problem with multiple resource capacity constraints. It is hard to solve in many instances. Here we propose an algorithm to solve (AP) approximately, which is derived from generalized linear programming [19].

Step 1: Solve the K th master linear program:

PROBLEM (D^K)

$$d^K = \min \sum_{k=1}^{K}\sum_{i=1}^{n} c_i^k \alpha_k \tag{15}$$

subject to

$$\sum_{k=1}^{K}\sum_{i=1}^{n} r_i X^k_{i0,0}\alpha_k \le R \tag{16}$$

$$\sum_{k=1}^{K} \alpha_k = 1 \tag{17}$$

$$\alpha_k \ge 0 \ , \ k = 1,2,\ldots,K$$

where α_k for $k=1,2,\cdots,K$ are the decision variables and d^K is the value of objective function. Here c_i^k and $X^k_{i0,0}$ for $k=1,2,\cdots,K$ are known parameters in this linear program. Let λ^K denote the negative of the shadow price of the resource constraint (16) in (D^K).

Step 2: Solve the following sub-problem.

$$L(\lambda^K) = \min \sum_{i=1}^{n}\left\{ z_i(X_{i0,0}) + \lambda^K \sum_{i=1}^{n} r_i X_{i0,0} \right\} - \lambda^K R$$
$$= \sum_{i=1}^{n}\left\{ z_i(X_{i0,0}^{K+1}) + \lambda^K \sum_{i=1}^{n} r_i X_{i0,0}^{K+1} \right\} - \lambda^K R \tag{18}$$

subject to

$$(1-\alpha)M_i \le X_{i0,0} \le (1+\alpha)M_i, i = 1,\ldots,n \tag{19}$$

where $X_{i0,0}^{K+1}$ denotes the solution to this sub-problem. To solve that, we must firstly, for $i = 1,\ldots,n$, solve the following newsvendor problems for all j in $J(i)$, named as following.

PROBLEM (SP):

$$L_i(\lambda^K) = \min_{X_{j0,0}} E_{D_{j0}}\left\{ \sum_{j\in J(i)} z_j(X_{j0,0}) + \lambda^K \sum_{j\in J(i)} r_i X_{j0,0} \right\}$$
$$= E_{D_{j0}}\left\{ \sum_{j\in J(i)} z_{j0}(X_{j0,0}^{K+1}) + \lambda^K \sum_{j\in J(i)} r_i X_{j0,0}^{K+1} \right\}$$
$$= \min \sum_{j\in J(i)} \left\{ O_j \int_0^{X_{j0,0}} (X_{j0,0} - D) f_j(D) dD \right.$$
$$\left. + U_j \int_{X_{j0,0}}^{\infty} (D - X_{j0,0}) f_j(D) dD + \lambda^K r_i X_{j0,0} \right\} \tag{20}$$

subject to

$$(1-\alpha)M_i \le \sum_{j\in J(i)} X_{j0,0} \le (1+\alpha)M_i \tag{21}$$
$$X_{j0,0} \ge 0, \quad j \in J(i)$$

where $X_{j0,0}^{K+1}$ denotes the solution to (SP). Note that (SP) is a newsvendor problem with two constraints. The solution procedure can be summarized as follows:

(1) For each item j, calculate the optimal unconstrained order quantity $X_{j0,0}^{u}$ via (12). If $(1-\alpha)M_i \le \sum_{j\in J(i)} X_{j0,0}^{u} \le (1+\alpha)M_i$, then stop. $X_{j0,0}^{u}$ is the optimal solution. Let $X_{j0,0}^{K+1} = X_{j0,0}^{u}$.

(2) If $\sum_{j\in J(i)} X_{j0,0}^{u} < (1-\alpha)M_i$, solve (LP) with $w_j = 1$ and $C = (1-\alpha)M_i$. Let $X_{j0,0}^{K+1}$ be the optimal solution of (LP).

(3) If $\sum_{j\in J(i)} X_{j0,0}^{u} > (1+\alpha)M_i$, solve (LP) with $w_j = 1$ and $C = (1+\alpha)M_i$. Let $X_{j0,0}^{K+1}$ be the optimal solution of (LP).

(4) After $X_{j0,0}^{K+1}$ for all $j \in J(i)$ are calculated, $L_i(\lambda^K)$ can be obtained from

$$L_i(\lambda^K) = \lambda^K \sum_{j\in J(i)} r_i X_{j0,0}^{K+1} + \sum_{j\in J(i)} \left\{ O_j\left(X_{j0,0}^{K+1} - m_{j0}\right) \right.$$
$$\left. + \left(O_j + U_j\right)\sigma_{i0} G\left(\frac{X_{j0,0}^{K+1} - m_{j0}}{\sigma_{i0}} \right) \right\} \tag{22}$$

Therefore we have

$$X_{i0,0}^{K+1} = \sum_{j\in J(i)} X_{j0,0}^{K+1} \text{ for } i = 1,\ldots,n \tag{23}$$

and

$$L(\lambda^K) = \sum_{i=1}^{n} L_i(\lambda^K) - \lambda^K R \tag{24}$$

Step 3: For a preset value ε, if $(L(\lambda^K) - d^K)/L(\lambda^K) \le \varepsilon$, then $X_{i0,0}^{K+1}$ is the solution of (AP). Otherwise go to Step 1 with adding $X_{i0,0}^{K+1}$ and c_i^{K+1} for $i = 1,2,\ldots,n$ to (15). Here c_i^{K+1} is given by

$$c_i^{K+1} = z_i(X_{i0,0}^{K+1}) = \sum_{j\in J(i)} \left\{ O_j\left(X_{j0,0}^{K+1} - m_{j0}\right) \right.$$
$$\left. + \left(O_j + U_j\right)\sigma_{i0} G\left(\frac{X_{j0,0}^{K+1} - m_{j0}}{\sigma_{i0}} \right) \right\} \tag{25}$$

To find the initial solution when K =0, we set arbitrarily λ^0 with a small value and insert it into (SP). Then solve the optimal solution $X_{i0,0}^{1}$ and c_i^1 for $i = 1,2,\ldots,n$ through the procedures shown above.

C. Solution Procedure for (DP)

After obtaining $X_{i0,0}$,we can solve (DP) for $t = 1,2,\ldots,T$. Note that (DP) is a multi-item newsvendor model with one capacity constraint and at each time point t, the value of $X_{i0,0}$ and $X_{jk,t'=k}$ for $k = 1,2,\ldots,t-1$ are already known. Without capacity constraint the optimal order quantity $X_{jt,t'=t}^{u}$ for item j is such that

$$X_{jt,t'=t}^{u} = m_{jt} + \sigma_{it}\Phi^{-1}\left(\frac{U_j}{O_j + U_j} \right) - \sum_{k=1}^{t-1} X_{jk,t'=k} \tag{26}$$

By using the Lagrangian multiplier method, we can obtain the optimal order quantity such that

$$X_{jt,t'=t} = m_{jt} + \sigma_{it}\Phi^{-1}\left(\frac{U_j - \lambda}{O_j + U_j} \right) - \sum_{k=1}^{t-1} X_{jk,t'=k} \tag{27}$$

The solution procedure can be summarized as follows:

Step 1: For each item j, determine the optimal unconstrained order quantity $X^{u}_{jt,t'=t}$ via (26). If $\sum_{j\in J(i)} X^{u}_{jt,t'=t} \le X_{i0,t'=t}$, then stop. The solution is optimal. Let $X_{jt,t'=t} = X^{u}_{jt,t'=t}$.

Step 2: Choose an initial value of $\lambda > 0$.

Step 3: Determine $X_{jt,t'=t}$, for all $j \in J(i)$ via (27).

Step 4: If

(a) $\sum_{j\in J(i)} X_{jt,t'=t} = X_{i0,t'=t}$,then stop. The $X_{jt,t'=t}$ are optimal.

(b) $\sum_{j\in J(i)} X_{jt,t'=t} < X_{i0,t'=t}$, then go to step 3 with a smaller value of λ.

(c) $\sum_{j\in J(i)} X_{jt,t'=t} > X_{i0,t'=t}$, then go to step 3 with a larger value of λ.

Correspondingly the expected cost v_t for $t = 1,2,\ldots,T$ can be obtained from

$$v_t = \sum_i \sum_{j\in J(i)} O_j \left(X_{jt,t'=t} + \sum_{k=1}^{t-1} X_{jk,t'=k} - m_{jt} \right) + \left(O_j + U_j\right)\sigma_{it} G\left(\frac{X_{jt,t'=t} + \sum_{k=1}^{t-1} X_{jk,t'=k} - m_{jt}}{\sigma_{it}} \right) \tag{28}$$

IV. Numerical Examples

In this section, we consider an example with $N = 7$ items, $n = 3$ product families; the number of time points in the planning horizon (or the number of sub-periods in the production period) $T = 3$, $t = 0,1,2,3$; $t' = 0,1,2,3$. Under PPP the production plan for families is determined at time $t = 0$, while plans for items are determined at time $t = 1,2,3$. Here the resource refers to the working time and its capacity in production period $R = 500000$ s. The target sales quantities of families M_i for $i = 1,2,\ldots,n$ are given by Table I. The allowable change ratio $\alpha = 0.2$. Typically we set U_j to be two times as large as O_j. The parameters we used are listed in Table I and Table II. We fix ε at 0.01%.

We program the algorithm described in Section III with MATLAB 7.1. The results are shown in Table III and Table IV. The objective value of the approximate solution is $v_3 = 8498.590$.

TABLE I.

Parameter values for Families and items

i	j	M_i	σ_{i0}	m_{j0}	U_j	O_j	r_i
1	1	1500	150	900	24.9	12.45	100
	2			800	15	7.5	
2	3	2500	280	1200	14.3	7.15	64
	4			2300	8.6	4.3	
3	5	800	80	400	30	15	120
	6			300	10	5	
	7			200	8	4	

TABLE II.

Standard deviations and Means of demand estimates for families and items

i	j	σ_{i1}	σ_{i2}	σ_{i3}	m_{j1}	m_{j2}	m_{j3}
1	1	120	90	60	300	600	900
	2				200	400	700
2	3	224	168	112	400	1000	1200
	4				600	1800	2400
3	5	64	48	32	100	200	400
	6				100	250	350
	7				60	140	210

TABLE III.

Production plan for families (time $t = 0$)

i	j	$X_{j0,0}$	$X_{i0,0}$	$X_{i0,1}$	$X_{i0,2}$	$X_{i0,3}$
1	1	953	1800	600	600	600
	2	847				
2	3	1071	3000	1000	1000	1000
	4	1929				
3	5	434	1002	334	334	334
	6	334				
	7	234				

TABLE IV.

PRODUCTION PLAN FOR ITEMS (TIME $t = 1,2,3$)

i	j	$X_{j1,1}$	$X_{j2,2}$	$X_{3,3}$
1	1	351.687	287.078	287.078
	2	251.687	187.078	287.078
2	3	421.561	318.472	251.375
	4	573.711	686.229	751.343
3	5	127.566	93.108	193.108
	6	127.566	143.108	93.108
	7	87.566	73.108	63.108

Let v_{30} denote the expected cost under traditional planning method. By substituting $X_{j0,0}$ for $X_{jt,t'=t} + \sum_{k=1}^{t-1} X_{jk,t'=k}$, m_{j3} for m_{jt} , σ_{j3} for σ_{jt} respectively in (28), we obtain $v_{30} = 9074.972$. Then we calculate the reduction of the expected cost W and the ratio of reduction θ as follows.

$$W = v_3 - v_{30} = 9074.972 - 8498.590 = 576.382$$

$$\theta = \frac{v_3 - v_{30}}{v_{30}} = 0.063 = 6.3\%$$

The results above show that the expected cost using PPP is 6.3% less than that using traditional planning method. It suggests that PPP can significantly decrease the expected cost compared with traditional one.

V. THE IMPACT OF PREDICTION ON THE PERFORMANCE OF PLAN POSTPONEMENT

In the above sections, we assume that the improvement of the forecast accuracy is represented by the decrease of the variances of demand estimates in sequential time points. This assumption can be reasonable in many cases [17]. Here we investigate the effect of the change of the means of demand estimates on the performance of PPP. For simplicity, we assume that the average demands of items decrease or increase simultaneously with the same proportion (denoted by β) in relation to their initial evaluations. We also assume the estimate made at time point $t = 3$ be a proper substitute for the actual demand. We consider the two following cases:

(a) The demand changes are not observed until at the last planning time point $t = 3$. Therefore we have $m_{j1} = m_{j0}/3$, $m_{j2} = 2m_{j0}/3$, $m_{j3} = (1+\beta)m_{j0}$, for $j = 1,\ldots,N$.

(b) The demand changes are observed at each planning time point (that is, at $t = 1,2,3$). We have $m_{j1} = (1+\beta)\cdot(m_{j0}/3)$, $m_{j2} = (1+\beta)\cdot(2m_{j0}/3)$, $m_{j3} = (1+\beta)\cdot m_{j0}$, for $j = 1,\ldots,N$.

Case (a) demonstrates the situation in which the initial estimate of the average demand made at time $t = 0$ seems not necessary to be revised at the first two time points in the planning horizon until at the last time point. Case (b) means that the change trend of demand is noted at the first time t=1 and then observed during the whole planning horizon. To compare these two cases, we keep the other parameters of the model unchanged except the value of the means of demand estimates and calculate the corresponding expected cost. Fig. 2 and Fig. 3 display the value of expected cost reduction W with different value of β in case (a) and case (b). Fig. 4 and Fig. 5 show the corresponding change ratio θ.

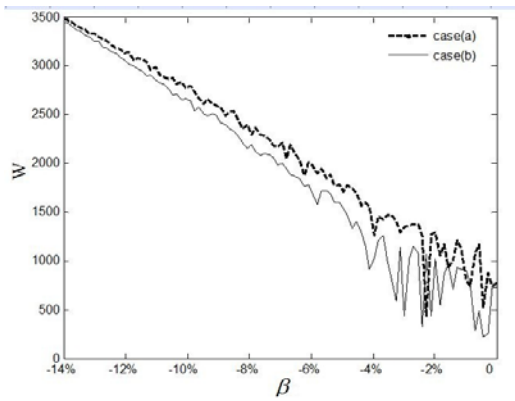

Figure 2. The relation between the reduction of expected cost and the changing proportion of the demand mean in negative direction direction.

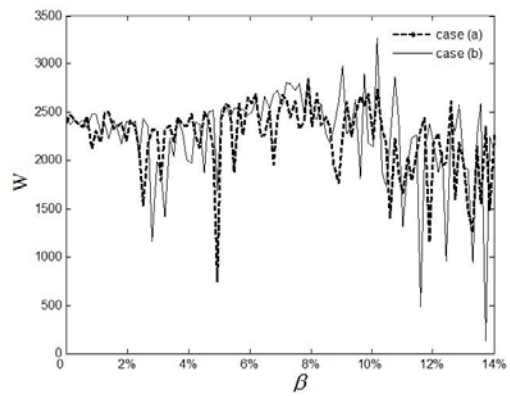

Figure 3. The relation between the reduction of expected cost and the changing proportion of the demand mean in positive direction.

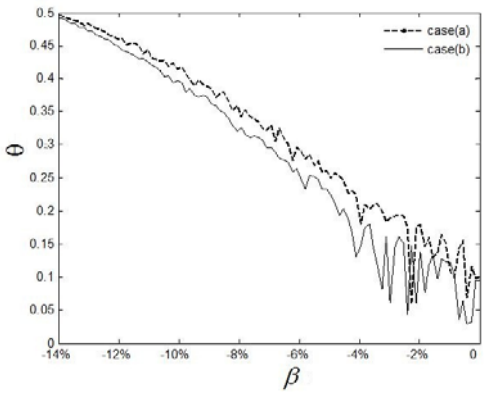

Figure 4. The relation between the ratio of reduction and the changing proportion in the negative direction.

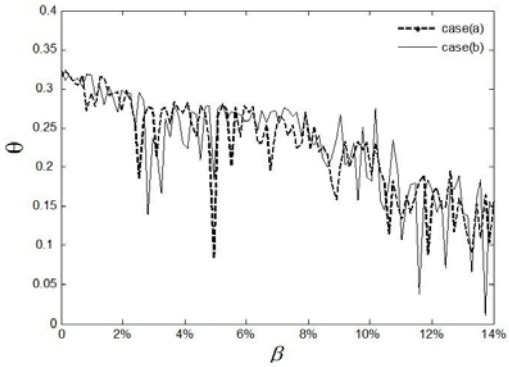

Figure 5. The relation between the ratio of reduction and the changing proportion in the positive direction.

From Fig. 2 and Fig. 4, it can be seen that if in the planning horizon the demand estimate made at initial time point is larger than the actual demand, which means the demand is overestimated at the initial time point (that is, $\beta < 0$),PPP can perform well and the corresponding cost reduction keeps increasing as the changing proportion (the absolute value of β) is increased. On the contrary, Fig. 3 and Fig. 5 show that once the demand is underestimated that means the demand estimate made at initial time point is less than the actual demand(that is, $\beta > 0$) postponement strategy can only achieve very little cost reduction. Fig. 3 and Fig. 5 also show that when β is a small positive value, PPP can still obtain a limited cost reduction while as changing proportion becomes larger the cost reduction becomes irregular. The reason is that since the production capacity is limited when the demand is overestimated at initial time point there will be sufficient production capacity to fulfill the adjusted production plan whereas if the demand is initially underestimated there is not enough production capacity left to modify the production plan unless the changing proportion is quite small.

From Fig. 2 to Fig. 5, we can see that in the situation $\beta < 0$ the performance of PPP in case (a) is better than that in case (b). However, in the situation $\beta > 0$ it is not obvious whether case (a) is still better than case (b). This result shows that as long as the capacity is sufficient PPP can work well no matter when the demand change is observed.

VI. Conclusion

In this paper we analyze the production planning problem involving postponement strategy, which is named as Production Planning Postponement (PPP). We develop a model for evaluating the expect cost of PPP and the impact of demand prediction on the performance of PPP. The results of a numerical example show that if the demand is overestimated at initial time PPP can perform well and the cost reduction keeps increasing as the changing proportion is increased whereas if the demand is initially underestimated PPP can only achieve very little cost reduction. The reason for this phenomenon is that in the former situation there is some surplus capacity which can be used in the subsequent production periods whereas in latter situation there is not any production capacity left to modify the production unless the changing proportion is quite small.

There are several possibilities for furthering our research on this topic area. First, one could research the impact of the correlation between the demands for the different types of products on the performance of PPP. Note that in this paper we only investigate the case of positive correlation since we assume that all the demand changes are in the same direction. Generally speaking, when the demand correlation is negative PPP will especially be beneficial due to postponement enables the exploitation of pooling. However such analysis is technically complex since the independent newsvendor approach can not be used. Second, one could consider the time value of postponement. Note that in this paper the time point is not a decision variable. By assuming the accuracy of demand estimate is a function of time, one can try to find optimal time points to divide the whole planning horizon with different production differentiations. A third extension would be to include demand substitution, which means a certain proportion of customers would like to choose a similar product in the second specification if the initially desired one is not available.

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