

# Research on Project Scheduling Problem with Resource Constraints

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**Abstract**—Resource-constrained project scheduling problem is the traditional optimization problem considering constraint conditions of resource supply, which makes the problem more difficult to solve in real life. Resource-constrained project scheduling problem is a typical NP-hard problem, which is quite popular issue in recent years. This article firstly analyzes the literature review of resource-constrained project scheduling problem. And then, the mathematical model of this problem is established, and the solution via the genetic algorithm is also developed. Finally, the numerical experiment is adopted to verify the effectiveness and efficiency of the proposed model and algorithm.

**Index Terms**—Resource-constrained, Project Scheduling, Genetic Algorithm, NP-hard Problem

## I. INTRODUCTION

The market competition is quite aggressive under the circumstances of economic globalization. Besides the aggressive market competition, the emerging competitors arouse the enterprises' potential ability which they would have to cater to the market requirements for the sake of a better operation and development. The diversities of product demand lead to more customization and personalization, which requires enterprises to manage the production activities based on the project unit. However, the project structure is usually complicated and also the project constraints become more important. Therefore, group members in a project are difficult to propose a reasonable scheduling solution to balance the resource requirement and to alleviate the contradiction between project duration and cost in a short period of time. As a result, there are a number of researches on how to develop and optimize the project scheduling problem.

This article mainly concentrates on the optimization problem of resource-constrained project scheduling. And genetic algorithm is developed to find out the global optimum solution. And thus it could deduce the optimal task duration under resource constraints and the corresponding scheduling sequences of each task.

Resource-constrained Project Scheduling Problem (RCPSp) was first introduced by Kelley in 1963. The author illustrated an important quality of a class of problems, namely, the NP-complete problems [1]. The focus was to study how to identify, deal with and understand the essence of NP-complete problems. In addition, two aspects of optimization have mainly concentrated on the algorithms for solving NP-hard problem and the targets of optimization problems. Algorithms for solving project scheduling problems under resource constraints are mainly divided into two kinds of aspects: one is the precise algorithm, and the other is heuristic algorithm. Pattenon et al. [2] introduced priority tree oriented algorithm for a group of priority scheduling problem. Mingzzi et al. [3] presented a new 0-1 linear programming formulation of the problem that requires an exponential number of variables, corresponding to all feasible subsets of activities that can be simultaneously executed without violating resource or precedence constraints. Peter Brucker et al. [4] proposed a branch and bound algorithm the resource constraint for the resource-constrained project scheduling problem. Mao et al. [5] regarded the duration of each activity in the project is also related to its actual start time in a schedule. As an extension, the branch- and- bound algorithm put forward can solve this kind of most general project scheduling problems with optimality. These algorithms are for the final algorithm target through the accurate solution.

However, the precise algorithm is only suitable for project management problem which is small-scale and clear structure. For the complex project management problem, it is difficult to obtain the ideal results and would be at expense of the more calculation time by using precise algorithm. Therefore, the problem of project management mainly depends on the approximation algorithm, which mainly refers to the heuristic algorithm. The heuristic algorithm would improve control strategy based on heuristic information during searching process, which could make the search towards the most promising solution. Additionally, the heuristic information is associated with the control information of the specific

problem. The difference between heuristic algorithm and the precise one is that the former could choose the least cost node as the next search node it will move to through one heuristic function, when the current search node needs to choose the next search node. The heuristic algorithm could produce the optimal solution to one search problem rapidly, which is especially suitable for multiple resource constraints of the project scheduling problem.

The Genetic Algorithm (GA) is one kind of heuristic algorithm. It is a simulation of the biological theory of evolution by natural selection and genetic mechanism of the process of biological evolution calculation model, and is one random search method evolved from biological evolution (survival of the fittest genetic mechanism). Its main characteristic is that it copes with structural object directly, while there is no limitation of function derivation and continuity, which has inherent implicit parallelism and better ability of global searching. It uses the probability optimization technology to automatically access and guide optimal search space, which adaptively adjust the search direction and do not need to determine the rules. This algorithm makes the solution improved in the process of competition in order to obtain the satisfactory solution or the optimization solution [6]. It has widely used in many kinds of fields, such as function optimization [7, 8], computer aid design [9], intelligent instrument design [10], scheduling optimization [11, 12], steady-state operation optimization [13], robotic assembly line balancing problem [14], process planning [15], the two-dimensional assortment problem [16], intrusion detection [17], lubrication pump stacking design [18], electronic commerce [19], and so on. In this article, we use GA to solve the project scheduling problem with resource constraints so as to find out the optimal scheduling scheme.

## II. RELATED WORKS

Resource-Constrained Project Scheduling Problem is a hot issue attracted by a lot of researchers from all over the world. For example, Stinson et al. [20], Christofides et al. [21], Demeulemeester and Herroelen [22] and so on presented a branch and bound approach to solve the problem and the differences among them lie in branch schemes as well as elimination rules and other details. Mori and Tseng [23] proposed a genetic algorithm for multi-mode resource-constrained project scheduling problems and compared it with a stochastic scheduling method proposed by Drexl and Gruenewald [24]. Xu et al. [25] illustrated how to combine the idea of rollout with priority rule heuristics and justification for the RCPSP and examined the resulting solution quality and computational cost. They presented empirical evidence that these procedures are competitive with the best solution procedures described in the literature. In addition, Jarboui et al. [26] designed a combinatorial particle swarm optimization (CPSO) algorithm in order to solve MRCPSP. The results that have been obtained using a standard set of instances, after extensive experiments,

proved to be very competitive in terms of number of problems solved to optimality.

Most of the heuristics methods used for solving RCPSP belong to the class of priority rule based methods. Several approaches in this class have been proposed in the literature. For example, Wiley et al. [27] developed a method utilizing the work breakdown structure (WBS) and Dantzig-Wolfe decomposition to generate feasible aggregate level multi-project program plans and schedules. The Dantzig-Wolfe procedure provided a means of generating interim solutions and their appropriate funding profiles. The decision maker may then choose any one of these solutions besides the optimal solution based upon his/her own experience and risk tolerance. Lova et al. [28] analyzed the effect of the two components of a heuristics based on priority rules-schedule generation scheme and priority rule-over two measures of performance-mean project delay and multiproject duration increase. They then considered two approaches: single-project and multi-project.

In addition, Joglekar and Ford [29] integrated a traditional control theory based derivation of optimal resource allocation and a system dynamics model. They used the control theory model to derive an optimal allocation policy, which they described with a Resource Allocation Policy Matrix. The matrix was useful in explaining differences in project performance and developing an intuitive understanding of the characteristics and impacts of different allocation policies. The results show that and how foresighted policies can improve schedule performance without increasing the total amount of resource. Lambrechts et al. [30] built a robust schedule that meets the project deadline and minimize the schedule instability cost. They described how stochastic resource breakdown can be modeled, which reaction was recommended, when resource infeasibility occurred due to a breakdown, and how one can protect the initial schedule from the adverse effects of potential breakdowns. The computational results showed that protection of the baseline schedule, coupled with intelligent schedule recovery, yielded significant performance gains over the use of deterministic scheduling approaches in a stochastic setting.

## III. THE MATHEMATICAL MODEL OF PROJECT SCHEDULING PROBLEM

Resource-constrained project scheduling optimization problem mainly concentrates on the resource optimization as well as the time optimization. If one project can manipulate the unlimited resources, it can induce one the shortest duration accordingly. Generally, resources one project can utilize are limited. Consequently, the effect of resource-constrained on the project duration is very important. However, once the activities of the project are confirmed, the demand of activities on the resources is stable to some extent. The optimization of duration-resource is not to reduce the resource demand of activities, but to balance the resources according to the time slot. Generally speaking, the optimization of duration-resource can be divided into

two categories: ① The limited resource-the shortest duration; ② The fixed duration-the resource balance.

Resources can be divided into two categories according to availability: ① renewable resources, ② non-renewable resources. Non-renewable resource is not flexible. How many non-renewable resources required for the completion of one project is fixed. The resource amount cannot be less than this number; otherwise the activity cannot be completed, which does not depend on the time. Nevertheless, renewable resource can be reused, which is a kind of resources relevant to time. In a period of time, there can be multiple activities demanding on one the same resources simultaneously, while the demand may exceed the total amount of resources. If the resources are not renewable, the demand must be satisfied. If the resources are renewable, it can be solved by the adjustment of activity time parameters.

The limited resource-the shortest duration mainly refers to the project duration as short as possible through scheduling under circumstances of the limited resource constraints.

(1) Premise conditions

The premise of the model has the following five ones:

- ① The number of activities is determined.
- ② There allows the activity only one execution mode which does not have the preemptive right.
- ③ The activity constraint relation is determined, which is a constraint of non-delay finish-start.
- ④ The activity requires a variety of renewable resources, while the demand and supply of each resource is determined.
- ⑤ There are only renewable resource constraints.

(2) The establishment of mathematical model

The project duration is the latest completion time of the last activity. Therefore, the model is described as below:

$$\text{Min}\{\text{Max}\{FT_j\}\}. \quad (1)$$

$$S.T. ST_j \geq 0, \forall j. \quad (2)$$

$$ST_j \geq FT_h, \forall h \in P_j, \forall j. \quad (3)$$

$$\sum_{A_j \in I_T} r_{jk} \leq R_K, \forall k, t. \quad (4)$$

where the project consists of  $J-1$  tasks.  $ST_j$  is the start time of the task  $j$ ,  $FT_j$  is the finish time of the task  $j$ . The task  $h$  is a precedent task of task  $j$ .  $r_{jk}$  indicates the amount of resource  $k$  required by task  $j$  at time  $t$ . Formula (1) is the objective function, the goal of which is to make the model have the shortest duration time. Formula (2) indicates that the start time of each activity is non-negative. Formula (3) indicates the constraints of precedence relations between activities. Formula (4) indicates that resources used by activities must not exceed the total amount provided, which denotes resource constraint.

#### IV. GENETIC ALGORITHM FOR SOLVING PROJECT SCHEDULING PROBLEM

##### A. Design of Coding

The algorithm coding process is the activity scheduling process referring to the precedence relations and task priority illustrated by adjacency matrix which is converted by AON network diagram of project tasks. The exhaustive process could be divided into three steps.

(1) Convert AON network diagram of project tasks into adjacency matrix stored in computer, which describes the constraints of project task scheduling.

(2) The chromosome coding

The chromosome is compiled by task priority, which is randomly generated by a set of integers. Each gene in it indicates the corresponding activity priority. Chromosome coding generated randomly by computer must be  $\{Pri|i \in \text{random}(1, n+1)\}$ , which must be identical, because of the uniqueness of the project task priority.

(3) Generation of activity scheduling

The crucial procedure of obtaining fitness value is to generate scheduling process of project tasks through task priority and task adjacency matrix. In order to ensure the precedence relationships between tasks, the process is divided into three steps.

1) Set the task sequence on the basis of the task priority.

2) Take the task which the current priority number is minimum, and then determine whether the precedent task is complete, if so, take the task into the scheduled tasks  $Z$ , update the set  $U$ ; otherwise, put back the set  $U$ .

3) To determine whether there exists one task in set  $U$ , if not, output the scheduling sequence; otherwise, return to the second step.

##### B. Design of Fitness Function

The fitness value produced by fitness function is the unique evaluation criteria for judging the individual quality, and determines the performance of genetic algorithm.

The optimization problem of project scheduling process under resource constraints is discussed in this article, the target of which is to pursue the shortest total duration of project under resource constraints. The exhaustive steps are as follows.

1) Set start time of the first task be 1, according to the previous scheduling result.

2) Schedule the next task, and determine whether the previous task is the precedent task of the current one. If so, the initial start time of the current task is the finish time of the previous one, otherwise, the initial start time of the current task is the start time of the previous task.

3) Determine whether there exist resource conflicts among tasks, if so, the task start time will delay one day, and repeat the third step, otherwise, go to the fourth step.

4) Determine whether all the tasks have been scheduled, if not, go to the second step, otherwise, go to the fifth step.

5) Determine the completion time of all the tasks. Take the maximum one as the project duration output.

C. Design of Genetic Operator

(1) The selection operator

The algorithm adopts the roulette wheel method for the selection operator on the basis of fitness value deduced by the above procedures. The roulette wheel method could be described as a popular method that it is a benchmark, which is used to identify individuals from groups, and is the regulation of natural selection. The probability of each individual survived to next generation is equal to that of the ratio of individual fitness value by the whole group fitness value. The higher fitness value, the greater probability the next generation can inherit; vice verse.

(2) The crossover operator

Crossover operation is similar to animal breeding, obtaining new individual. Generally, crossover recombination of the genes produces a new excellent individual. This article deploys the two-point crossover operation. However, the traditional two-point crossover operation would result in a mismatch value in this problem. Consequently, the traditional two-point crossover operation needs to be improved. The main solution is the substitution for repetitious sequences of descendent generation from the traditional two-point crossover operation.

(3) The mutation operator

The probability of mutation operation is generally lower. Additionally, the number of such operation in the genetic process is very few, which will not affect the result of genetic selection. This article deploys random mutation through inputting mutation probability by the operator manually.

D. Termination Conditions

This article deploys the traditional genetic iteration method as iteration termination condition. i.e. when the next generation is created, judge whether it is the maximum generation. If true, the algorithm is over, otherwise, come into the next iteration. Note that the maximum generation is set by the operator according to the individual problem requirement in advance.

V. ONE CASE STUDY

This article uses cosmetics R&D as a case to illustrate project scheduling problem. The development process of a new kind of cosmetics generally contains 9 tasks, which are demand analysis, product function position, odor allocation, color design, market analysis, product synthesis, promotion strategies, market promotion, and consumer feedback. Some relationships between tasks are as follows. The product synthesis must be operated until the completion of the product function position, odor allocation, and color design, and the precedent tasks of product synthesis are product function position, odor allocation, and color design. Similarly, promotion strategies must be operated until the completion of market analysis and product synthesis, and the precedent tasks of promotion strategies are market analysis, and product synthesis. The detailed tasks and their relationships, resource demands and durations are

illustrated in Table 1. AON network diagram is shown in Figure 1. Resources supply is 12 units per time unit.

TABLE I.  
THE PROJECT TASK INFORMATION

Number	Procedure	Precedent Task	Resource Demand	Duration
1	Start	0	0	0
2	Demand Analysis	1	3	5
3	Product Function Position	2	6	8
4	Odor Allocation	2	2	5
5	Color Design	2	8	3
6	Market Analysis	2	5	8
7	Product Synthesis	3,4,5	9	7
8	Promotion Strategies	6,7	3	5
9	Market Promotion	8	5	7
10	Consumer Feedback	9	6	7
11	End	10	0	0

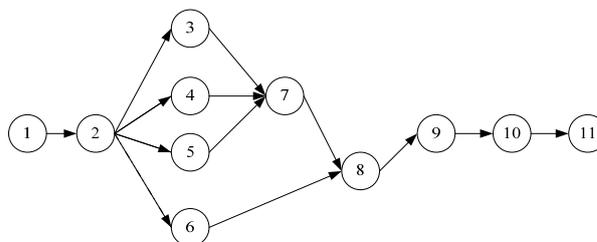


Figure 1. AON network diagram

The algorithm realization is coded by C++ language, running on a VC++6.0 environment, at Windows XP operation system. Parameter settings are shown in Table 2.

TABLE II.  
INPUT PARAMETERS OF GENETIC ALGORITHM

Name	Parameter	Value
Algorithm parameters	Population size	20
	Maximum iteration	100
	Crossover probability	0.3
	Mutation probability	0.015
Constraints parameters	Resource	12

The final scheduling scheme is illustrated in Table 3, and its minimum project duration is 44 time units.

TABLE III.  
THE FINAL SCHEDULING SOLUTION

Number	Procedure	Start time	Finish time
1	Start	0	0
2	Demand Analysis	0	5
3	Product Function Position	5	13
4	Odor Allocation	13	18
5	Color Design	13	16
6	Market Analysis	5	13
7	Product Synthesis	18	25
8	Promotion Strategies	25	30

9	Market Promotion	30	37
10	Consumer Feedback	37	44
11	End	44	44

## VI. CONCLUSIONS

In this article, we study project scheduling problem with resource constraints. Firstly, after analysis, the model of this problem is set up. Subsequently, a novel genetic algorithm used to solve this problem is developed. Finally, one numerical experiment is given so as to verify the effectiveness and efficiency of the proposed model and algorithm.

The future work includes two aspects: (1) performance analysis of genetic algorithm for solving this problem comparing to other heuristic ones such as particle swarm optimization, artificial immune system, and so on need further study; (2) more complex scheduling models considering other dynamic factors including disruptions is another direction needing further explored.

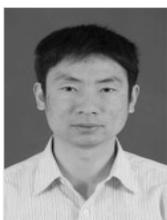
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