Optimization of Two-Stage Cylindrical Gear Reducer with Adaptive Boundary Constraints

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Abstract—**An optimization mathematical model for designing two-stage cylindrical gear reducer is firstly constructed based upon the drive principles and design criteria of cylindrical gear transmission. Then some techniques for calculating the key performance parameters, such as profile factor and dynamic load coefficient, etc., and revising the corresponding boundary constraints dynamically during the optimization process are researched and implemented. Finally, dedicated interactive optimization design software is developed based on VC++/MATLAB mixed programming. Compared with the conventional optimization methods with fixed performance parameter, the proposed method can accurately calculate all the key parameters and revise the corresponding constraint functions adaptively in each iteration step. Optimization examples illustrate that the proposed gear optimization method is more effective and reasonable.**

Index Terms—**optimization design, gear reducer, adaptive boundary constraints, performance parameters**

I. Introduction

With the advantages of large transmission power and high efficiency, gear drives are widely applied in the fields of construction machinery, auto industry, etc., and play an important part in the modern industries. Gears are the core components of the various kinds of transmission systems such as speed increaser and reducer, and their reliabilities can directly decide the performances of the entire system. To make sure the strength, service life and reliability, gears are usually designed with generous safety tolerances by initial means, which can produce a transmission system with large volume and heavy weight. And the accompanied strong inertial forces will bring heavy shocks and impacts, which have an adverse impact on the running of the transmission system. Therefore, the optimal designs of gear drives have been taken seriously by researchers and designers. Optimal researches about shortening the center distance, reducing the volume, enhancing the load capacity and lengthening the lifespan have vital economical significances.

Deep researches have been taken on the optimal problems of mechanical engineering with various optimization algorithms by many scholars at home and abroad [1~3]. A multi-objective optimal design of gear transmission by GA (Genetic Algorithm) considering EHL (Elasto-Hydrodynamic Lubrication) status is carried out by Yin et al.[4], and BP artificial neural network is used to approximate the numerous key coefficients, such as the internal dynamic factor, load factors, form factor, stress correction factor and so on, during the optimization process. A novel mesh adjustable finite-element algorithm is used by Niu et al. [5] in order to optimize the gear dimensions and maximize the torque output. Optimal designs are also carried out by Savage et al. [6] for single mesh spur gear reductions aiming at the optimal design of system life, system volume and system weight and optimal results are obtained. And based on the optimization of double public gear speed change transmission system, a gear optimization software is developed by Jin et al. [7] using VC++ to improve the optimization efficiency. The similar mathematical models are established with the objective function aiming at the minimum center distance or volume by the above scholars. But when the calculation of the numerous key coefficients is involved, the coefficients are usually regarded as constants or approximated by curves, due to the complex and cumbersome calculation process. In fact, these coefficients have close links with teeth number, module, and helix angle etc., and they also have great influences on gear strength calculation. During the process of iterative optimization, the values of the coefficients will change depending on the design variables such as teeth number, module, helix angle and so on. So the coefficients need to update to accommodate the new constraint conditions after every iterative step. Therefore, the optimal results are not accurate by the previous methods which regard the coefficients as constants or approximate with curves.

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In this paper, a new optimal method of gear drive based on adaptive constraint conditions is presented to overcome the limitations of the previous optimization methods. During the optimization process in this method, the key coefficients related to the constraint conditions can be updated with the changes of the design variables. Hence the adaptive update of the constraint conditions can be realized. And the implementation procedure can be described as the following steps. First of all, the values of all the key coefficients are calculated based on the design variables according to the strength criterions, and the constraint conditions can be formed with the current coefficients. Then the new design variables can be produced after an iterative step, which will cause the changes of the coefficients. These iterations will continue until the objective function is achieved and the constraint conditions are satisfied. Using this dynamic calculation method, the coefficients can be updated in every iterative step and the limitations of regarding the coefficients as constants or simulating with curves can be solved. Taking the complex calculation process into consideration, a calculation unit is developed to calculate the coefficients. And the MATLAB optimization toolbox is also used to optimize a two-stage cylindrical gear reducer with the objective function of minimum center distance. Based on the unit, the dynamic calculation can be realized during the optimization iterations. And the VC++ platform is used to encapsulate the MATLAB optimization unit, and the optimization software for cylindrical gear reducer is developed to improve the design and optimization efficiency. In the end, an optimal design for two-stage cylindrical gear reducer is carried out, and the optimization results show the feasibility and accuracy compared with other optimization methods.

II. Optimization Methodology

A. Mathematical Model

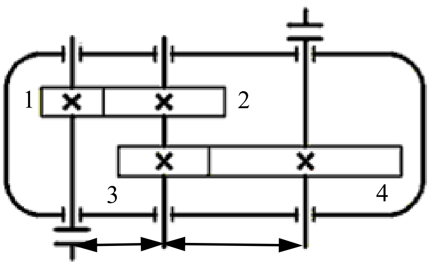

Figure 1. Structure sketch of a two-stage reducer

Fig. 1 shows the structure sketch of a two-stage cylindrical gear reducer with external mesh. The high-speed stage consists of gear 1 and 2, while the low-speed stage consists of gear 3 and 4. Center distances of the two stages are a_1 and a_2 respectively. The total center distance of two stages is the sum of a_1 and a_2. Whatever the optimization algorithm is utilized to solve the problem, the correct mathematical model should firstly be established, which includes the determinations of appropriate design variables, expected objective function and detailed constraint conditions [8,9].

a) Design variables

During the optimization process, the selections of design variables have crucial influences on the accuracy of the optimization results. Whether there are links among the design variables or not combined with the determination of objective function need to be considered as to the selections of the variables [10]. For example, in an optimal design which aims at the minimum volume, the facewidths of two stages should be regarded as two isolated optimization variables. While in an optimal design aiming at the minimum total center distance, the limitations of facewidths don't contribute to the optimal objective. Since the objective function is for the minimum total center distance of two stages in this paper, facewidths are not selected as design variables. And width coefficients of two stages are regarded as constants according to the design need.

The teeth numbers are integer variables while modules are discrete. In order to solve the optimal problem according to the general nonlinear programming method, both the teeth numbers and the modules are regarded as continuous variables in this paper. After the optimization process, the teeth numbers and modules need to be rounded as integer and discrete variables respectively according to the actual situations. And the strengths of two stages also need to be checked again in order to satisfy the strength criterions.

Therefore, based on the above discussions, the design variables are defined as the following: pinion teeth number z_1 of high-speed stage, module m_1, helix angle β_1, transmission ratio u_1; pinion teeth number z_3, module m_2, and helix angle β_2. The seven design variables are shown in (1).

$$\begin{aligned} X &= \left[x_1, x_2, x_3, x_4, x_5, x_6, x_7\right]^T \\ &= \left[z_1, m_1, \beta_1, u_1, z_3, m_2, \beta_2\right]^T \end{aligned} \tag{1}$$

b) Objective function

As to the optimal problem of gear transmission, there are several objective functions which aim at different targets such as the minimum center distance, the minimum volume and the maximum load capacity. And in this paper, the objective function is selected as the total minimum center distance, which seeks the minimum sum of center distances. The objective function can be expressed as shown in (2) using the design variables in (1).

$$f\left(X\right) = x_1 x_2 \frac{x_4 + 1}{2\, cos\left(x_3\right)} + x_5 x_6 \frac{\frac{u_\Sigma}{x_4} + 1}{2\, cos\left(x_7\right)} \tag{2}$$

Where, u_Σ stands for the total transmission ratio.

c) Constraint conditions

In this optimal design of two-stage cylindrical gear reducer, the constraint conditions mainly consist of four aspects, including strength conditions, size conditions, lubrication conditions and limitations of all design variables. The strength conditions can be defined as: the calculated contact fatigue strengths and bending fatigue strengths of two stages should be less than the allowable contact and bending fatigue strengths respectively. The size conditions are to make sure that there is no interference between the wheel's addendum circle of high-speed stage and the output shaft, and at the same time, there is also no interference between the input shaft and the pinion's addendum circle of low-speed stage. The lubrication conditions are to guarantee the essential lubrication status for the two wheels of two stages to assure that four gears can be lubricated. The limitation conditions can be described as: there is no undercut for the two pinions and the values of modules and helix angles need to be limited in an accepted range. Therefore, the constraint conditions of two-stage optimization problem are not simple superposition of those of single–stage.

When to determine the conditions in detail, the four types of constraint conditions can be obtained based on the gear meshing theory and strength calculation criterions. Taking the high-speed stage for example, the conditions can be expressed as the followings.

Strength constraint conditions

The calculated contact stresses of high-speed stage need to be less than the allowable stresses respectively, as shown in (3).

$$c_1(X) = \sigma_{Hi} - [\sigma_{Hi}] \le 0 (i = 1,2) \tag{3}$$

The calculated bending stresses of high-speed stage should be less than the allowable values respectively as shown in (4).

$$c_2(X) = \sigma_{Fi} - [\sigma_{Fi}] \le 0 (i = 1,2) \tag{4}$$

Size constraint conditions

To make sure there is no interference between the wheel's addendum circle of high-speed stage and the output shaft as (5) shown.

$$c_3(X) = \frac{x_2(x_1 x_4 + 2ha^*)}{2\cos(x_3)} - x_5 x_6 \frac{\frac{u_\Sigma}{x_4} + 1}{2\cos(x_7)} + S_1 \le 0 \tag{5}$$

Where, S_1 stands for the minimum space between the wheel's addendum circle of high-speed stage and the axis of output shaft, and h_a* denotes the addendum coefficient.

Lubrication conditions

All the gears at the two stages need to be lubricated in the running process. In order to achieve this demand, the high-speed stage transmission ratio u_1 and low-speed stage transmission ratio u_2 should satisfy the following relation [11], as shown in (6).

$$u_1 = (1.2 \sim 1.4) u_2 \tag{6}$$

After a transformation, the lubrication conditions can be expressed as (7) and (8) shown.

$$c_4(X) = 1.2 u_\Sigma - x_4^2 \le 0 \tag{7}$$

$$c_5(X) = x_4^2 - 1.4 u_\Sigma \le 0 \tag{8}$$

Limitations of design variables

Make sure there is no undercut for pinion, as shown in (9).

$$c_6(X) = x_1 - \frac{2ha^*}{\sin^2\left(\arctan \frac{\tan \alpha_n}{\cos(x_3)}\right)} \le 0 \tag{9}$$

Where, α_n denotes the normal pressure angle.

Maximum teeth number of pinion should be limited to 40, as shown in (10)

$$c_7(X) = x_1 - 40 \le 0 \tag{10}$$

Module of high-speed stage should be limited from 2 to 10, as (11) and (12) shown.

$$c_8(X) = 2 - x_2 \le 0 \tag{11}$$

$$c_9(X) = x_2 - 10 \le 0 \tag{12}$$

And helix angle should be limited from 8 to 15 degrees, as shown in (13) and (14).

$$c_{10}(X) = \frac{8\pi}{180} - x_3 \le 0 \tag{13}$$

$$c_{11}(X) = x_3 - \frac{15\pi}{180} \le 0 \tag{14}$$

B. *Implementation of Adaptive Constraints*

In the previous methods, the key coefficients related to the stress calculations are usually regarded as constants, or calculated by some simplified approximate functions. During the calculations, many charts and curves are needed in order to obtain precise values, and some coefficients should be determined based on the estimations, which results in a barrier to the optimization process. And the estimated coefficients will also cause the wrong optimization results to a large extent.

In order to improve the accuracy of the optimization results, dynamic calculations of the key coefficients during the iterative steps are achieved in this paper. As a result, the adaptive constraint conditions can be realized. In each iterative step, the coefficients are calculated based on the teeth number, module, helix angle etc. Then, the constraint conditions can be determined following the current coefficients and the design variables can be produced under the current conditions. These steps will loop until the minimum total center distance is obtained and all the constraint conditions are satisfied. Since the coefficients are updated with changes of the design variables, the limitations in the previous methods can be overcome. Therefore, the optimization results are more accurate compared with other methods.

III. SOFTWARE IMPLEMENTATION PROCESS

Many equality and non-equality constraints are involved in the optimal design of two-stage cylindrical gear reducer, and this problem is a typical constraint nonlinear programming problem. To solve this type of problems, many optimization algorithms can be used, for example, the exterior point method, the interior point method, complex method and feasible direction method etc. And much commercial numerical software such as MATLAB can support existing optimization algorithms [12]. Once the objective function, constraint conditions and the initial values are defined, the optimal design can be preceded by invoking an optimization function, which will improve the efficiency greatly. Hence the optimization programs are coded in MATLAB and dynamic exchange is achieved using the data interaction technique during the optimization process.

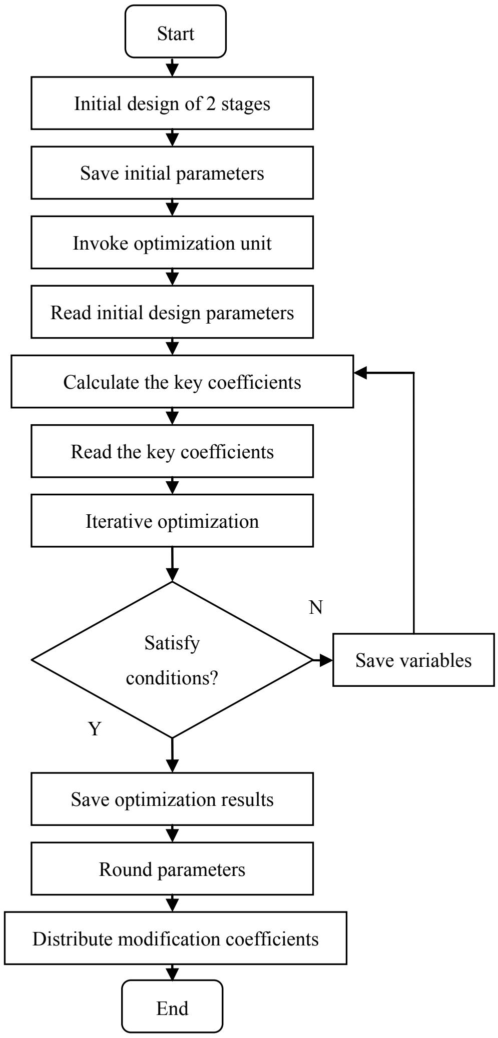

Figure 2. Flow chart of the optimization process

Fig. 2 shows the implementation process. At first, the optimization function fmincon in MATLAB optimization toolbox is used to perform an optimization of two-stage gear reducer according to the existing programs. And the command mcc-build is used to compile the codes into dynamic link libraries in order to be invoked in other programming environments such as C and C++. The optimization unit can be formed in this way. Then, the parameters, including the teeth number, module, helix angle and working parameters, are written into disk files to be read by the optimization unit. In each iterative step, the unit will read the parameters from the files and form new constraint conditions. After the current step, design variable are saved to files again. And based on these variables, the key coefficients can be calculated in order to adjust with the variables, which can realize the dynamic optimization. Once the optimization result is obtained, design variables are rounded according to the actual situation and modification coefficients can be distributed in order to fit a specified center distance. Based on this process, the software can be developed.

IV. EXAMPLE AND ANALYSIS

A. Initial Conditions

Taking a certain two-stage helical cylindrical gear reducer as example, an optimal design is carried out. The working parameters of this reducer are known as the following: input power P_I=10KW, the input rotate speed n_1=2250r/min, the output rotate speed n_2=187.5r/min and total transmission ratio u_Σ=12.

B. Initial Design

The initial design parameters such as the precision classes (*PC*) of high-speed stage (stage1) and low-speed stage (stage2), materials, contact fatigue strength σ_{Hlim} and bending fatigue strength σ_{Flim} of the gears, facewidth coefficient Φ_d and the comprehensive coefficient *K* can be tabulated in Table 1.

TABLE I.
MATERIALS AND WORKING PARAMATERS

Item	PC	Materials	σ_{Hlim} (MPa)	σ_{Flim} (MPa)	Φ_d	K
stage1	8	Carburized steel	1200/1200	600/600	1.0	2.5
stage2	8	Carburized steel	1200/1200	600/600	0.8	2.5

Based on the parameters in Table 1, initial designs of high and low speed stage are respectively performed using the conventional design method, and the design results can also be obtained accordingly. Table 2 shows the design results, including teeth numbers, modules, helical angles, etc., of high and low speed stages.

TABLE II.
INITIAL DESIGN RESULTS

Item	z_1	z_2	m_1	β_1 (°)	b_1(mm)	b_2(mm)
Value	26	133	3	8.97309	80	75
Item	z_3	z_4	m_2	β_2 (°)	b_3(mm)	b_4(mm)
Value	34	80	4.5	5.28352	125	120

C. Optimal Results

After the initial designs, the center distances of two stages are accordingly optimized using the design and optimization software. In order to show the differences between the previous optimization method and the adaptive optimization method, two experiments using the same parameters are preceded respectively.

During the entire optimization process, all the design variables are regarded as continuous variables. Since the teeth number is an integer variable while module is discrete, essential round should be done after the optimization, and the center distance also need to be adjusted according to the actual situation. Based on the specified center distance, the modification coefficients can be distributed on the principles of equal strength or equal sliding ratio. Then the ultimate optimization results can be obtained.

Table 3 has shown the different optimization results of two-stage gear reducer using the two optimization methods.

TABLE III.
COMPARISON BETWEEN THE TWO METHODS.

Parameter Value	z_1	z_2	m_1	β_1 (°)	b_1(mm)	b_2(mm)	z_3	z_4	m_2	β_2 (°)	b_3(mm)	b_4(mm)	a_Σ(mm)
Previous Method	25	93	3	11.40534	75	70	31	99	3.5	11.69013	95	90	412.88
Adaptive Method	23	92	2.75	14.85120	70	65	30	93	3.0	14.71726	80	75	354.35

After the optimizations using the two methods, safety factors of tooth contact strength and root bending strength are checked again. Combined with the initial safety factors, these factors can be summarized as Table 4 shown.

TABLE IV.
COMPARISON OF THE SAFETY FACTORS.

Safety factors	Tooth contact strength				Root bending strength			
	High-speed stage		Low-speed stage		High-speed stage		Low-speed stage	
	Pinion S_{H1}	Wheel S_{H2}	Pinion S'_{H1}	Wheel S'_{H2}	Pinion S_{F1}	Wheel S_{F2}	Pinion S'_{F1}	Wheel S'_{F2}
Initial Design	1.188	1.249	1.348	1.392	3.394	3.476	3.963	4.010
Previous Method	1.127	1.173	1.215	1.259	3.507	3.625	3.435	4.469
Adaptive Method	1.039	1.084	1.004	1.013	3.528	3.666	2.488	2.514

From the data in Table 2 and Table 3, we can see that compared with the initial design, the teeth numbers of pinions, modules and facewidths have reduced slightly using the previous method and the adaptive optimization method. As a result, the total center distance has fallen. Meanwhile, the optimal effect of adaptive optimization method is more obvious than that of previous method. The total minimum center distance using adaptive method is 354.35mm, which is less than 28.99% from that using the initial method, and less than 14.18% from that of previous method.

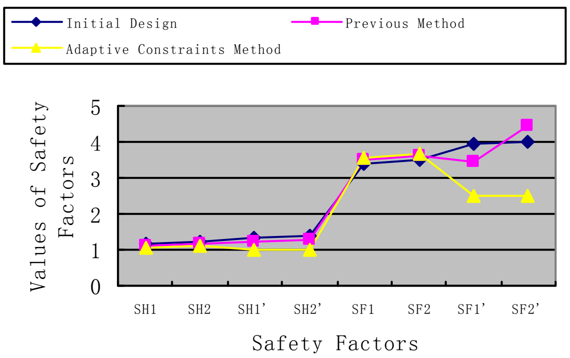

Figure 3. Line graph of safety factors.

And from Fig. 3, it can be seen that all the safety factors, except for the bending strength safety factors of high-speed stage, have reduced using the adaptive optimization method. And after the optimizations, the contact strength safety factors are all greater than 1.0 and the bending strength safety factors are greater than 1.3, which indicates that the optimization results can guarantee the minimum center distance as well as satisfy all the constraint conditions.

Therefore, the results have shown that the optimization results of gear reducer obtained by the previous methods are not sufficient and there is still large space to be optimized. Only by using the method of variable coefficients optimization and adjusting the key coefficients based on the newly produced design variables, can the optimization be carried out completely. As a result, the optimization results are more accurate and the optimization effects of gear reducer are better.

V. CONCLUSIONS

The following conclusions can be drawn from this work:

(1) Using the method of adaptive boundary constraint conditions, the key coefficients in every iterative step can be updated with changes of design variables. This can make sure the dynamic calculations of constrain conditions as well as the accuracy of the optimization results.

(2) The design and optimization software for two-stage cylindrical gear reducer developed by VC++ and MATLAB can be applied in the areas of initial design, strength checks and optimization. The faster design and

optimization can be achieved and the design efficiency is improved accordingly.

(3) From the experimental results, it can be seen obviously that the total center distance obtained by the variable coefficients optimization method is smaller than that obtained by the previous method which regards the coefficients as constants or approximates with curves. The result indicates the feasibility and accuracy of this method.

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