Using Finite Element Software to Simulation Fracture Behavior of Three-point Bending Beam with Initial Crack

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Abstract— **using computation software to simulate fracture behaviors of structures can shorten the time to estimate safety of engineering structures and reduce a lot of cost of experiment. However three dimensional numerical simulation for fracture process is a challenging issue, because the convergence and accuracy are not easily obtained and high computational cost is generated. In this paper, three new fracture mechanics models, including cohesive model, virtual crack closure technique (short for VCCT) and extended finite element method (short for XFEM) have been utilized to simulate and analysis the fracture process of the three-point bending beam with initial notch based on the finite element software platform – Abaqus version 6.11. The numerical results have shown that three different methods can be used to simulate different fracture toughness materials and fracture types. Meanwhile,based on Abaqus software platform and parallel computing technology, this type of simulations could be completed in personal computers.**

Index Terms—**concrete, initial crack, numerical simulation, crack propagation**

I. INTRODUCTION

Concrete as a modern building material is the most widely used. As concrete have the advantages with abundant availability, simple process, well fireproof, lower cost and wide adaptability [1], concrete has been widely used in our country. It has been reported that every year 2 billion cubic meters concrete is used for various engineering projects in our country. However, concrete is a brittle material with high compressive strength, low tensile strength and poor toughness, it makes that cracks of different degrees and different forms occur in using process and even in construction process. And it makes the durability of concrete structure deteriorates and lifetime shortens. In modern times profuse experiments and engineering practices show that: cracking of concrete structure is almost inevitable; According to current economy and technique level, it is reasonable that concrete cracking could be controlled in the harm-allowable range [2]. Simulating concrete fracture behavior is the main way to achieve aforesaid aims. And a lot of scholars at home and abroad have done researches on concrete fracture behavior, and have developed many effectual concrete fracture models. Xu Shi-liang[3-6] and Jeng proposed double-K fracture criterion for fracture mechanics of concrete. Tang Chun-an focused on fracture mechanism and numerical simulation of concrete and rock [7] using Realistic Failure Process Analysis software (short for RFPA). Ref. [8] implemented new damage model coupled plasticity for concrete to simulate damage behavior of concrete in FEM software Abaqus via UMAT subroutine. And some scholars used extended finite element method (XFEM) to study on fracture damage behavior of concrete [9-10]. Jin Feng[11] imported the subroutine named User Defined Element (UEL) to implement XFEM to simulate the fracture of concrete, based on FEM software Abaqus.

Abaqus offers the technologies necessary to include fracture and failure in the syringe design process. The extended finite element method (XFEM) allows for the analysis of crack initiation and propagation along an arbitrary, solution dependent path without the need for remeshing. To compare with the advantages in predicting concrete fracture behavior for various concrete fracture models, this paper uses cohesive zone model, XFEM and virtual crack closure technique (VCCT) to predict the 3D crack propagation for three-point bending concrete beam with initial crack based the finite element software platform –Abaqus version 6.11, and compares with the features in simulating soften behavior for various concrete fracture models.

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II. INTRODUCTION OF EXTENDED FINITE ELEMENT METHOD

A. *Theory Of Extended Finite Element Method*

The extended finite element method was first introduced by Belytschko and Black [12]. It is an extension of the conventional finite element method based on the concept of partition of unity, which allows local enrichment functions to be easily incorporated into a finite element approximation. The presence of discontinuities is ensured by the special enriched functions in conjunction with additional degrees of freedom. Crack modeling based on XFEM allows for simulation of both stationary and moving cracks. Evaluation of crack contour integrals with XFEM is available for three dimensional analyses. Simulation of propagating cracks with XFEM does not require initial crack and crack path definitions to conform to the structural mesh. The crack path is solution dependent - i.e., it is obtained as part of the solution. Cracks are allowed to propagate through elements allowing for modeling of fracture of the bulk material.

For the purpose of fracture analysis, the enrichment functions typically consist of the near-tip asymptotic functions that capture the singularity around the crack tip and a discontinuous function that represents the jump in displacement across the crack surfaces. The approximation for a displacement vector function with the partition of unity enrichment [8] is

$$\mathbf{u}=\sum_{I=1}^{N}N_I(x)[\mathbf{u}_I+H(x)\mathbf{a}_I+\sum_{\alpha=1}^{4}F_\alpha(x)\mathbf{b}_I^\alpha] \tag{1}$$

where $N_I(x)$ are the usual nodal shape functions; $\mathbf{u}_I$ is the usual nodal displacement vector associated with the continuous part of the finite element solution; the second term is the product of the nodal enriched degree of freedom vector, $\mathbf{a}_I$ and the associated discontinuous jump function $H(x)$ across the crack surfaces; and the third term is the product of the nodal enriched degree of freedom vector, $\mathbf{b}_I^\alpha$, and the associated elastic asymptotic crack-tip functions, $F_\alpha(x)$. The first term on the right-hand side is applicable to all the nodes in the model; the second term is valid for nodes whose shape function support is cut by the crack interior; and the third term is used only for nodes whose shape function support is cut by the crack tip. $H(x)$ can be written as equation 2,

$$H(x)=\begin{cases}1 & if\,(x-x^*)\cdot n\geq 0\\ -1 & otherwise\end{cases} \tag{2}$$

where $\mathbf{x}$ is a sample (Gauss) point, $\mathbf{x}^*$ is the point on the crack closest to $\mathbf{x}$, and $\mathbf{n}$ is the unit outward normal to the crack at $\mathbf{x}^*$ (fig 1).

The asymptotic crack tip functions in an isotropic elastic material, $F_\alpha(x)$, which are given by

$$F_\alpha(x)=[\sqrt{r}\sin\frac{\theta}{2},\sqrt{r}\cos\frac{\theta}{2},\sqrt{r}\sin\theta\cdot\sin\frac{\theta}{2},\sqrt{r}\sin\theta\cdot\cos\frac{\theta}{2}] \tag{3}$$

Where (r,θ) a polar coordinate system with its origin at the crack is tip and $\theta=0$ is tangent to the cracking tip.

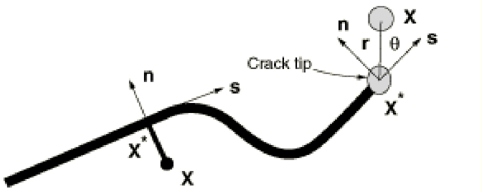

Figure 1. Illustration of normal and tangential coordinates for a smooth crack

Crack initiation refers to the beginning of degradation of the cohesive response at an enriched element. The process of degradation begins when the stresses or the strains satisfy specified crack initiation criteria. Crack initiation criteria are available based on the following:
The maximum principal stress criterion:

$$f=\left\{\frac{\langle\sigma_{\max}\rangle}{\sigma_{\max}^0}\right\} \tag{4}$$

The maximum principal strain criterion:

$$f=\left\{\frac{\langle\varepsilon_{\max}\rangle}{\varepsilon_{\max}^0}\right\} \tag{5}$$

The maximum nominal stress criterion:

$$f=\max\left\{\frac{\langle t_n\rangle}{t_n^0},\frac{t_s}{t_s^0},\frac{t_t}{t_t^0}\right\} \tag{6}$$

The maximum nominal strain criterion:

$$f=\max\left\{\frac{\langle\varepsilon_n\rangle}{\varepsilon_n^0},\frac{\varepsilon_s}{\varepsilon_s^0},\frac{\varepsilon_t}{\varepsilon_t^0}\right\} \tag{7}$$

The quadratic traction-interaction criterion:

$$f=\left\{\frac{\langle t_n\rangle}{t_n^0}\right\}^2+\left\{\frac{t_s}{t_s^0}\right\}^2+\left\{\frac{t_t}{t_t^0}\right\}^2 \tag{8}$$

The quadratic separation-interaction criterion:

$$f=\left\{\frac{\langle\varepsilon_n\rangle}{\varepsilon_n^0}\right\}^2+\left\{\frac{\varepsilon_s}{\varepsilon_s^0}\right\}^2+\left\{\frac{\varepsilon_t}{\varepsilon_t^0}\right\}^2 \tag{9}$$

Concrete is considered as brittle material in this paper, and the maximum principal stress of material is set as the value of commence of damage. $\sigma_{\max}^0$, represents the maximum principal stress. The symbol $\langle\ \rangle$ represents the Macaulay bracket with the usual interpretation. The Macaulay brackets are used to signify that a purely compressive stress state does not initiate damage. Damage is assumed to initiate when the maximum principal stress ratio (as defined in the expression 1 above) reaches a value of one. In the soften segment of material, a scalar damage variable, D has been used to represent damage evolution. It initially has a value of 0. If damage evolution is modeled, D monotonically evolves from 0 to

1 upon further loading after the initiation of damage. The normal and shear stress components are affected by the damage according to express 10-express 12.

$$t_n = \begin{cases} (1-D)T_n, & T_n \geq 0 \\ T_n, & T_n < 0 \end{cases} \tag{10}$$

$$t_s = (1-D)T_s \tag{11}$$

$$t_t = (1-D)T_t \tag{12}$$

Where T_n, T_s and T_t are the normal and shear stress components predicted by the elastic traction-separation behavior for the current separations without damage.

B. *Cohesive Zone Model*

Cohesive zone model was proposed by Dugdale and Barenblatt. Based on the fundamental idea, Hillerborg proposed classical fictitious crack model (FCM) in 1976 and applied it for concrete's study. The major advantage of FCM is that strain-softening curve is introduced to simulate crack occurrence and growth using deformation-damage model[13]. Because of the advantages, cohesive zone model is built into many FEM softwares, such as Ls-dyna and Abaqus.

In cohesive zone model, it is assumed that there is a viscous region around crack-tip. In the viscous region, viscous curve relationship exists in adhesion force and relative distance between two faces. When adhesion force reaches the peak, the crack appears. When the crack grows in a certain extent, the two faces separate. Cohesive element is the application of cohesive zone model in Abaqus. When the load is at low level, cohesive element provides adhesion constraint for subdomain, the structure is in intact state. When the load increases gradually, cohesive element is damaged gradually, the crack begins to occur and propagate, and the response between interfaces is calculated using the constraint equation defined previously. In Abaqus constraint equation is introduced into global balance equations using penalty method and is solved using incremental iteration approach is used [14].

C. *Virtual Crack Closure Technique*

VCCT was first introduced by Rybicki and Kanninen[15] in the year 1977. Initially it was called Modified Crack-Closure Integral (MCCI), subsequently renamed to Virtual Crack Closure Technique. Raju[16] wanted to explain legitimately VCCT with mathematics in 1987, and provided the computational formula according to higher order element and singular element.

VCCT is based on the assumption that the strain energy released when a crack is extended by a certain amount is the same as the energy required to close the crack by the same amount. Assuming that the crack closure is governed by linear elastic behavior, the energy to close the crack (and, thus, the energy to open the crack) is calculated from the following equations: (Δa is the crack extended length)

$$\int_0^{\Delta a} \sigma_{yy}^{(1)}(x)\Delta v^2(x)dx = F_{y1}^{(1)} v_{1,1'}^{(2)} \tag{13}$$

III. THREE-POINT BENDING NUMERICAL SIMULATIONS OF CONCRE BEAM WITH INITIAL CRACK

Three-point-bending concrete beam with initial crack under ultimate loading has been simulated using XFEM, cohesive model and VCCT on the FEM software platform Abaqus version 6.11. The model region of concrete beam is 7.2m×0.6m×0.4m with a initial crack. The initial crack is 0.01m×0.4m in the middle of concrete beam's bottom face, shown in Fig 2.

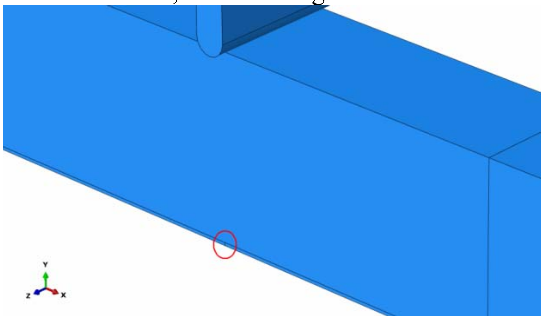

Figure 2. The concrete beam with initial crack

According to experiment results, material is C30 concrete and material parameters are shown as table 1-3.

TABLE Ⅰ.

PARAMETERS OF C30 CONCRETE USING XFEM

Young's Modulus (MPa)	**Poisson's Ratio**	**Density (Kg/m^3)**	**MAXPS Damage (MPa)**	**G_I (N/m)**	**G_{II} (N/m)**	**G_{III} (N/m)**
26800	0.19	2500	2.9	250	250	250

TABLE Ⅱ.

MATERIAL PARAMENTERS USING COHESIVE ZONE MODEL

K_{nn}	**K_{ss}**	**K_{tt}**	**MAXS Damage (MPa)**			**G_{IC}**	**G_{IIC}**	**G_{IIIC}**
(MPa)	**(MPa)**	**(MPa)**	**N_{max}**	**T_{max}**	**S_{max}**	**(J)**	**(J)**	**(J)**
26800	26800	26800	1.0	1.0	1.0	50	50	50

TABLE III.
MATERIAL PARAMENTERS USING VCCT

G_{IC} (N/m)	G_{IIC} (N/m)	G_{IIIC} (N/m)
50	50	50

The two steadies and punch are defined as rigid body constraint. The connection between two steadies, punch and concrete beam are defined as finite sliding, surface to surface contact using Coulomb friction coefficient 0.2. The boundary condition includes two fixed displacement at two steadies' reference points. The loading is loaded at the reference point of punch and the magnitude of loading is negative Y displacement 0.05m. The loading can cause that crack penetrate through concrete beam. Then the ultimate load of concrete beam could be obtained from reaction force curve. There are 22470 C3D8R (8-node linear brick, reduced integration with hourglass control) solid elements in FEM model, shown in Fig 3. Meanwhile multiple CPUs parallel computing technique is employed to reduce computation time [17, 18].

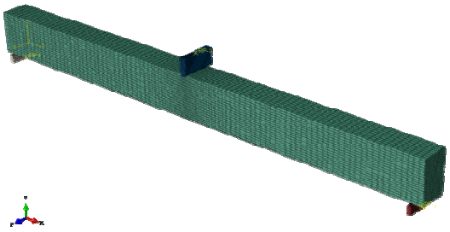

Figure 3. The mesh of FEM model

IV. RESULTS ANALYSIS OF NUMERICAL SIMULATION

Fig 4-7 show max principal stress contour in crack propagation under 0.05m displacement loading. Fig 8 shows the initial crack finally penetrate through concrete beam. Max principal stress concentrates at the middle of concrete beam. In three-point bending experiment, the stresses in the middle of concrete beam are biggest and mainly tensile stresses, moreover the initial crack presents in the middle of concrete beam, stress concentration has been strengthened. The crack spreads according to mode I initially, shown as max principal stress contour Fig 5, the maximum of max principal stress presents at near cracking tip and parallel to X direction. But the cracking spreads as some angle with initial cracking during crack propagation; it is result of two steadies' boundary conditions.

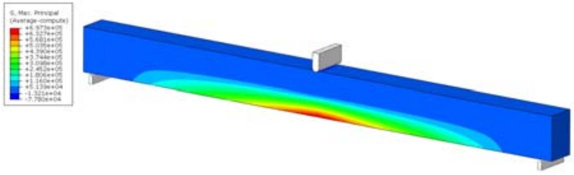

Figure 4. Max principal stress contour in crack propagation at time 0.01

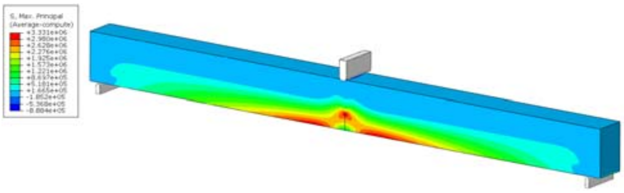

Figure 5. Max principal stress contour in crack propagation at time 0.1

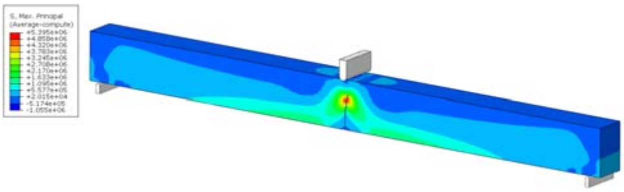

Figure 6. Max principal stress contour in crack propagation at time 0.1012

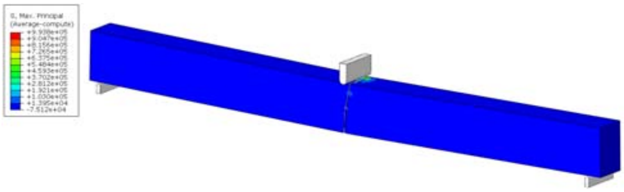

Figure 7. Max principal stress contour in crack propagation at time 1.0

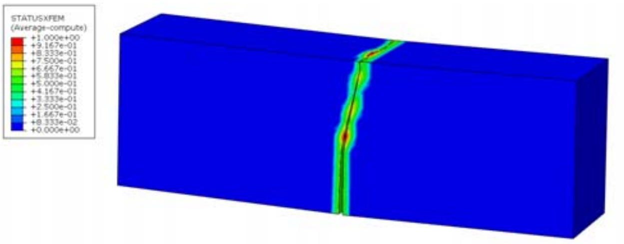

Figure 8. Crack penetrate through concrete beam

Fig 9-12 show Mises stress contour using cohesive zone model in crack propagation. And Fig 13-16 show Max principal logarithmic strain. They indicate the fracture process of three-point bending concrete beam with initial crack. Compared with XFEM, the result using cohesive zone model is different: Crack always propagates along the pre-existing crack propagation path and couldn't deviate from the pre-existing crack propagation path.

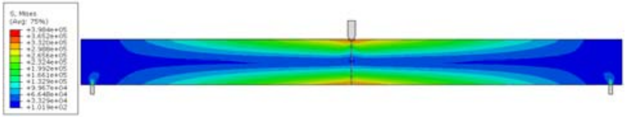

Figure 9. Mises stress contour using Cohesive at time 0.01

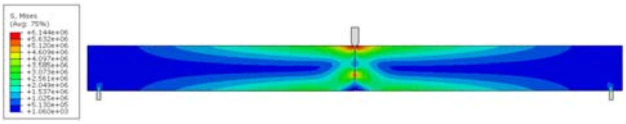

Figure 10. Mises stress contour using Cohesive at time 0.055

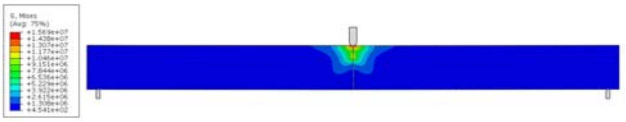

Figure 11. Mises stress contour using Cohesive at time 0.1

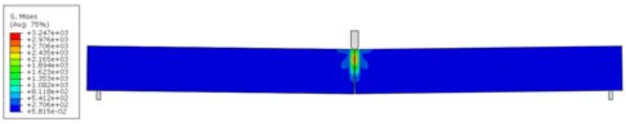

Figure 12. Mises stress contour using Cohesive at time 1.0

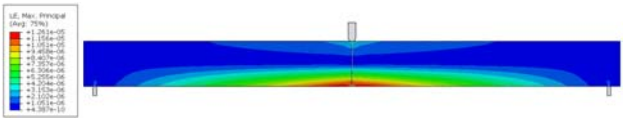

Figure 13. Max principal LE contour using Cohesive at time 0.01

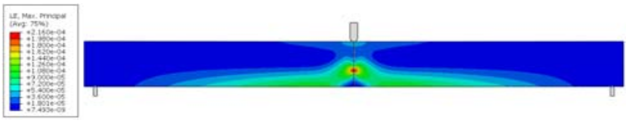

Figure 14. Max principal LE contour using Cohesive at time 0.055

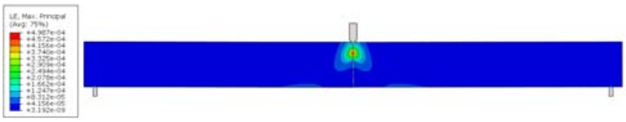

Figure 15. Max principal LE contour using Cohesive at time 0.1

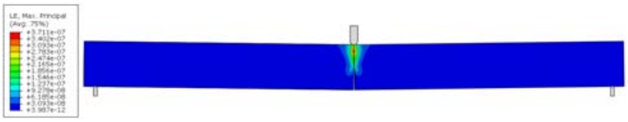

Figure 16. Max principal LE contour using Cohesive at time 1.0

Fig 17-20 show Mises stress contour using VCCT in crack propagation. And Fig 21-24 show Max principal logarithmic strain. They also indicate the fracture process of three-point bending concrete beam with initial crack. Because VCCT assumes that the crack closure is governed by linear elastic behavior, the result using VCCT seems more rigid compared with cohesive zone model. In loading process, the reaction force immediately declines after reaching peak, the opening displacement of crack is also relatively smaller, and the fracture process shows more brittle fracture feature.

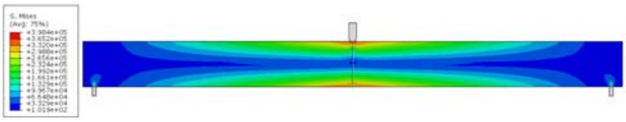

Figure 17. Mises stress contour using VCCT at time 0.01

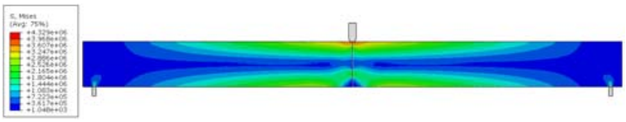

Figure 18. Mises stress contour using VCCT at time 0.04461

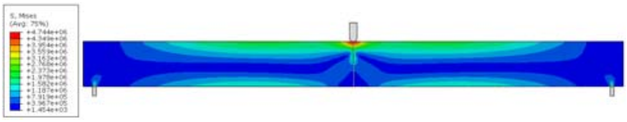

Figure 19. Mises stress contour using VCCT at time 0.04464

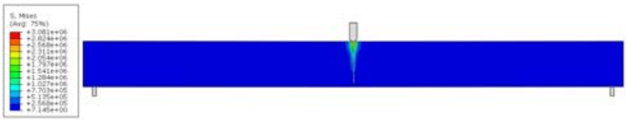

Figure 20. Mises stress contour using VCCT at time 1.0

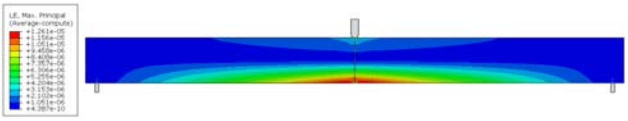

Figure 21. Max principal LE contour using VCCT at time 0.01

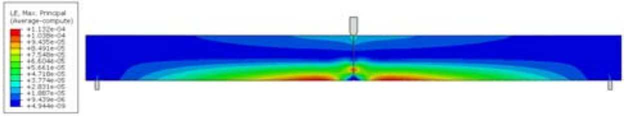

Figure 22. Max principal LE contour using VCCT at time 0.04461

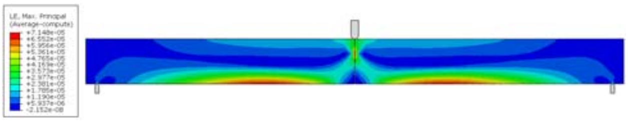

Figure 23. Max principal LE contour using VCCT at time 0.04464

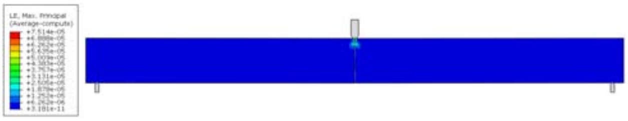

Figure 24. Max principal LE contour using VCCT at time 1.0

In order to describe the process of crack fracture more reasonably, this paper uses P-CMOD curves to evaluate three finite element methods to simulate crack fracture behavior. P-CMOD curves using VCCT, XFEM and cohesive zone model are shown in Fig 25. it has been found that the fracture toughness of cohesive model is the biggest, but using VCCT method is more advisable to simulate the brittle fracture behavior.

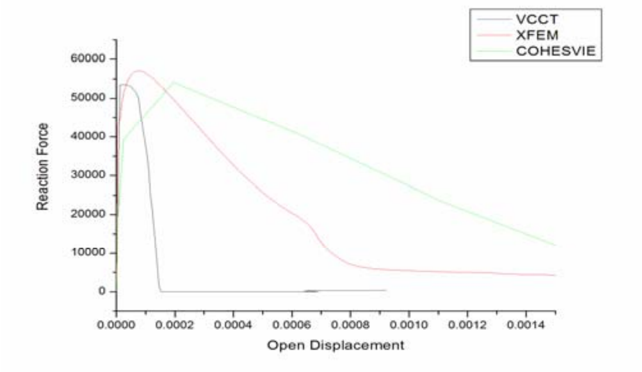

Figure 25. The P-CMOD curve using VCCT/XFEM/COHESIVE

V. CONCLUSION

In this paper, three numerical simulation technologies have been developed using FEM software Abaqus. The three numerical simulation technologies are XFEM approach, cohesive zone method and VCCT method. Three-point bending numerical simulations of C30 concrete beam with initial crack under different loadings have been researched using above three computer simulation technology. There are five significant conclusions can be concluded:

a) Using computer simulation technology and computer simulation software could present the fracture process of concrete, Meanwhile cohesive zone model method, XFEM approach and VCCT method could simulate soft behavior when concrete is fracturing.

b) Extended finite element method could simulate arbitrary cracking extent path of three-point-bending concrete beam.

c) Using numerical simulation method and computer simulation technology the carrying capacity of concrete beam could be obtained, which provides some foundations for realistic concrete structures design.

d) Using VCCT method to predict the fracture process of concrete could make soft behavior more distinct. VCCT method is more suited to simulate the brittle fracture process of concrete.

e) For the problem of huge DOFs in the FEM models, in this paper multiple CPUs parallel computing technique is employed based on Abaqus software platform. It improves the efficiency, reduces computation time, and makes the simulations possible in personal computers.

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