An Integrated Method for DS/AHP under Ambidextrous Decision Information

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***Abstract*—In order to widen individual preference information extracting way and enhance practical values of the traditional DS/AHP method, multiplicative preference relations and fuzzy preference relations are reviewed as key extracting decision information way of pairwise comparison. After that, a basic model for deriving basic probability assignment (BPA) functions is constructed by means of minimizing the value of total deviation degree, which is able to attach importance to efficient elements and neglect inefficient elements of knowledge matrices in the process of calculating BPA functions. Based on the proposed basic model, an integrated method for DS/AHP is established to gradually integrate evidence pieces over all of criteria and that over all of group members by fusing ambidextrous decision information with criteria priority values (CPVs) and member priority values (MPVs). A numerical example about textbook choice is utilized to illustrate the whole procedure of the proposed method finally.**

***Index Terms*—DS/AHP; multiplicative preference relation; fuzzy preference relation; integrated method; multiple criteria decision making**

I. Introduction

In order to deal with Multiple Criteria Decision Making (MCDM) problems with incomplete information, Beynon et al. introduced a seminal method named DS/AHP by incorporating Dempster–Shafer theory (DST) with Analytic Hierarchy Process (AHP)[1]. The DS/AHP method allows a decision maker to make preference judgments on groups of decision alternatives rather on individual alternative or through pairwise comparisons of alternatives. Their results are a body of evidence consisting of preference levels on groups of decision alternatives and a level of concomitant ignorance in forms of Basic Probability Assignments (BPA), and enable levels of belief and plausibility to be identified on the best decision alternatives existing within varying sized groups of decision alternatives.

In terms of the seminal DS/AHP method, Beynon et al. developed the measurement of local ignorance level in judgments made, together with the elucidation of an associated measure of non-specificity, and through the evaluation of the limits on these measures, subsequent index values were constructed too[2]. After that, Beynon et al. presented an approach to evidence aggregation that from members in a decision making group when their importance in the group was non-equivalent, and the key idea in this approach was that the individual importance had been investigated previously, with arguments given as to the need for and against the utilization of group member importance weights[3].

Inspired by the DS/AHP, much attention has been paid to develop DS/AHP methods or applied it to solve real world problems. Hua et al. introduced a seeming method named “DS-AHP” for solving the MCDM problems with incomplete decision matrix. The DS-AHP method could identify all possible focal elements from the incomplete decision matrix and be possible to deal with various decision matrixes, either complete or incomplete, crisp or fuzzy, certain or uncertain, by allowing decision makers to describe their evaluations on decision alternatives in a flexible, natural and reliable manner[4]. Ju et al. established a method that incorporating DS/AHP with extended TOPSIS to solve group MCDM problems with incomplete information. In their method, the positive ideal solution vector was defined as the maximum plausibility of all emergency alternatives with respect to each criterion, and the negative ideal solution vector was defined as the minimum belief of all emergency alternatives with respect to each criterion[5]. In addition, the DS/AHP method was as a tool to identify inter-group alliances as well as introduced a ‘majority rule’ approach to decision making through consensus building[6], to solve problems regarding the multiple attribute decision under uncertainties[7], and to solve vendor selection problems[8-10].

Obviously, the DS/AHP method has been established for only more than ten years, but it has been attracted many researchers to attend it in MCDM fields due to its advantage upon solving uncertainty abilities. In the DS/AHP method, Beynon et al. utilized knowledge matrix to extract decision makers’ preference information,

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and employed the eigenvector of knowledge matrix to transform preference information into BPA functions which could be combined by the Dempster combination rule in DST. There is only a format that represents the preference information provided for decision makers in DS/AHP method. But in practical applications, decision makers can participate in decision tasks at different time and various locations, they also have different cultural and educational backgrounds[11]. Thus there is a need to provide different preference formats for them to express their preferences[12]. More specifically, the knowledge matrix in DS/AHP method is multiplicative preference relations which are a type of AHP judgment matrix with incomplete information. For pairwise comparison, recent research demonstrates the way for extracting preference information also contains fuzzy preference relations except for multiplicative preference relations[13]. Each of them could be applied to extract preference information from particular decision makers in special region, since every member has different experiences, faces various circumstances, and etc. Consequently, multiplicative preference relations in the form of AHP judgment matrix (knowledge matrix) applied by Beynon et al. could be adopt to extract special preference information well, but it will be inefficient if decision makers are not used to this kind of information extracting way.

In order to widen individual preference information extracting way and enhance practical values of DS/AHP, this paper constructs a basic model for deriving BPA functions from ambidextrous preference information extracting way and introduces its corresponding integrated method for extending traditional DS/AHP method.

II. TRADATIONAL DS/AHP MEHTHOD

The DS/AHP method is a subjective probability quantification which is able to consider the closed world case of judgment making based around DST. In this section, we firstly describe some related basic notions in the DS/AHP method such as frame of discernment, basic probability assignment, belief function, and plausibility function, and secondly introduce main thoughts existing in the DS/AHP method.

Definition 1 Suppose a possible hypothesis of variable is θ_n, each of possible hypotheses is exclusive, then a finite nonempty exhaustive set of all possible hypotheses $\Theta = \{\theta_1, \cdots, \theta_N\}$ is called frame of discernment.

Definition 2 Suppose Θ is a frame of discernment, each subset θ of Θ is mapping to a number $m(\theta)$ ($m(\theta) \in [0,1]$), and the function m also meets the following requirement

$$m(\phi) = 0, \sum_{\theta \subseteq \Theta} m(\theta) = 1, \tag{1}$$

then $m(\cdot)$ is defined as basic probability assignment (BPA) on the power set of 2^{Θ}. BPA shows the degree of support a particular subset of Θ from a certain evidence source. Specially, if the BPA function associated with subset θ of Θ is larger than 0 ($m(\theta) > 0$, $\theta \subseteq \Theta$), then this subset θ is defined as a focal element.

Definition 3 Suppose Θ is a frame of discernment, $m: 2^{\Theta} \to [0,1]$ is the a BPA function of Θ, and θ is a subset of Θ, then $Bel(\theta)$ is belief function to measure the total amount of probability among the element θ by adding the BPAs of all the subsets of θ. It can also be expressed by the following formula:

$$Bel(\theta) = \sum_{X \subseteq \theta} m(X). \tag{2}$$

Belief function $Bel(\theta)$ shows the total degree of trust in the element θ and all its subsets.

Definition 4 Suppose Θ is a frame of discernment, $m: 2^{\Theta} \to [0,1]$ is a BPA function of Θ, and θ is a subset of Θ, then $Pl(\theta)$ is plausibility function to measure the maximal probability among the element θ. It can also be expressed by the following formula:

$$Pl(\theta) = 1 - Bel(\bar{\theta}) = \sum_{X \cap \theta \neq \phi} m(X). \tag{3}$$

Plausibility function $Pl(\theta)$ shows the total degree of trust in not denying the subset θ.

Definition 5 The same evidence frame of discernment Θ could generate different BPA functions from different sensors. Let $m'(\cdot)$ and $m''(\cdot)$ respectively denote BPA functions from two different sensors, θ' and θ'' denote focal elements existing in two sensors, so the Dempster combination rule could be defined as

$$\begin{cases} m(\theta) = \dfrac{\sum_{\theta' \cap \theta'' = \theta} m'(\theta')m''(\theta'')}{1 - \sum_{\theta' \cap \theta'' = \phi} m'(\theta')m''(\theta'')}, \theta \neq \phi; \\ m(\phi) = 0. \end{cases} \tag{4}$$

Let $S = (A, C, E)$ be a group MCDM problem, where $A = \{a_i \mid i = 1, \cdots, I\}$ is a non-empty finite set of decision alternatives, $C = \{c_j \mid j = 1, \cdots, J\}$ is a non-empty finite set of decision criteria, $D = \{d_k \mid k = 1, \cdots, K\}$ is a non-empty finite set of decision makers. Generally, individual knowledge on each criterion is different and each criterion has distinct dominance to decision goals, so let priority values associated with C and D respectively be denoted by $W = \{w_k^{(j)} \mid j = 1, \cdots, J, k = 1, \cdots, K\}$ and $R = \{r_k \mid k = 1, \cdots, K\}$. With above definitions and notions, the key thoughts of DS/AHP can be describe as below[3].

Firstly, the decision group is informed there is no discussion between the group members during their judgment making. For a single member the first stage of decision process is the gauging of the level of knowledge they have to each criterion, and determine the associated criteria priority values (CPVs) , i.e., $W = \{w_k^{(j)} \mid \forall j, \forall k\}$.

Secondly, each decision maker is asked to extract the initial focal elements with reference to Θ on a particular criterion which should not include the same decision alternatives in more than one identified group of decision alternatives and different levels of preference should be given to the identified groups of decision alternatives. A seven-scale unit with the scale values range from "moderately preferred" up to "extremely preferred" is available when discerning levels of preference on identified groups of decision alternatives.

Thirdly, for each criterion all of focal elements are identified and given the scale values respectively by every member in decision group, where no attempt is made to include any preference values from the direct comparison between identified groups of decision alternatives. All of knowledge matrices for each criterion are established by every group member. After that, the BPA function associated with the criterion could be easily computed by considering its associated CPV.

Fourthly, the criterion BPA function associated with the judgments made by a particular decision maker over different criteria should be combined to construct a BPA function representing overall opinion by Dempster combination rule as shown in (4).

Finally, overall BPA functions associated with each decision maker are combined by also utilizing Dempster combination rule, in which the effect of a discount rate is considered for individual BPA functions via member priority values (MPVs). With the final BPA function, the belief function and the plausibility function are calculated and the optimal choice is able to be made accordingly.

III. AMBIDEXTROUS DECISION INFORMATION

When group members are presented with a number of elements (criteria or alternatives) which have to be ranked with respect to a preference scale, it is assumed that they can compare each pair of elements and provide an ordinal preference judgment whether an element is preferred to another one or both elements are equally preferred. In pairwise comparison prioritization process, it is also assumed that group members are able to express the strength of their preferences by providing additional cardinal information[14].

Pairwise comparison as a crucial part of AHP provides a comprehensive and rational framework to structure a decision problem. In traditional AHP, pairwise judgments are structured in a pairwise comparison matrix and a prioritization procedure is applied to derive its priorities. If comparison judgments are cardinally consistent then the constructed pairwise comparison matrix is also consistent and all prioritization methods give the same result. However, as decision makers are often biased in their subjective comparisons, some level of inconsistency of their preference judgments may exist.

In traditional AHP, multiplicative preference relation is employed to perform pairwise comparison and construct pairwise comparison matrix. In this kind of pairwise comparison, the preference of d_k on A with respect to c_j is described by a positive matrix, i.e.,

$$E_k^{(j)} = (e_{st}^{(k,j)})_{I\times I} = \begin{bmatrix} 1 & e_{12}^{(j,k)} & \cdots & e_{1I}^{(j,k)} \\ 1/e_{12}^{(j,k)} & 1 & \cdots & e_{2I}^{(j,k)} \\ \vdots & \vdots & \ddots & \vdots \\ 1/e_{1I}^{(j,k)} & 1/e_{2I}^{(j,k)} & \cdots & 1 \end{bmatrix}, \quad (5)$$

where $e_{st}^{(j,k)}$ denotes a relative priority of a_s with respect to a_t. The element $e_{st}^{(j,k)}$ is measured by a ratio scale, and $e_{st}^{(j,k)} \in \{1/9, 1/8, \cdots, 1/2, 1, 2, \cdots, 8, 9\}$. $e_{st}^{(j,k)} = 1$ ($e_{st}^{(j,k)} = 1$) denotes there's indifference between a_s and a_t, $e_{st}^{(j,k)} = 9$ or $e_{st}^{(j,k)} = 1/9$ denotes that a_s is unanimously preferred to a_t, and $e_{st}^{(j,k)} \in \{2, \cdots, 8\}$ or $e_{st}^{(j,k)} \in \{1/2, \cdots, 1/8\}$ denotes intermediate evaluations. The measurement above is multiplicative reciprocal, i.e., $e_{st}^{(j,k)} e_{ts}^{(j,k)} = 1$, $\forall i, \forall j$; $e_{ss}^{(j,k)} = 1$, $\forall s$. The consistence in multiplicative preference relations is defined as a strong condition for preference transitivity, i.e., $E_k^{(j)} = (e_{st}^{(j,k)})_{I\times I}$ is perfectly consistent if

$$e_{sz}^{(j,k)} e_{zt}^{(j,k)} = e_{st}^{(j,k)}, \ \forall s, \forall t, \forall z. \quad (6)$$

Pairwise comparison is also carried out by utilizing fuzzy theory in the field of decision making. Fuzzy theory is very helpful in dealing with fuzziness of human judgment quantitatively, and a number of results have been published. Its applications to group decision making also have been made by many researchers. Fuzzy preference relations as a dominant way to order individual preferences has been widely executed and obtained lots of interesting results especially in group decision making[15]. In this kind of pairwise comparison, the preference of d_k on A with respect to c_j is described by a matrix, i.e.,

$$F_k^{(j)} = (f_{st}^{(j,k)})_{I\times I} = \begin{bmatrix} 0.5 & f_{12}^{(j,k)} & \cdots & f_{1I}^{(j,k)} \\ 1-f_{12}^{(j,k)} & 0.5 & \cdots & f_{2I}^{(j,k)} \\ \vdots & \vdots & \ddots & \vdots \\ 1-f_{1I}^{(j,k)} & 1-f_{2I}^{(j,k)} & \cdots & 0.5 \end{bmatrix}. \quad (7)$$

$F_k^{(j)}$ is with membership function $\mu_k^{(j)}: A \times A \to [0,1]$, where $\mu_k^{(j)}(a_s, a_t) = f_{st}^{(j,k)}$ denotes the preference degree of a_s over a_t; $f_{st}^{(j,k)} = 0.5$ or $f_{ts}^{(j,k)} = 0.5$ denotes there is indifference between a_s and a_t, $f_{st}^{(j,k)} = 1$ or $f_{ts}^{(j,k)} = 0$ denotes a_s is unanimously preferred to a_t, $0.5 < f_{st}^{(j,k)} < 1$ or $0 < f_{ts}^{(j,k)} < 0.5$ denotes a_s is preferred to a_t. Above measurement is assumed as additive reciprocal, i.e., $f_{st}^{(j,k)} + f_{ts}^{(j,k)} = 1$, $\forall s, \forall t$; $f_{ss}^{(j,k)} = 0.5$, $\forall s$. The definition of consistency is proposed on fuzzy preference relations, i.e., $F_k^{(j)} = (f_{st}^{(j,k)})_{I\times I}$ is perfectly consistent if

$$f_{st}^{(j,k)} f_{zt}^{(j,k)} f_{ts}^{(j,k)} = f_{zs}^{(j,k)} f_{tz}^{(j,k)} f_{st}^{(j,k)}, \ \forall s, \forall t, \forall z. \quad (8)$$

Multiplicative preference relations and fuzzy preference relations as above brief description both have been applied in many areas. However, it is difficult to extract complete preferences from decision makers because of their time pressure or limited expertise associated with problem domain. Thus, it is natural that decision makers may provide their preference with incomplete information in practical problems. The DS/AHP is a significant method for solving MCDM problems in multiplicative preference relation with incomplete information, but it is unable to solve the problems in fuzzy preference relation with incomplete information and the problems that the double types of preference information existing simultaneously.

IV. BASIC MODEL FOR DERIVING BPA FUNCTION

In DS/AHP method, each group member is asked to extract initial focal elements with reference to Q on every criterion and judge different preference levels for the identified groups of decision alternatives separately. Decision information originating from each one on a particular criterion as mentioned above could be denoted by a knowledge matrix $G_k^{(j)} = (g_{st}^{(j,k)})_{I_k^{(j)} \times I_k^{(j)}}$, i.e.,

$$G_k^{(j)}=\begin{bmatrix} 1 & 0 & \cdots & 0 & g_{1I_k^{(j)}}^{(j,k)} \\ 0 & 1 & \ddots & \vdots & g_{2I_k^{(j)}}^{(j,k)} \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & 1 & g_{(I_k^{(j)}-1)I_k^{(j)}}^{(j,k)} \\ 1/g_{1I_k^{(j)}}^{(j,k)} & 1/g_{2I_k^{(j)}}^{(j,k)} & \cdots & 1/g_{(I_k^{(j)}-1)I_k^{(j)}}^{(j,k)} & 1 \end{bmatrix},\ \forall j, \forall k. \quad (9)$$

In (9), all of elements in $G_k^{(j)}$ are judged by d_k on c_j; $g_{st}^{(j,k)}$ is an evaluated value that a focal element with reference to Θ, described by multiplicative preference relation; $I_k^{(j)}-1$ is the amount of focal elements identified by d_k on c_j whose intersection are empty. Note that, $g_{st}^{(j,k)}=0$ ($s,t \neq I_k^{(j)}$, $s \neq t$) for the reason that any preference values from the direct comparison between identified groups of decision alternatives are not allowed to judge.

Similarly, as the principles of constructing knowledge matrices in DS/AHP, another pairwise comparison with fuzzy preference relations could be introduced to establish knowledge matrices too. Suppose that $h_{st}^{(j,k)}$ is an evaluated value with fuzzy preference relation judged by d_k on c_j lying in [0,1], so a knowledge matrix with fuzzy preference relations $H_k^{(j)}=(h_{st}^{(j,k)})_{I_k^{(j)}\times I_k^{(j)}}$ judged by d_k on c_j is able to be constructed as below.

$$H_k^{(j)}=\begin{bmatrix} 0.5 & 0 & \cdots & 0 & h_{1I_k^{(j)}}^{(j,k)} \\ 0 & 0.5 & \ddots & \vdots & h_{2I_k^{(j)}}^{(j,k)} \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & 0.5 & h_{I_k^{(j)}I_k^{(j)}}^{(j,k)} \\ h_{1I_k^{(j)}}^{(j,k)} & 1-h_{2I_k^{(j)}}^{(j,k)} & \cdots & 1-h_{I_k^{(j)}I_k^{(j)}}^{(j,k)} & 0.5 \end{bmatrix},\ \forall j, \forall k. \quad (10)$$

It should be emphasized that above double kinds of knowledge matrices ($G_k^{(j)}$ and $H_k^{(j)}$) are both employed to extract individual decision information from group members, but each of them is suitable for different situation. If group members are used to give judgments for pairwise comparison with multiplicative preference relations, then $G_k^{(j)}$ is adopt; else if group members are used to give judgments for that with fuzzy preference relations, then $H_k^{(j)}$ is adopt.

DS/AHP uses the right eigenvector method to obtain BPA functions associated with knowledge matrix $G_k^{(j)}$. That is, BPA functions are the normalized elements of the eigenvector associated with the largest eigenvalue from the knowledge matrix. However this way is unadapt to compute BPA functions associated with $H_k^{(j)}$. To simplify above problems and give a coherent integration method, the knowledge matrices with multiplicative preference relations $G_k^{(j)}$ are able to be transformed into the knowledge matrices with fuzzy preference relations $H_k^{(j)}$ by utilizing below equation.

$$h_{st}^{(j,k)} = g_{st}^{(j,k)}\Big/\left(1+g_{st}^{(j,k)}\right),\ \forall s, \forall t. \quad (11)$$

Above transforming equation is actually a theorem which has been proved[16]. Now the problems are translated into how to obtain BPA functions from knowledge matrices with fuzzy preference relations.

With respect to $H_k^{(j)}$, suppose that its corresponding BPA functions is a variable cluster of $m_k^{(j)}(t)$ that associated with focal elements ($t=1,\cdots,I_k^{(j)}-1$) and the frame of discernment Θ ($t=I_k^{(j)}$), and satisfies

$$m_k^{(j)}(t) \geq 0,\ \forall t;\ \sum_{t=1}^{I_k^{(j)}} m_k^{(j)}(t) = 1. \quad (12)$$

Considering inconsistency characteristics existing in actual preference judgments and definitions of fuzzy preference relations, we could construct below equation.

$$h_{st}^{(j,k)} \approx \frac{m_k^{(j)}(t)}{m_k^{(j)}(s)+m_k^{(j)}(t)},\ s=I_k^{(j)} \text{ or } t=I_k^{(j)}. \quad (13)$$

Based on (13), the deviation degree between $h_{st}^{(j,k)}$ and $m_k^{(j)}(t)\big/(m_k^{(j)}(s)+m_k^{(j)}(t))$ are given by following (14).

$$d_{st}^{(j,k)} = m_k^{(j)}(t) - h_{st}^{(j,k)}\left(m_k^{(j)}(s)+m_k^{(j)}(t)\right),\ s=I_k^{(j)} \text{ or } t=I_k^{(j)} \quad (14)$$

From the view of satisfying the overall consensus, the smaller is the value of total deviation degree, the better is BPA functions. Thus, we construct a multiple objective constrained optimization model as below.

$$\min\ d^{(j,k)} = \sum_{s=I_k^{(j)}}^{I_k^{(j)}} \sum_{t=1}^{I_k^{(j)}} \left| m_k^{(j)}(t) - h_{st}^{(j,k)}\left(m_k^{(j)}(s)+m_k^{(j)}(t)\right) \right| + \sum_{s=1}^{I_k^{(j)}} \sum_{t=I_k^{(j)}}^{I_k^{(j)}} \left| m_k^{(j)}(t) - h_{st}^{(j,k)}\left(m_k^{(j)}(s)+m_k^{(j)}(t)\right) \right| \quad (15)$$

$$s.t.\ \sum_{t=1}^{I_k^{(j)}} m_k^{(j)}(t) = 1;\ m_k^{(j)}(t) \geq 0,\ \forall t.$$

Since $d^{(j,k)}$ is a continuous function on the bounded convex polyhedron and has the lower boundary point (i.e., 0), (15) has an optimal solution. Consequently, above problem (15) can be transformed into the following linear goal programming problem.

$$\min\ d^{(j,k)} = \sum_{s=I_k^{(j)}}^{I_k^{(j)}} \sum_{t=1}^{I_k^{(j)}} (e_{st}^{+}+e_{st}^{-}) + \sum_{s=1}^{I_k^{(j)}} \sum_{t=I_k^{(j)}}^{I_k^{(j)}} (e_{st}^{+}+e_{st}^{-}) \quad (16)$$

$$s.t. \begin{cases} \sum_{s=I_k^{(j)}}^{I_k^{(j)}} \sum_{t=1}^{I_k^{(j)}} \left\{ m_k^{(j)}(t) - h_{st}^{(j,k)}\left(m_k^{(j)}(s)+m_k^{(j)}(t)\right) - e_{st}^{+} + e_{st}^{-} \right\} + \\ \sum_{s=1}^{I_k^{(j)}} \sum_{t=I_k^{(j)}}^{I_k^{(j)}} \left\{ m_k^{(j)}(t) - h_{st}^{(j,k)}\left(m_k^{(j)}(s)+m_k^{(j)}(t)\right) - e_{st}^{+} + e_{st}^{-} \right\} = 0; \\ \sum_{t=1}^{I_k^{(j)}} m_k^{(j)}(t) = 1;\ m_k^{(j)}(t) \geq 0,\ \forall t; \\ e_{st}^{+} e_{st}^{-} = 0;\ e_{st}^{+} \geq 0,\ e_{st}^{-} \geq 0,\ \forall s, \forall t. \end{cases}$$

The problem (16) can be solved by utilizing the existing goal programming methods[17], and its optimal solution is the BPA functions.

V. INTEGRATED METHOD

An integrated method for DS/AHP is able to be reasonably established by the ambidextrous decision information and the basic model for deriving BPA functions. The procedure of integrated method is shown as Fig.1.

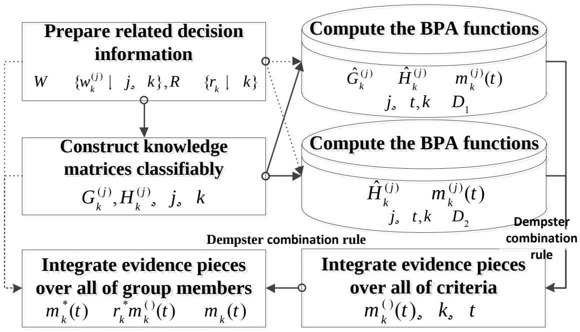

Fig 1. Procedure of integrated method

Step 1 Prepare related decision information. For the decision problem, evaluation criteria should be analyzed and a group of decision makers also should be employed. After that, the CPVs $W = \{w_k^{(j)} \mid \forall j, \forall k\}$ and the MPVs $R = \{r_k \mid \forall k\}$ need to be determined by utilizing weight deriving methods such as AHP, Delphi, and et al. Besides, group members need to be classified into couple subgroups in terms of their preferences for giving decision information ($D = D_1 \cup D_2$), one is suitable to multiplicative preference relations ($k \in D_1$), and the other is suitable to fuzzy preference relations ($k \in D_2$).

Step 2 Construct knowledge matrices classifiably. On a particular criterion, members in different subgroup are all asked to extract initial focal elements with reference to Θ which should not include the same alternatives in more than one identified group of alternatives and different levels of preference should be given to the identified groups of alternatives either by multiplicative preference relations or by fuzzy preference relations. Assemble above achieved information and establish all of knowledge matrices for each criterion associated with every group member, i.e., $G_k^{(j)}$ and $H_k^{(j)}$, $\forall k, \forall j$.

Step 3 Compute the BPA functions. Owning to each group member has distinct level of knowledge to each criterion, the BPA functions associated with each one should be combined with MPV information. Obviously, (9) and (10) do not consider MPVs, resulting in (15) and (16) cannot reflect distinct knowledge levels of group members. Actually (15) and (16) are the simplistic forms to solve all of group members have complete decision information on each criterion. In order to fuse the MPV information, knowledge matrices ($G_k^{(j)}$ and $H_k^{(j)}$, $\forall j, \forall k$) should be transformed by following (17) and (18) as the way used in traditional DS/AHP method.

$$\hat{G}_k^{(j)} = \begin{bmatrix} 1 & 0 & \cdots & 0 & w_k^{(j)} g_{1I_k^{(j)}}^{(j,k)} \\ 0 & 1 & \ddots & \vdots & w_k^{(j)} g_{2I_k^{(j)}}^{(j,k)} \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & 1 & w_k^{(j)} g_{(I_k^{(j)}-1)I_k^{(j)}}^{(j,k)} \\ 1/(w_k^{(j)} g_{1I_k^{(j)}}^{(j,k)}) & 1/(w_k^{(j)} g_{2I_k^{(j)}}^{(j,k)}) & \cdots & 1/(w_k^{(j)} g_{(I_k^{(j)}-1)I_k^{(j)}}^{(j,k)}) & 1 \end{bmatrix} \quad (17)$$

$$\hat{H}_k^{(j)} = \begin{bmatrix} 0.5 & 0 & \cdots & 0 & w_k^{(j)} h_{1I_k^{(j)}}^{(j,k)} \\ 0 & 0.5 & \ddots & \vdots & w_k^{(j)} h_{2I_k^{(j)}}^{(j,k)} \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & 0.5 & w_k^{(j)} h_{(I_k^{(j)}-1)I_k^{(j)}}^{(j,k)} \\ w_k^{(j)} h_{1I_k^{(j)}}^{(j,k)} & 1-w_k^{(j)} h_{2I_k^{(j)}}^{(j,k)} & \cdots & 1-w_k^{(j)} h_{(I_k^{(j)}-1)I_k^{(j)}}^{(j,k)} & 0.5 \end{bmatrix} \quad (18)$$

The knowledge matrices with multiplicative preference relations $\hat{G}_k^{(j)}$ ($k \in D_1$) should be transformed into that with fuzzy preference relations $\hat{H}_k^{(j)}$ by utilizing (11), and its BPA functions ($m_k^{(j)}(t)$, $\forall j, \forall t, \forall k$) resulting from $\hat{H}_k^{(j)}$ is able to be computed by (16).

Step 4 Integrate evidence pieces over all of criteria with respect to each group member. The BPA functions associated with the judgments made by a particular group member over different criteria are combined to construct a BPA function representing the overall opinion by Dempster combination rule as shown in (4). As a result, we can achieve each group member BPA function associated with all of criteria judgments, which is denoted by $m_k^{(\cdot)}(t)$, $\forall k, \forall t$.

Step 5 Integrate evidence pieces over all of group members. Following [3], a discount rate where their evidence is not as important as others is considered and all weights are adjusted by the largest weight amongst them, i.e., $r_k^* = r_k / \max(r_k, k = 1, \cdots, K)$. These values r_k^* ($\forall k$) are defined discount rates of group members from the most important one in group. So individual BPA function representing the preference judgments of the k^{th} member who is assigned discount rate w_k^* is given by

$$m_k^*(t) = r_k^* m_k^{(\cdot)}(t), \ \forall k, t \neq \Theta. \quad (19)$$

That is the adjusted BPA values are on all the focal elements except that assigned to ignorance (frame of discernment). The adjusted BPA functions defined by $m_k^{g}(t)$ for d_k is given by

$$\begin{cases} m_k^{g}(t) = \dfrac{m_k^*(t)}{\sum_{t \neq \Theta} m_k^*(t) + m_k^{(\cdot)}(\Theta)}, \ \forall k, t \neq \Theta; \\ m_k^{g}(\Theta) = \dfrac{m_k^{(\cdot)}(\Theta)}{\sum_{t \neq \Theta} m_k^*(t) + m_k^{(\cdot)}(\Theta)}, \ \forall k, t = \Theta. \end{cases} \quad (18)$$

The overall BPA functions associated with each group member are combined by also utilizing Dempster combination rule, and with the final BPA function the belief function and the plausibility function are calculated to assist in choosing the optimal alternative. As a general principle, the alternative with both the maximum belief function and the maximum plausibility function must be the optimal one.

VI. A NUMERICAL EXAMPLE

In order to illustrate the whole procedure of the proposed method, a numerical example about textbook choice that introduced in [3] is utilized. Ten available textbooks should be evaluated over three different criteria by four decision makers. The textbooks could be seen as decision alternatives which labeled $a_1, \cdots, a_{10}$, the criteria are denoted by c_1, c_2, c_3, group members are denoted by

$d_1,\ldots,d_4$. Decision information used in this example is shown as Fig.2.

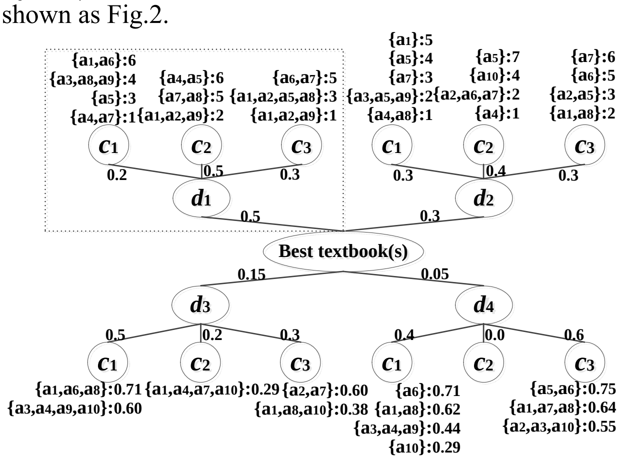

Fig 2. "Best textbook(s)" decision problem

As shown in Fig.2, the traditional DS/AHP model includes the judgments of all group members and is presented in a hierarchical structure. d_1 and d_2 give the information with multiplicative preference relations ($d_1,d_2 \in D_1$), and d_3 and d_4 give the information with fuzzy preference relations ($d_3,d_4 \in D_2$). From the focus of the problem 'Best textbook(s)' there is a group member level, where the hierarchy then partitions into the judgments made by the individuals. Since the decision information transformation and the BPA function computation are central issues in this paper, we first briefly exposit the procedure from knowledge matrix to BPA functions on the judgments made by individual d_1 (the area encircled with a dashed line in Fig. 2).

With respect to d_1 , it's corresponding CPVs are respectively $w_1^{(1)} = 0.2$, $w_1^{(2)} = 0.5$ and $w_1^{(3)} = 0.3$. On criterion c_1 the individual d_1 has identified four distinct groups of alternatives. These groups $\{a_1,a_6\}$, $\{a_3,a_8,a_9\}$, $\{a_5\}$ and $\{a_4,a_7\}$ have been assigned the scale values 6,4, 3 and 1. As a result, the knowledge matrix with multiplicative preference relations extracted by d_1 on c_1 is constructed as (19). $G_1^{(1)}$ should be integrated with its MPV information ($w_1^{(1)} = 0.2$) by (17) and transformed into a knowledge matrix with fuzzy preference relations $\hat{H}_1^{(1)}$ by (11). $\hat{H}_1^{(1)}$ is constructed as (20).

$$G_1^{(1)} = \begin{bmatrix} 1 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 \\ 1/6 & 1/4 & 1/3 & 1/1 & 1 \end{bmatrix} \quad (19)$$

$$\hat{H}_1^{(1)} = \begin{bmatrix} 0.50 & 0.00 & 0.00 & 0.00 & 0.55 \\ 0.00 & 0.50 & 0.00 & 0.00 & 0.44 \\ 0.00 & 0.00 & 0.50 & 0.00 & 0.38 \\ 0.00 & 0.00 & 0.00 & 0.50 & 0.17 \\ 0.45 & 0.56 & 0.63 & 0.83 & 0.50 \end{bmatrix} \quad (20)$$

Similarly, $\hat{H}_1^{(2)}$ and $\hat{H}_1^{(3)}$ extracted by d_1 on c_2 and c_3 and integrated with their MPVs, knowledge matrices with fuzzy preference relations, are constructed as below.

$$\hat{H}_1^{(2)} = \begin{bmatrix} 0.50 & 0.00 & 0.00 & 0.00 & 0.55 \\ 0.00 & 0.50 & 0.00 & 0.00 & 0.44 \\ 0.00 & 0.00 & 0.50 & 0.00 & 0.38 \\ 0.00 & 0.00 & 0.00 & 0.50 & 0.17 \\ 0.45 & 0.56 & 0.63 & 0.83 & 0.50 \end{bmatrix} \quad (21)$$

$$\hat{H}_1^{(3)} = \begin{bmatrix} 0.50 & 0.00 & 0.00 & 0.00 & 0.55 \\ 0.00 & 0.50 & 0.00 & 0.00 & 0.44 \\ 0.00 & 0.00 & 0.50 & 0.00 & 0.38 \\ 0.00 & 0.00 & 0.00 & 0.50 & 0.17 \\ 0.45 & 0.56 & 0.63 & 0.83 & 0.50 \end{bmatrix} \quad (22)$$

For d_1 , the BPA functions representing exact belief levels in the preferences to the identified focal elements on each criterion are able to be calculated by (16), and their corresponding evidence pieces over all of criteria could be integrated by Dempster combination rule as shown in (4). Above derived BPA functions is as below.

$$\begin{cases} m_1^{(1)}(a_5) = 0.171, m_1^{(1)}(a_1,a_6) = 0.085, m_1^{(1)}(a_4,a_7) = 0.513, \\ m_1^{(1)}(a_3,a_8,a_9) = 0.128, m_1^{(1)}(\Theta) = 0.103 \end{cases}$$

$$\begin{cases} m_1^{(2)}(a_4,a_5) = 0.122, m_1^{(2)}(a_7,a_{10}) = 0.146, m_1^{(2)}(\Theta) = 0.366, \\ m_1^{(2)}(a_1,a_2,a_9) = 0.366 \end{cases}$$

$$\begin{cases} m_1^{(3)}(a_{10}) = 0.545, m_1^{(3)}(a_6,a_7) = 0.109, m_1^{(3)}(a_1,a_2,a_5,a_8) = 0.182, \\ m_1^{(3)}(\Theta) = 0.164 \end{cases}$$

$$\begin{cases} m_1(a_1) = 0.072, m_1(a_4) = 0.045, m_1(a_1,a_6) = 0.022 \\ m_1(a_5) = 0.137, m_1(a_6) = 0.015, m_1(a_1,a_2) = 0.030 \\ m_1(a_8) = 0.038, m_1(a_9) = 0.034, m_1(a_4,a_5) = 0.009 \\ m_1(a_7) = 0.187, m_1(a_4,a_7) = 0.136, m_1(a_1,a_2,a_9) = 0.027 \\ m_1(a_{10}) = 0.127, m_1(a_7,a_{10}) = 0.011, m_1(a_3,a_8,a_9) = 0.034 \\ m_1(a_6,a_7) = 0.018, m_1(a_1,a_2,a_5,a_8) = 0.030, m_1(\Theta) = 0.027 \end{cases}$$

Similarly, we can achieve all of BPA functions associated with each group member on all of criteria judgments. As shown in step 5, a discount rate is integrated and the Dempster combination rule is also used to obtain overall BPA functions associated with decision group. Taking the final BPA functions into (2) and (3), the belief function and the plausibility function of each alternative are calculated as Table 1.

TABLE 1

BELIEF AND PLAUSIBILITY FUNCTIONS

	a_1	a_2	a_3	a_4	a_5
$Bel(q)$	0.494	0.001	0.001	0.074	0.000
$Pl(q)$	0.499	0.001	0.001	0.074	0.000
	a_6	a_7	a_8	a_9	a_{10}
$Bel(q)$	0.001	0.031	0.039	0.000	0.354
$Pl(q)$	0.001	0.031	0.043	0.000	0.355

With respect to the 'best textbook(s)' problem, the results in Table 1 identify a reduced number of textbooks that are considered best in terms of them being the adopted textbook(s) for a university course. It follows, if a single textbook is required the textbook a_1 is identified as the best, based on either the belief or plausibility values. Alternatively, a consideration set of more than one textbook could be identified from which further inspection and selection of those textbooks may be undertaken.

VII. CONCLUSIONS

This paper proposes an integrated method for DS/AHP under ambidextrous decision information, which is able to widen individual preference information extracting way and enhance practical values of DS/AHP method. Firstly some related basic notions in the DS/AHP method such as frame of discernment, basic probability assignment, etc. are described, and main thoughts existing in the DS/AHP method are introduced as well. After that multiplicative preference relations and fuzzy preference relations are reviewed as key extracting decision information way of pairwise comparison. With ambidextrous incomplete preference relations, a basic model for deriving BPA functions is constructed by means of minimizing the value of total deviation degree. An integrated method for DS/AHP is established based on the ambidextrous decision information and the basic model for deriving BPA functions by fusing CPV and MPV information. Finally a numerical example about textbook choice is utilized to illustrate the whole procedure of the proposed method. The proposed method is actually a development on extracting decision information in terms of the traditional DS/AHP thoughts, thus it is able to absorb incomplete information which is the inherent advantage of DS/AHP. Besides, group members have opportunities to select an appropriate information extracting way according to their preferences, as a result the decision qualities are able to be guaranteed. A linear goal programming is established to attach importance to efficient elements in knowledge matrices and neglect inefficient elements in the process of calculating BPA functions, resulting in the decision precision could be enhanced and the validity could be ensured too. It should be emphasized that, knowledge matrices with multiplicative preference relations are recommended to be transformed into that with fuzzy preference relations in this paper, aiming at setting up a new mode to solve BPA calculating problems reasonably. In above process, the transformation from fuzzy preference relations to multiplicative preference relations is also allowable, which induces the linear goal programming to extract BPA functions should to be adjusted in terms of its definition.

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REFERENCES

[1] J.B.Malcolm, B.Curry, P.Morgan, "The Dempster-Shafer theory of evidence: an alternative approach to multicriteria decision modelling", Omega, 2000, Vol.28, pp.37-50.
[2] J.B.Malcolm, "Understanding local ignorance and non-specificity within the DS/AHP method of multi-criteria decision making", European Journal of Operational Research, 2005, Vol.163, pp.403-417.
[3] J.B.Malcolm, "A method of aggregation in DS/AHP for group decision-making with the non-equivalent importance of individuals in the group", Computers & Operations Research, 2005, Vol.32, pp.1881-1896.
[4] Z.S.Hua, B.G.Gong, X.Y.Xu, "A DS–AHP approach for multi-attribute decision making problem with incomplete information", Expert Systems with Applications, 2008, Vol.34, pp.2221-2227.
[5] Y.Ju, A.Wang, "Emergency alternative evaluation under group decision makers: A method of incorporating DS/AHP with extended TOPSIS", Expert Systems with Applications, 2012, Vol.39, pp.1315-1323.
[6] Beynon M, "The Role of the DS/AHP in Identifying Inter-Group Alliances and Majority Rule Within Group Decision Making", Group Decision and Negotiation, 2006, Vol.15, pp.21-42.
[7] Y.X.Li, Y.Jiang, G.X.Zhao, "Personnel Selection Based on DS/AHP", Psychological Exploration, Vol.30, 2010, pp.72-77.
[8] C.Y.Liang, Z.M.Chen, Y.Ding, "The Application of DS/AHP Method in Vendor Selection", Operations Research and Management Science, Vol.14, 2005, pp.33-38.
[9] J. Han, M.L. Song. "Efficiency Evaluation Information System Based on Data Envelopment Analysis". Journal of Computers, Vol.6, 2011, pp.1857-1861.
[10] S.M. Yang, Q. Zhu, Z.B. Liu. "The Comprehensive Evaluation of New Energy Industry Developing Capability Based on Wavelet Neural Network Model". Journal of Computers, Vol.7, 2012, pp.439-443.
[11] Z.P.Fan, J.Ma, Y.P.Jiang, et al, "A goal programming approach to group decision making based on multiplicative preference relations and fuzzy preference relations", European Journal of Operational Research, 2006, Vol.174, pp.311-321.
[12] V.E.Herrera, S.Alonso, F.Chiclana, et al, "A consensus model for group decision making with incomplete fuzzy preference relations", Fuzzy Systems, IEEE Transactions on, 2007, Vol.15, pp.863-877.
[13] Z.P.Fan, Y.Zhang, "A goal programming approach to group decision-making with three formats of incomplete preference relations", Soft Computing, 2010, Vol.14, pp.1083-1090.
[14] T.L.Saaty, "Rank from comparisons and from ratings in the analytic hierarchy/network processes", European Journal of Operational Research, 2006, Vol.168, pp.557-570.
[15] Z.P.Fan, G.F.Hu, S.H.Xiao, "A method for multiple attribute decision-making with the fuzzy preference relation on alternatives", Computers & Industrial Engineering, 2004, Vol.46, pp.321-327.
[16] Z.S.Xu, J.Chen, "Some models for deriving the priority weights from interval fuzzy preference relations", European Journal of Operational Research, 2008, Vol.184, pp.266-280.
[17] M.J.Schniederjans, "Goal programming: methodology and applications", Springer, 1995.

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