Equivalence between Recursive and Analytical Evidential Reasoning Algorithms

Xinning Wang
School of Naval Architecture, Ocean and Civil Engineering, Shanghai JiaoTong University, Shanghai, China
Navy Submarine Academy, Qingdao, Shandong, China
Email: wxn_qd@163.com

***Abstract*—Due to having the explicit evidential reasoning (ER) aggregation function, the analytical ER algorithm has been extensively applied to decision problems under uncertainty. However, there are some different opinions to the validity of the analytical ER algorithm. In this paper, a new method is proposed for proving the equivalence between the recursive and analytical ER algorithms, in such a way that is different from and, it is believed, more rigorous than that of Wang et al (2006). The new method is based directly on Dempster-Shafer's combination rule and mathematics induction principle. It allows to consider simultaneously the combination and normalization of evidence. In addition, the iterative relationship of the normalization factors between two algorithms is derived. The paper further demonstrates the validity of the analytical ER algorithm theoretically and clarifies the relationship between the recursive and analytical ER algorithms.**

***Index Terms*—Dempster-Shafer theory; combination and normalization of evidence; multiple attribute decision analysis; evidential reasoning approach**

I. Introduction

The development of methods for dealing with uncertainty has received considerable attention in the last three decades and several numerical and symbolic methods have been proposed for handling uncertain information [29]. Due to the power of the D-S theory in handling uncertainties [1,5,6,9,32], so far, it has found wide applications in many areas such as expert systems [5,8], uncertainty reasoning [3,13,15], pattern classification [2,4,10,11], multiple attribute decision analysis [5,14,19-21,24-31], and regression analysis [16,20].

In the last decade, an evidential reasoning (ER) approach, which is a recursive style in nature, has been developed for multiple attribute decision analysis (MADA) under uncertainty [14,24,25,27,28]. This approach is developed on the basis of decision theory and Dempster-Shafer (D-S) theory of evidence [7,17]. Extensive research dedicated to the ER approach has been conducted in recent years. Firstly, the rule and utility-based information transformation techniques were proposed within the ER modeling framework [26]. This work enables the ER approach to deal with a wide range of MADA problems having precise data, random numbers and subjective judgments with probabilistic uncertainty in a way that is rational, transparent, reliable, systematic and consistent. Then, the in-depth research into the ER algorithm has been conducted by treating the unassigned belief degree in two parts, one caused by the incompleteness and the other caused by the fact that each attribute plays only one part in the whole assessment process because of its relative weight [27]. This work leads to a rigorous yet pragmatic ER algorithm that satisfies several common sense rules governing any approximate reasoning based aggregation procedures. The ER approach has thus been equipped with the desirable capability of generating the upper and lower bounds of the degree of belief for incomplete assessments, which are crucial to measure the degree of ignorance. Thirdly, the analysis process of the ER approach was fully investigated, which reveals the nonlinear features of the ER aggregation process [28]. Fourthly, the ER approach was further developed to deal with MADA problems with both probabilistic and fuzzy uncertainty [30]. This work leads to a new fuzzy ER algorithm that aggregates multiple attributes using the information contained in the fuzzy belief matrix, which can model precise data, ignorance and fuzziness under the unified framework of a distributed fuzzy belief structure. Fifthly, the ER approach was reanalyzed explicitly in terms of D-S theory and a general scheme of attribute aggregation was proposed for the purpose of dealing with MADA problems [14]. This work interprets the ER approach using the discounting operator and relaxes the constraints that the ER approach need to satisfy so that four synthesis axioms proposed by Yang and Xu in [27] can hold. Thus, this work provides convenience to develop new aggregation schemes. Recently, a new analytical ER algorithm was developed to deal with environmental impact assessment (EIA) problems [19]. In this work, the equivalence between the two algorithms was proven based on Yen's combination rule, which normalizes the combination of multiple piece of evidence at the end of the combination process. Due to having the explicit ER aggregation function, the analytical ER algorithm has been applied to decision problems extensively, such as environmental impact assessment (EIA) [19], constructing belief-rule-based systems [31], pipeline leak detection [23], bridge condition assessment [21], etc.

However, recently, Gao and Ni (2007) pointed out that Yen's combination rule was correct only under strict conditions but not under general conditions, and the analytical ER algorithm was incorrect because it was

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based on Yen's combination rule [12]. In a very recent reply by Wang [22], he re-examined the numerical illustrations provided in [12] and argued that the analytical ER algorithm is correct.

In this paper, we clarify the relationship between the recursive and analytical ER algorithms theoretically. Additionally, a different method is investigated to prove the equivalence between them. In the process of the proof, instead of using Yen's combination rule, we prove the equivalence between the recursive and analytical ER algorithms based directly on Dempster-Shafer's combination rule and mathematics induction principle, where the combination and normalization of evidence are considered simultaneously.

The rest of this paper is organized as follows. In Section 2, we present the background of this paper. Next, in Section 3, we present the proof of the equivalence between the recursive and analytical ER algorithms. Finally, we conclude the paper in section 4.

II. BACKGROUND

In [19], the recursive ER algorithm is represented as follows:

$$\{H_n\}: m_{n,I(i+1)} = K_{I(i+1)}(m_{n,I(i)}m_{n,i+1} + m_{n,I(i)}m_{H,i+1} + m_{H,I(i)}m_{n,i+1}), \quad (1)$$

$$m_{H,I(i)} = \bar{m}_{H,I(i)} + \tilde{m}_{H,I(i)}, i = 1,\ldots,L. \quad (2)$$

$$\{H\}: \tilde{m}_{H,I(i+1)} = K_{I(i+1)}(\tilde{m}_{H,I(i)}\tilde{m}_{H,i+1} + m_{H,I(i)}\bar{m}_{H,i+1} + \bar{m}_{H,I(i)}\tilde{m}_{H,i+1}), \quad (3)$$

$$\{H\}: \bar{m}_{H,I(i+1)} = K_{I(i+1)}(\bar{m}_{H,I(i)}\bar{m}_{H,i+1}), \quad (4)$$

$$K_{I(i+1)} = \left[1 - \sum_{n=1}^{N}\sum_{t=1,t\neq n}^{N} m_{n,I(i)}m_{t,i+1}\right]^{-1}. \quad (5)$$

$$\{H_n\}: \beta_n = \frac{m_{n,I(L)}}{1-\bar{m}_{H,I(L)}}, n = 1,\ldots,N, \quad (6)$$

$$\{H\}: \beta_H = \frac{\tilde{m}_{H,I(L)}}{1-\bar{m}_{H,I(L)}}. \quad (7)$$

where $m_{n,I(i)}$ denotes the combined probability mass generated by aggregating the first *i* attributes; $K_{I(i+1)}$ denotes the normalization factor of the recursive ER algorithm by the first $i+1$ attributes; $m_{n,I(i)}m_{n,i+1}$ measures the relative support to the hypothesis that the general attribute should be assessed to the grade H_n by both the first *i* attributes and the (i+1)th attribute; $m_{n,I(i)}m_{H,i+1}$ measures the relative support to the hypothesis by the first *i* attributes only; $m_{H,I(i)}m_{n,i+1}$ measures the relative support to the hypothesis by the (i+1)th attribute only. It is assumed in the above equations that $m_{n,I(1)} = m_{n,1}(n = 1,\ldots,N)$, $m_{H,I(1)} = m_{H,1}, \bar{m}_{H,I(1)} = \bar{m}_{H,1}$ and $\tilde{m}_{H,I(1)} = \tilde{m}_{H,1}$.

The analytical ER algorithm, which was proven to be equivalent to the recursive ER algorithm in [19], can be represented as follows:

$$\{H_n\}: m_n = k[\prod_{i=1}^{L}(m_{n,i} + \bar{m}_{H,i} + \tilde{m}_{H,i}) - \prod_{i=1}^{L}(\bar{m}_{H,i} + \tilde{m}_{H,i})], \quad (8)$$

$$\{H\}: \tilde{m}_H = k\left[\prod_{i=1}^{L}(\bar{m}_{H,i} + \tilde{m}_{H,i}) - \prod_{i=1}^{L}\bar{m}_{H,i}\right] \quad (9)$$

$$\{H\}: \bar{m}_H = k\prod_{i=1}^{L}\bar{m}_{H,i}, n = 1,\ldots N. \quad (10)$$

$$k = [\sum_{n=1}^{N}\prod_{i=1}^{L}(m_{n,i} + \bar{m}_{H,i} + \tilde{m}_{H,i}) - (N-1)\prod_{i=1}^{L}(\bar{m}_{H,i} + \tilde{m}_{H,i})]^{-1}, \quad (11)$$

$$\{H_n\}: \beta_n = \frac{m_n}{1-\bar{m}_H}, \quad (12)$$

$$\{H\}: \beta_H = \frac{\tilde{m}_H}{1-\bar{m}_H}, \quad (13)$$

where $m_{n,i}$ is the probability mass assigned to grade H_n, $m_{H,i}$ is the probability mass assigned to the whole set *H*, which is split into two parts: $\bar{m}_{H,i}$ and $\tilde{m}_{H,i}$, where $\bar{m}_{H,i}$ is caused by the relative importance of the attribute e_i and $\tilde{m}_{H,i}$ by the incompleteness of the assessment on e_i for a_l, k denotes the normalization factor for combining L pieces of evidence in the analytical ER algorithm.

Based on Yen's combination rule, where the normalization in the evidence combination rule of the Dempster-Shafer (D-S) theory of evidence can be applied at the end of the evidence combination process without changing the combination result [32], the equivalence between the recursive and analytical ER algorithms has been proven by Wang et al in [19]. That is

$$(m_1 \oplus m_2) \oplus m_3(E) = N[(m_1 \otimes m_2) \otimes m_3(E)](E \in \Theta), \quad (14)$$

where $\oplus$ and $\otimes$ denote Dempster-Shafer's combination rule with normalization and without normalization, respectively, N denotes the normalization process, Θ is the set of evaluation grades, m_1, m_2 and m_3 are the three basic probability assignment functions on the frame of discernment Θ and $m_1(A \mid A \in \Theta) \geq 0$, $m_1(B \mid B \in \Theta) \geq 0$, $m_1(C \mid C \in \Theta) \geq 0$.

In Wang's proof process, the normalization was applied after all factors were combined. That is, the combination and the normalization of evidence were separated from each other. However, Gao and Ni pointed out that Yen's combination rule was incorrect through two examples and further demonstrated that Yen's combination rule was correct under strict conditions [12]. The proof of the rule in [32] was revisited in [12]. The proof by Yen (1990) is examined as follows:

Suppose three basic probability assignment functions, m_1, m_2 and m_3 are to be combined. First, the result generated by combing m_1 and m_2 using Dempster's rule was calculated by

$$m_1 \oplus m_2(C) = \frac{m_{12}^{'}(C)}{1-K_{12}}, \quad (15)$$

where $m_{12}^{'}(C) = \sum_{A\cap B=C} m_1(A)m_2(B)$ and $K_{12} = \sum_{A\cap B=\varnothing} m_1(A)m_2(B)$. Therefore, Yen had

$$(m_1 \oplus m_2) \oplus m_3(E) = \frac{\frac{1}{1-K_{12}}\sum_{C\cap D=E} m_{12}^{'}(C)m_3(D)}{1-\frac{1}{1-K_{12}}\sum_{C\cap D=\varnothing} m_{12}^{'}(C)m_3(D)}. \quad (16)$$

Substituting $m_{12}^{'}(C)$ with $\sum_{A\cap B=C} m_1(A)m_2(B)$, he obtained

$$(m_1 \oplus m_2) \oplus m_3(E) = \frac{\frac{1}{1-K_{12}}\sum_{A\cap B\cap D=E} m_1(A)m_2(B)m_3(D)}{1-\frac{1}{1-K_{12}}\sum_{A\cap B\cap D=\varnothing, A\cap B\neq\varnothing} m_1(A)m_2(B)m_3(D)} \quad (17)$$

However, in [12], they pointed out that (16) was in general not equivalent to (17) because (16) was equivalent to

$$(m_1 \oplus m_2) \oplus m_3(E) = \frac{\frac{1}{1-K_{12}}\left[\sum_{A\cap B\cap D=E} m_1(A)m_2(B)m_3(D)\right]}{1-\frac{1}{1-K_{12}}\left[\sum_{A\cap B\cap D=\varnothing, A\cap B\neq\varnothing} m_1(A)m_2(B)m_3(D)\right]} \quad (18)$$

and (17) was equivalent to (18) only under the following assumptions

$$\sum_{A\cap B\cap D=E, A\cap B\neq C} m_1(A)m_2(B)m_3(D) = 0, \quad (19)$$

$$\sum_{A\cap B\cap D=\varnothing, A\cap B\neq C} m_1(A)m_2(B)m_3(D) = 0 \quad (20)$$

Based on the above analysis, Gao and Ni drawn two conclusions. The first was that Yen's combination rule was incorrect. The second was that the analytical ER algorithm was incorrect because it was based on Yen's combination rule. Yet, Wang re-examined the numerical illustrations provided in [12] and argued that the analytical ER algorithm is correct. The details can be found in [22].

However, we do not agree with the conclusion that the analytical ER algorithm is incorrect. What we can say is that to prove the equivalence between the recursive and analytical ER algorithms it is not necessary to use Yen's combination rule. We think it is not appropriate to say that: "the analytical ER algorithm is not correct because it is based on Yen's combination rule". This corresponds to say that: "the conclusion *X* is proven by incorrect method *Y*, so the conclusion *X* is incorrect". It is possible that the conclusion *X* is in fact correct whilst the method *Y* may be wrong (or not rational).

In the next section, instead of using Yen's combination rule, we apply Dempster-Shafer's combination rule and mathematics induction principle to prove the equivalence between the recursive and analytical ER algorithms, in which the combination and normalization of evidence are considered simultaneously.

III. The Proof Of The Equivalence Between The Recursive And Analytical Er Algorithms

In the following, for simplicity, we use $K_l, l = 1,\ldots,L$ to denote the normalization factor for combining l pieces of evidence in the analytical ER algorithm. Therefore, K_L is equal to k in (11).

First of all, let us combine two factors with normalization. The combined probability masses generated by aggregating the two factors using Dempster-Shafer's combination rule are given as follows.

$$\begin{aligned}
K_{I(2)} &= \left[1-\sum_{n=1}^{N}\sum_{t=1,t\neq n}^{N} m_{n,I(1)}m_{t,2}\right]^{-1} \\
&= \left[1-\sum_{n=1}^{N}\sum_{t=1,t\neq n}^{N} m_{n,1}m_{t,2}\right]^{-1} \\
&= \left[1-\sum_{n=1}^{N}\left(\sum_{t=1}^{N} m_{n,1}m_{t,2} - m_{n,1}m_{n,2}\right)\right]^{-1} \\
&= \left\{1-\sum_{n=1}^{N}\left[m_{n,1}\left(\sum_{t=1}^{N} m_{t,2} - m_{n,2}\right)\right]\right\}^{-1} \\
&= \left\{1-\sum_{n=1}^{N}\left[m_{n,1}(1-m_{H,2}-m_{n,2})\right]\right\}^{-1} \\
&= \Big[m_{H,1}(1-\sum_{n=1}^{N} m_{n,2}) + \\
&\quad \sum_{n=1}^{N}\prod_{i=1}^{2}(m_{n,i}+m_{H,i}) - Nm_{H,1}m_{H,2}\Big] \\
&= \Big[m_{H,1}m_{H,2} + \sum_{n=1}^{N}\prod_{i=1}^{2}(m_{n,i}+m_{H,i}) \\
&\quad -Nm_{H,1}m_{H,2}\Big]^{-1} \\
&= \Big[\sum_{n=1}^{N}\prod_{i=1}^{2}(m_{n,i}+m_{H,i}) - \\
&\quad (N-1)\prod_{i=1}^{2} m_{H,i}\Big]^{-1} \\
&= \Big[\sum_{n=1}^{N}\prod_{i=1}^{2}(m_{n,i}+\tilde{m}_{H,i}+\bar{m}_{H,i}) \\
&\quad -(N-1)\prod_{i=1}^{2}(\tilde{m}_{H,i}+\bar{m}_{H,i})\Big]^{-1} \\
&= K_2.
\end{aligned}$$

Then we have

$$m_{n,I(2)} = K_{I(2)}(m_{n,1}m_{n,2} + m_{n,1}m_{H,2} + m_{H,1}m_{n,2})$$
$$= K_{I(2)}\left[m_{n,1}(m_{n,2} + m_{H,2}) + m_{H,1}m_{n,2}\right]$$
$$= K_{I(2)}\left[m_{n,1}(m_{n,2} + m_{H,2}) + m_{H,1}\right.$$
$$\left.(m_{n,2} + m_{H,2}) - m_{H,1}m_{H,2}\right]$$
$$= K_{I(2)}\left[(m_{n,1} + m_{H,1})(m_{n,2} + m_{H,2})\right.$$
$$\left.-m_{H,1}m_{H,2}\right]$$
$$= K_{I(2)}\left[\prod_{i=1}^{2}(m_{n,i} + m_{H,i}) - \prod_{i=1}^{2}m_{H,i}\right]$$
$$= K_{I(2)}\left[\prod_{i=1}^{2}(m_{n,i} + \bar{m}_{H,i} + \tilde{m}_{H,i})\right.$$
$$\left.-\prod_{i=1}^{2}(\bar{m}_{H,i} + \tilde{m}_{H,i})\right]$$
$$= K_{2}\left[\prod_{i=1}^{2}(m_{n,i} + \bar{m}_{H,i} + \tilde{m}_{H,i})\right.$$
$$\left.-\prod_{i=1}^{2}(\bar{m}_{H,i} + \tilde{m}_{H,i})\right]$$
$$\tilde{m}_{H,I(2)} = K_{I(2)}(\tilde{m}_{H,1}\tilde{m}_{H,2} + \tilde{m}_{H,1}\bar{m}_{H,2} + \bar{m}_{H,1}\tilde{m}_{H,2})$$
$$= K_{2}\left[\tilde{m}_{H,1}(\bar{m}_{H,2} + \tilde{m}_{H,2}) + \bar{m}_{H,1}\tilde{m}_{H,2}\right]$$
$$= K_{2}\left[\tilde{m}_{H,1}(\bar{m}_{H,2} + \tilde{m}_{H,2}) + \bar{m}_{H,1}(\tilde{m}_{H,2} +\right.$$
$$\left.\bar{m}_{H,2}) - \bar{m}_{H,1}\bar{m}_{H,2}\right]$$
$$= K_{2}\left[(\tilde{m}_{H,2} + \bar{m}_{H,2})(\tilde{m}_{H,1} + \bar{m}_{H,1}) - \bar{m}_{H,1}\bar{m}_{H,2}\right]$$
$$= K_{2}\left[\prod_{i=1}^{2}(\tilde{m}_{H,i} + \bar{m}_{H,i}) - \prod_{i=1}^{2}\bar{m}_{H,i}\right].$$
$$\bar{m}_{H,I(2)} = K_{I(2)}\bar{m}_{H,1}\bar{m}_{H,2} = K_{2}\prod_{i=1}^{2}\bar{m}_{H,i}\ .$$

Suppose the following equations are true while combining the first $(l-1)$ evidence using Dempster-Shafer's combination rule, $3 \le l \le L$,

$$m_{n,I(l-1)} = K_{l-1}\left[\prod_{i=1}^{l-1}(m_{n,i} + \bar{m}_{H,i} + \tilde{m}_{H,i})\right.$$
$$\left.-\prod_{i=1}^{l-1}(\bar{m}_{H,i} + \tilde{m}_{H,i})\right].$$
$$\tilde{m}_{H,I(l-1)} = K_{l-1}\left[\prod_{i=1}^{l-1}(\tilde{m}_{H,i} + \bar{m}_{H,i}) - \prod_{i=1}^{l-1}\bar{m}_{H,i}\right],$$
$$\bar{m}_{H,I(l-1)} = K_{l-1}\prod_{i=1}^{l-1}\bar{m}_{H,i}\ .$$

where K_{l-1} denotes the normalization factor for combining $l-1$ pieces of evidence in the analytical ER algorithm.

Then the results combining $m_{I(l-1)}$ with the lth evidence m_l can be formulated as follows:

$$K_{I(l)} = \left[1 - \sum_{n=1}^{N}\sum_{t=1,t\neq n}^{N} m_{n,I(l-1)}m_{t,l}\right]^{-1}$$
$$= \left[1 - \sum_{n=1}^{N}(m_{n,I(l-1)}\sum_{t=1,t\neq n}^{N} m_{t,l})\right]^{-1}$$
$$= \left\{1 - \sum_{n=1}^{N}\left[m_{n,I(l-1)}\left(\sum_{t=1}^{N} m_{t,l} - m_{n,l}\right)\right]\right\}^{-1}$$
$$= \left\{1 - \sum_{n=1}^{N}\left[m_{n,I(l-1)}(1 - m_{H,l} - m_{n,l})\right]\right\}^{-1}$$
$$= \left[1 - \sum_{n=1}^{N} m_{n,I(l-1)} + m_{H,l}\sum_{n=1}^{N} m_{n,I(l-1)}\right.$$
$$\left.+\sum_{n=1}^{N} m_{n,I(l-1)}m_{n,l}\right]^{-1}$$
$$= \left(\frac{K_{l-1}}{K_l}\right)^{-1} = \frac{K_l}{K_{l-1}}$$

Note that $K_{I(l)}$ is not equal to K_l . The relationship of the normalization factors between them will be summarized in the following. We then have

$$m_{n,I(l)} = K_{I(l)}(m_{n,I(l-1)}m_{n,l} + m_{H,I(l-1)}m_{n,l} +$$
$$m_{H,l}m_{n,I(l-1)})$$
$$= K_{I(l)}\left[(m_{n,I(l-1)} + m_{H,I(l-1)})m_{n,l} +\right.$$
$$\left.m_{H,l}m_{n,I(l-1)}\right]$$
$$= K_{I(l)}\left[(m_{n,I(l-1)} + m_{H,I(l-1)})m_{n,l} +\right.$$
$$\left.m_{H,l}(m_{n,I(l-1)} + m_{H,I(l-1)}) - m_{H,I(l-1)}m_{H,l}\right]$$
$$= K_{I(l)}\left[(m_{n,I(l-1)} + m_{H,I(l-1)})(m_{n,l} +\right.$$
$$\left.m_{H,l}) - m_{H,I(l-1)}m_{H,l}\right]$$
$$= K_{I(l)}K_{l-1}\left[\prod_{i=1}^{l}(m_{n,i} + m_{H,i}) - \prod_{i=1}^{l}m_{H,i}\right]$$
$$= \frac{K_l}{K_{l-1}}K_{l-1}\left[\prod_{i=1}^{l}(m_{n,i} + m_{H,i}) - \prod_{i=1}^{l}m_{H,i}\right]$$
$$= K_l\left[\prod_{i=1}^{l}(m_{n,i} + m_{H,i}) - \prod_{i=1}^{l}m_{H,i}\right]$$
$$= K_l\left[\prod_{i=1}^{l}(m_{n,i} + \bar{m}_{H,i} + \tilde{m}_{H,i}) -\right.$$
$$\left.\prod_{i=1}^{l}(\bar{m}_{H,i} + \tilde{m}_{H,i})\right]$$

$$\begin{aligned}\tilde{m}_{H,I(l)} &= K_{I(l)}(\tilde{m}_{H,l}\bar{m}_{H,I(l-1)} + \tilde{m}_{H,l}\bar{m}_{H,I(l-1)} \\ &\quad +\bar{m}_{H,l}\tilde{m}_{H,I(l-1)}) \\ &= K_{I(l)}\left[\tilde{m}_{H,l}(\bar{m}_{H,I(l-1)} + \tilde{m}_{H,I(l-1)}) + \right. \\ &\quad \left. \bar{m}_{H,l}\tilde{m}_{H,I(l-1)}\right] \\ &= K_{I(l)}\left[\tilde{m}_{H,l}(\bar{m}_{H,I(l-1)} + \tilde{m}_{H,I(l-1)}) + \right. \\ &\quad \left. \bar{m}_{H,l}(\tilde{m}_{H,I(l-1)} + \bar{m}_{H,I(l-1)}) - \bar{m}_{H,l}\bar{m}_{H,I(l-1)}\right] \\ &= K_{I(l)}\left[(\tilde{m}_{H,I(l-1)} + \bar{m}_{H,I(l-1)})(\tilde{m}_{H,l} + \bar{m}_{H,l}) \right. \\ &\quad \left. -\bar{m}_{H,l}\bar{m}_{H,I(l-1)}\right] \\ &= K_l\left[\prod_{i=1}^{l}(\tilde{m}_{H,i} + \bar{m}_{H,i}) - \prod_{i=1}^{l}\bar{m}_{H,i}\right].\end{aligned}$$

$$\begin{aligned}\bar{m}_{H,I(l)} &= K_{I(l)}\bar{m}_{H,l}\bar{m}_{H,I(l-1)} \\ &= K_{I(l)}\bar{m}_{H,l}K_{l-1}\prod_{i=1}^{l-1}\bar{m}_{H,i} \\ &= K_{I(l)}K_{l-1}\prod_{i=1}^{l}\bar{m}_{H,i} \\ &= \frac{K_l}{K_{l-1}}K_{l-1}\prod_{i=1}^{l}\bar{m}_{H,i} \\ &= K_l\prod_{i=1}^{l}\bar{m}_{H,i}.\end{aligned}$$

Therefore, according to mathematical induction principle, the above equations are true for any $l \in \{1,\dots,L\}$. For $l=L$, we obtain the following normalized combined probability assignments generated by aggregating the L attributes:

$$\begin{aligned}m_{n,I(L)} &= K_L\left[\prod_{i=1}^{L}(m_{n,i} + \bar{m}_{H,i} + \tilde{m}_{H,i}) - \right. \\ &\quad \left. \prod_{i=1}^{L}(\bar{m}_{H,i} + \tilde{m}_{H,i})\right], n = 1,\dots,N\end{aligned},$$

$$\tilde{m}_{H,I(L)} = K_L\left[\prod_{i=1}^{L}(\tilde{m}_{H,i} + \bar{m}_{H,i}) - \prod_{i=1}^{L}\bar{m}_{H,i}\right]$$

$$\bar{m}_{H,I(L)} = K_L\prod_{i=1}^{L}\bar{m}_{H,i}.$$

According to the definition of the basic probability assignment function, we have

$$\begin{aligned}&\sum_{n=1}^{N}m(H_n) + m(H) \\ &= \sum_{n=1}^{N}m(H_n) + \bar{m}(H) + \tilde{m}(H) = 1\end{aligned},$$

from which we obtain

$$\begin{aligned}K_L &= \left[\sum_{n=1}^{N}\prod_{i=1}^{L}(m_{n,i} + \bar{m}_{H,i} + \tilde{m}_{H,i}) - \right. \\ &\quad \left. (N-1)\prod_{i=1}^{L}(\bar{m}_{H,i} + \tilde{m}_{H,i})\right]^{-1}\end{aligned}.$$

Furthermore, we can obtain the iterative relationship between $K_{I(l)}$ and K_l as follows:

$$K_{I(l)} = \begin{cases} K_l & l = 2 \\ K_l / K_{l-1} & 2 < l \le L \end{cases}$$

From the above analysis, it is shown that the recursive ER algorithm is equivalent to the analytical ER algorithm.

IV. CONCLUSIONS

In this paper, the equivalence between the recursive and analytical ER algorithms is re-investigated. In this investigation, a new method for proving the equivalence between the recursive and analytical ER algorithms has been proposed. Unlike previous approach described in [19], this method does not rely on Yen's combination rule, but on Dempster-Shafer's combination rule and mathematics induction principle, in which the combination and normalization of evidence can be considered simultaneously. Furthermore, the iterative relationship of the normalization factors between two algorithms has been derived. Hence, this method further consolidates the proof of the equivalence proposed by Wang et al in [19] and makes the analytical ER algorithm usable for conducting sensitivity analysis and optimization for the parameters of the ER algorithm, etc.

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Wang Xinning is a Ph.D candidate in the School of Naval Architecture, Ocean and Civil Engineering. His research interest covers intelligent algorithm, reliability and data fusion.

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