Fractional Order Correlation Algorithm's Accuracy Analysis

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***Abstract*—In order to improve the correlation accuracy of correlation algorithm, the fractional order correlation algorithm of uncertain time sequences is proposed in this paper. By taking advantage of the memory property of fractional order, the algorithm introduces the measurement of fractional order differential for the local trend of time sequence into the correlation algorithm and also analyzes the influences of differential order and noise upon correlation accuracy, provides selection relations between noise level and order. It has been proven with examples that the correlation accuracy of fractional order correlation algorithm has increased by two orders of magnitude as compared with relative and C-type correlation, and increased by one orders of magnitude as compared with slope and T-type correlation.**

***Index Terms*—fractional order, uncertainty, time series, association.**

I. Introduction

Time series is a set of chronological observations of a certain indicator. The fuzzy membership of the description information and the import of the error message caused series of the description uncertainty. The correlation analysis of the uncertain time series can help us to overcome the limitation of the cognitive ability and ultimately find out the internal relations of different objectives.

The application of correlation analysis of uncertain time sequences is mainly embodied in factor analysis, decision making and superiority analysis. Correlation analysis is widely applied in numerous fields such as economy, society, agriculture, mining, transportation, education, medical science, ecology, environment, water conservancy, hydrology, petroleum, geology and aviation, solving lots of practical problems unsolved in the past. Since small sample and little information are the objective state of correlation analysis in practical application, this paper will focus on the study of correlation model of uncertain time sequences with multiple small samples and little information[1,2].

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II. Review

A. Correlation Algorithm Review

Grey correlation analysis highlights development tendency while pays less attention to sample size and typical regularity of distribution. Therefore, grey correlation analysis has a great advantage for the correlation model of uncertain time sequences with multiple small samples and little information.

In recent years, gray correlation algorithm obtained a significant development, and many scholars have made great contributions[3-8]. From the relational degree itself, it experienced from the gray relational algorithm of no differential measurement information (such as Tang's correlation, the absolute correlation Ⅱ, the relative correlation, correlation interval Ⅰ, range correlation Ⅱ) to gray relational algorithm with first-order differential metrical information (such as absolute relational degree Ⅰ, slope correlation, and T-type correlation), and then turned to be the gray relational algorithm with second-order differential metrical information (B-type correlation, C-type correlation).

The abovementioned indicates that the introduction of the high order information and the fractional information into the associated metrics of the uncertainties series is the development trend of related algorithms.

B. Summary of Fractional Differential

Fractional calculus refers to the calculus with order of any real number order. For more than three centuries, many famous scientists did a lot of basic work on fractional calculus; however, fractional calculus really began to grow till the last 30 years. Oldham and Spanier[9] discussed mathematical calculations of the fractional number and their application in areas like physics, engineering, finance, biology, etc. In 1993, Samko made systematic and comprehensive exposition on fractional integral and derivative related properties and their applications. Many researchers have found that, fractional derivative model can more accurately describe

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the nature of a memory and the genetic material and the distribution process than integer order derivative model[10]. The overall and memory characteristic of fractional are widely used in physics, chemistry, materials, fractal theory[11], image processing[12] and other fields. Currently, the analysis of fractional differential has become a new active researched area that aroused great attention of domestic and foreign scholars, and turned to be the world's leading edge and hot research field.

III. Algorithm Principle

A. The Import of Fractional Order

Differential operations can enhance the high-frequency and weaken low-frequency of the signals. Fractional differential operation can nonlinearly improve more high-frequency and weaken less low-frequency of the signals with the growth of the order. From the perspective of the information extraction, the order of integer order operations is discrete whereas fractional one is continuous, and can provide more tracking correlation information to help the identification of the target tracking.

Each observed value on the tracking is the common result of variety of subjective and objective factors and the development of all previous observations; therefore tracking is of overall and memory characteristics. Fractional differential operator is intended differential operator with overall and memory characteristics whereas integral order doesn't have this feature. Therefore, from the description of the tracking it can conclude that, fractional differential could more accurately describe the memory nature of tracking comparing with the integral order one, and was imported to calculate the relevancy of the tracking. [13,14]

B. The Nature of Fractional Differential

Fractional differential operator can meet the exchange rate and the overlay standard

$$D^{v_1}D^{v_2}s(t)=D^{v_2}D^{v_1}s(t)=D^{v_1+v_2}s(t).$$

(0, 1) differential order measures the overall situation of the sequence, other differential order results can all be acquired through the iterate integer-order differential on it. First-order differential reflects the slope of the tracking, second-order differential reflects the curvature of the tracking, and they all response to the partial trends of the tracking. To give consideration to the measurements of both global and local trends, non-integral order emphasis on (0, 1) order, integer order taking into account of the first, second order, therefore, this paper is only analysis the (0, 3)-order differential related information.

C. Fractional Differential Difference Form

Since time series is discrete, when using the fractional differentials in it's associate calculation, the definition pattern of fractional differential algorithms must be change into the difference form. Then, we derive the fractional differential difference formula via Grümwald-Letnikov definition.

Known, v order fractional differential Grümwald-Letnikov definition is

$$ {}_{a}^{G}D_t^{v}s(t)=\lim_{h\to 0}s_t^{v}(t)=\lim_{\substack{h\to 0\\ nh\to t-a}}h^{-v}\sum_{r=0}^{n}C_r^{-v}s(t-rh) \tag{1}$$

Where in

$$C_r^{-v}=\frac{(-v)(-v+1)...(-v+r-1)}{r!}$$

According to Expression(1), if the persistent period of s(t) is: t ∈ [a ,t], divide [a ,t] into equal parts corresponding to one unit interval h=1, it can be got that

$$n=\left[\frac{t-a}{h}\right]^{h=1}=[t-a]$$

Then, v order fractional differential difference expression to unitary signal s(t) can be get

$$\frac{d^v s(t)}{dt^v}\approx s(t)+(-v)s(t-1)+\frac{(-v)(-v+1)}{2}s(t-2)$$
$$+\frac{(-v)(-v+1)(-v+2)}{6}s(t-3)+...$$
$$+\frac{\Gamma(-v+1)}{n!\Gamma(-v+n+1)}s(t-n)$$

From this differential expression, the difference coefficient of the fractional is

$$a_0=1,a_1=-v,a_2=\frac{(-v)(-v+1)}{2},$$
$$a_3=\frac{(-v)(-v+1)(-v+2)}{6},...,a_n=\frac{\Gamma(-v+1)}{n!\Gamma(-v+n+1)} \tag{2}$$

IV. Experiments of Existing Correlation Algorithm

A. Fractional Order Correlation Relative Correlation

Assume X_0 is the reference sequence, the degree of Relative correlation between $X_i, i=1,2,...,n$ and X_0 is,

$$r_{oi}=\frac{1+\left|s_0^{'}\right|+\left|s_i^{'}\right|}{1+\left|s_0^{'}\right|+\left|s_i^{'}\right|+\left|s_i^{'}-s_0^{'}\right|}$$

where:

$$\left|s_0^{'}\right|=\left|\sum_{k=2}^{n-1}y_0(k)+\frac{1}{2}y_0(n)\right|,$$

$$\left|s_i^{'}\right|=\left|\sum_{k=2}^{n-1}y_i(k)+\frac{1}{2}y_i(n)\right|,$$

$$\left|s_i^{'}-s_0^{'}\right|=\left|\sum_{k=2}^{n-1}(y_i(k)-y_0(k))+\frac{1}{2}(y_i(n)-y_0(n))\right|$$

$z_0(k)=\dfrac{x_0(k)}{x_0(1)}$, $z_i(k)=\dfrac{x_i(k)}{x_i(1)}$,

$y_0(k)=z_0(k)-z_0(1)$, $y_i(k)=z_i(k)-z_i(1)$,

$k=1,2,...,n$.

B. Slope Correlation

Assume X_0 is the reference sequence, the degree of Slope correlation between $X_i, i=1,2,...,n$ and X_0 is,

$$r(X_0,X_i)=\frac{1}{n-1}\sum_{t=1}^{n-1} r_i(t),$$

$$r_i(t)=\frac{1+\left|\dfrac{\Delta x_0(t)}{\overline{x_0}}\right|}{1+\left|\dfrac{\Delta x_0(t)}{\overline{x_0}}\right|+\left|\dfrac{\Delta x_0(t)}{\overline{x_0}}-\dfrac{\Delta x_i(t)}{\overline{x_i}}\right|},$$

$\overline{x_0}=\dfrac{1}{n}\sum_{t=1}^{n} x_0(t)$, $\Delta x_0(t)=x_0(t+1)-x_0(t)$,

$\overline{x_i}=\dfrac{1}{n}\sum_{t=1}^{n} x_i(t)$, $\Delta x_i(t)=x_i(t+1)-x_i(t)$

C. T-type Correlation

Assume X_0 is the reference sequence, the degree of T-type correlation between $X_i, i=1,2,...,n$ and X_0 is,

$$r(X_0,X_i)=\frac{1}{n-1}\sum_{k=1}^{n-1} r(k),$$

Where in,

$$r(k)=\begin{cases}\operatorname{sgn}(\Delta y_0(k)\cdot\Delta y_i(k))\cdot\dfrac{\min(\Delta y_0(k),\Delta y_i(k))}{\max(\Delta y_0(k),\Delta y_i(k))}\\ 0;\Delta y_0(k)\cdot\Delta y_i(k)=0\end{cases}$$

$\Delta y_0(k)=y_0(k+1)-y_0(k)$,

$\Delta y_i(k)=y_i(k+1)-y_i(k)$, $y_0(k)=\dfrac{x_0(k)}{D_0}$,

$y_i(k)=\dfrac{x_i(k)}{D_i}$ $D_0=\dfrac{1}{n-1}\sum_{k=2}^{n}\left|x_0(k+1)-x_0(k)\right|$,

$D_i=\dfrac{1}{n-1}\sum_{k=2}^{n}\left|x_i(k+1)-x_i(k)\right|$, $k=1,2,...,n$

D. C-type Correlation

Assume X_0 is the reference sequence, the degree of C-type correlation between $X_i, i=1,2,...,n$ and X_0 is,

$$r(X_0,X_i)=\frac{1}{n-2}\sum_{k=1}^{n-2} r_{0i}(k),$$

Where in, $r_{0i}(k)=[d_{0i}^{(0)}(k)+d_{0i}^{(1)}(k)+d_{0i}^{(i)}(k)]/3$,

$d_{0i}^{(0)}(k)=\dfrac{x_0(k)}{x_i(k)}$, $d_{0i}^{(1)}(k)=\dfrac{x_0(k+1)-x_0(k)}{x_i(k+1)-x_i(k)}$,

$$d_{0i}^{(2)}(k)=\frac{x_0(k+2)-2x_0(k+1)+x_0(k)}{x_i(k+2)-2x_i(k+1)+x_i(k)}$$

E. Experimental Analyses

Assume $X_1, X_2, ..., X_7$, $X_i=x_{i1}, x_{i2}, ..., x_{i26}$ are time sequences, where $x_{i,j}\in[0,1000]; i=1,2,...,7; j=1,2,...,26$, Assume X_1 is the reference sequence, see Figure1.

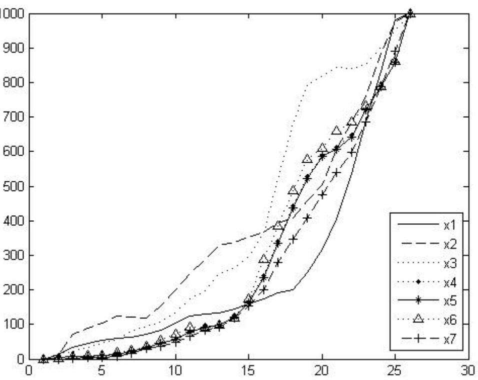

Figure 1. Sequences $X_1, X_2, ..., X_7$

Add noise with range of [-1, 1] into the reference sequences X_1 to get X_{11}; add noise with range of [-10, 10] into all the tracking to get X_{i2}, in which i=1,2,..., 7; add noise with range of [-100, 100] into all the tracking to get X_{i3}, in which i=1,2... 7. Calculate respectively the correlation between X_{i2} and X_{11}, X_{i3} and X_{11} (see figure 2, 3,4,5).

In the figure,rxhk stands for the correlation degree between xh and xk, in which r stands for correlation degree, h and k are the subscripts of the sequences generated after adding noise.

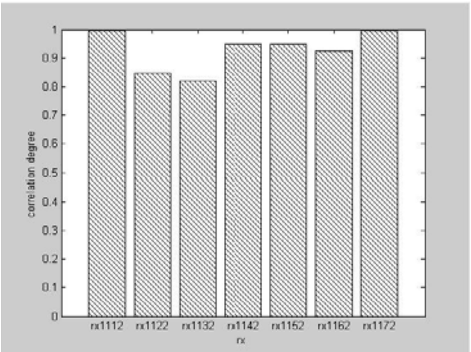

Figure 2. Relative Correlation between Sequences X_{i2} and X_{11}

It can be seen from figure 2 that the correlation degree of rx1112 is not obvious. Therefore, the conclusion can be drawn that the relative correlation is not accurate enough when noise signal amplitude ratio is 0.01.

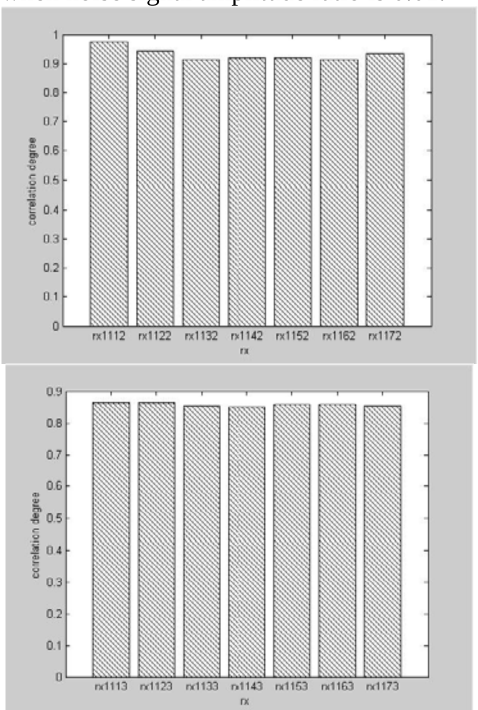

Figure 3. Slope Correlation between Sequences X_{i2} and X_{11}, X_{i3} and X_{11}

It can be seen from figure 3 that though the correlation value of rx1112 is greater than other correlation values, the correlation degree of rx1113 is not the greatest. Therefore, it can be said that when noise signal amplitude ratio is 0.01, slope correlation is accurate enough while when noise signal amplitude ratio is 0.1, it is far from accurate.

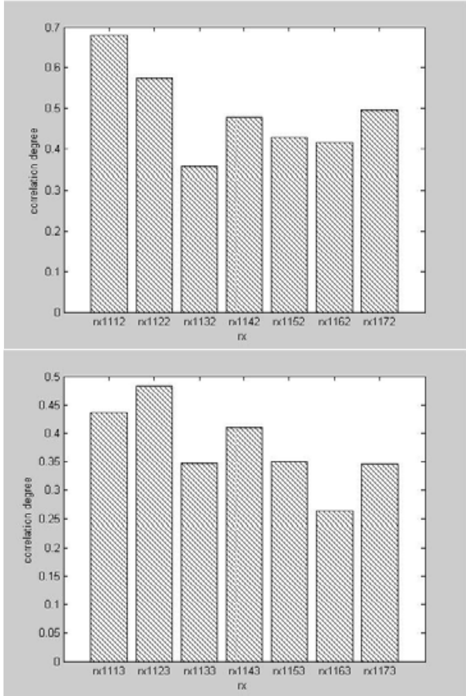

Figure 4. T-type Correlation between Sequences X_{i2} and X_{11}, X_{i3} and X_{11}

It is shown in figure 4 that though the correlation value of rx1112 is greater than other correlation values, the correlation degree of rx1113 does not reach the maximum. Therefore, the conclusion can be arrived at that when noise signal amplitude ratio is 0.01, T-type correlation is accurate enough while when noise signal amplitude ratio is 0.1, it is far from accurate.

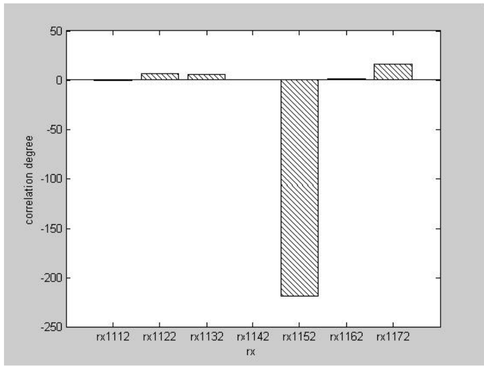

Figure 5. C-type Correlation between Sequences X_{i2} and X_{11}

It is shown in figure 5 that the correlation value of rx1112 is not the greatest. Therefore, it can be said that when noise signal amplitude ratio is 0.01, C-type correlation is not accurate enough.

F. Summary

It can be concluded from the analyses of figure 2, 3, 4 and 5 that relative correlation and C-type correlation are of poor correlation accuracy while T-type correlation and slope correlation can accurately correlate sequences whose noise signal amplitude ratio is 0.01.

V. Fractional Order Correlation Algorithm

A. Fractional Order Correlation

Assume X_0 is the reference sequence, the degree of fractional order correlation between $X_i, i=1,2,...,n$ and X_0 under v order is:

$$r(x_0(k), x_i(k), v) = \frac{\min\limits_i \min\limits_k \left| x^v{}_0(k) - x^v{}_i(k) \right| + 0.5 * \max\limits_i \max\limits_k \left| x^v{}_0(k) - x^v{}_i(k) \right|}{\left| x^v{}_0(k) - x^v{}_i(k) \right| + 0.5 * \max\limits_i \max\limits_k \left| x^v{}_0(k) - x^v{}_i(k) \right|}$$

$$r(X_0, X_i, v) = \frac{1}{n-5} \sum_{k=6}^{n} r(x_0(k), x_i(k), v), v \in (0,3)$$

Where in

$$x_i^v(k) = \sum_{d=j-5}^{j} x_i(d) * a_{(j-d+1)}; i = 0,1,...,n$$

Then calculate separately the curve of correlation degree (see figure 6,7) at the order of (0, 3) between X_{i2} and X_{11}, X_{i3} and X_{11}.

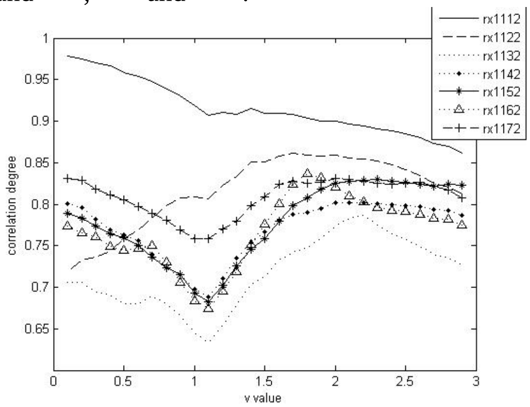

Figure 6. Fractional Order Correlation Curve of X_{i2} and X_{11}

From figure 6 it can be seen that the correlation value of rx1112 is far greater than the values of other curves when $v \in (0,3)$. This indicates that when noise signal amplitude ratio is 0.01, fractional order correlation is accurate enough.

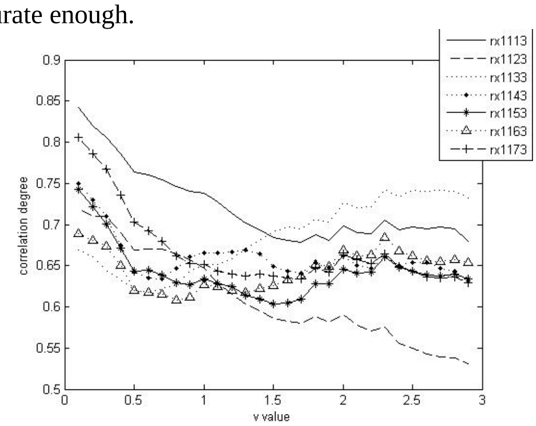

Figure 7. Fractional Order Correlation Curve of Sequences X_{i3} and X_{11}

It can be seen from figure 7 that the correlation value of rx1113 is greater than the values of other curves when $v \in (0,1.5)$ and that the correlation value of rx1113 does not reach the maximum when $v \in (1.5,3)$. This is because the increase of noise level has affected the correlation accuracy of the higher-order differential. When the order is set at 0.5, then the following fractional order correlation will be obtained (see figure 8).

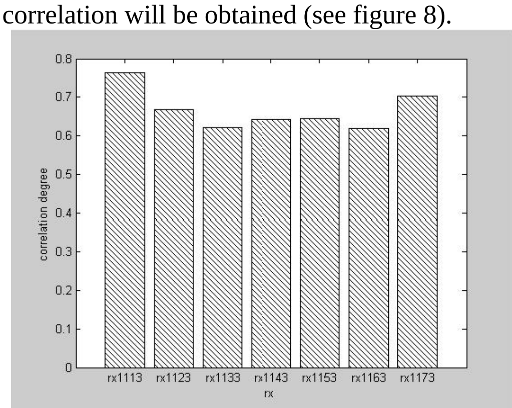

Figure 8. Fractional Order Correlation of Sequences X_{i3} and X_{11} when the order is 0.5

It can be seen from figure 8 that when noise signal amplitude ratio is 0.1 and the sequence order is 0.5, fractional order correlation is of high degree of accuracy.

B. Correlation Judgment

(1) Judgment for correlation values. The greater the correlation value is, the greater the correlation between sequences is; otherwise, the smaller the correlation between sequences is.

(2) Relationship between order and correlation. Comparing with high order differential, low order differential extract more of the low-frequency information and less high-frequency information. As for the time series, low order differential extract more long-term-effect information while high order differential extract more short-term-effect information.

In the circumstance of no noise, the correlation accuracy will increase as the order increases. On the other hand, noise level will influence the correlation accuracy of the algorithm. The addition of noise will affect the high-frequency information of the sequence and as noise level increases, its influence upon correlation accuracy will expand from high order to low order.

The selection relations between noise and order have been achieved through series of experiments. It is discovered that when noise signal amplitude ratio is 0.01, differential order is set at (0.5, 2); when noise signal amplitude ratio is 0.1, differential order is set at 0.5. When noise signal amplitude ratio is unknown, the order is set at 0.5 and if the correlation values approximate each other, then the order needs to be increased until the correlation values become distinctive between one another. Therefore, the correlation accuracy of fractional order algorithm is suitable at best for the distinction of sequence correlation when noise signal amplitude ratio is 0.1.

It can be concluded from the results of figure 2, 4 and 6 that the correlation accuracy of fractional order correlation algorithm has increased by two orders of magnitude as compared with relative correlation algorithm and C-type correlation algorithm while increased by one order of magnitude as compared with T-type correlation algorithm and slope correlation algorithm.

VI. CONCLUSIONS

This paper proposes the fractional order correlation algorithm of uncertain time sequences, and analyzes the influences of differential order and noise upon correlation accuracy, provides selection relations between noise level and order. The experiments proved that fractional order correlation algorithm has increased by two orders of magnitude as compared with relative and C-type correlation, and increased by one orders of magnitude as compared with slope and T-type correlation.

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