A Faithful Translation from Entity-relationship Schemas to the Description Logic $\text{ALENT}_+$

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Abstract— An interesting topic on designing entity relationship (ER) schemas is how to transform ER schemas into knowledge bases (KBs) in description logics (DLs). It is significance in translations that one can use automated DL reasoning services to support the development and maintenance of correct ER schemas. This paper proposed a faithful translation, which translates ER schemas and ER models into KBs in the description logic $\text{ALENT}_+$. The faithfulness preserves the satisfiability and the unsatisfiability, and therefore the translation is sound. The translation allows us to reduce reasoning on ER schemas to finite models reasoning on $\text{ALENT}_+$ KBs.

Index Terms— knowledge representation, ER schemas, ER models, $\text{ALENT}_+$, faithful translations

I. INTRODUCTION

Relational databases and description logics (DLs) are two important formalisms[1]. On the hand, the ER schema is the most widespread formalism for relational database schema design, which is usually defined using a graphical notation particularly useful for an easy visualization of the data dependencies [2-4]. On the other hand, DLs are equipped with capabilities to automatically reason on knowledge bases [5,6]. Thus, providing a formalization of the ER schema in terms of DLs will allow for supporting reasoning on the ER schema such as entity satisfiability, entity subsumption and consistency of the ER schema. Within this interesting topic, many researchers have already proposed some methods such as the formal framework for translating ER schemas [7], the translation from ER schemas into $\text{ALENT}_+$ knowledge bases [8] and the translation from ER schemas into $\text{DLR}$ knowledge bases [9].

This paper mainly focuses on the following four important questions:
  
  ◦ How to transform ER schemas into knowledge bases in DLs;
  ◦ How to transform ER models;
  ◦ How to automatically decide whether a given ER model satisfies a correct ER schema; and
  ◦ How to ensure the soundness and the completeness of the translations. To represent ER models and ER schemas in description logics, an ER schema can be taken as a logical theory, and an ER model can be taken as a model for a logical theory. To translate the ER schemas into description logics, we can translate the ER models into models for the logical theories in description logics. In this translation, a set of entities is taken as a concept, and so is a set of relationships. The objects are either entities or relationships. The roles are classified into two kinds: the roles correspond to the attributes in the ER schemas, and the roles correspond to the ER-roles in the ER schemas.

Our main contributions in this paper are to propose a description logic called $\text{ALENT}_+$ and a faithful translation from ER schemas into $\text{ALENT}_+$ knowledge bases. The faithfulness ensures the preservations of the satisfiability and the unsatisfiability, which means that our translation is sound and complete. By this translation, one can not only design a correct ER schema, but also obtain whether a given ER model satisfies the ER schema. $\text{ALENT}_+$ is quite expressive and includes a novel combination of constructs, including existential quantifications, existential number restrictions, and inverse roles. The significance of the translation is that it allows us to reduce reasoning on ER schemas to finite models reasoning on $\text{ALENT}_+$ knowledge bases.

The paper is organized as follows: the next section introduces the description logic $\text{ALENT}_+$, including its syntax and semantics; the third section takes ER schemas as logical theories and ER models as models for ER schemas, and translates them into $\text{ALENT}_+$ knowledge bases, and proves the faithfulness of the translation; the last section concludes the paper.

Note that, in this paper, we respectively apply boldface, italic and typewriter to represent symbols in ER schemas, symbols in ER models, and symbols in $\text{ALENT}_+$, for example, E, E, E.

II. DESCRIPTION LOGIC $\text{ALENT}_+$

To represent ER schemas and ER models in terms of DLs knowledge bases, we introduce a DL called
In $\text{ALENT}_+$, concepts are formed according to the following syntax:

<table>
<thead>
<tr>
<th>constructor</th>
<th>syntax</th>
<th>semantics</th>
</tr>
</thead>
<tbody>
<tr>
<td>atomic concept</td>
<td>$\emptyset$</td>
<td></td>
</tr>
<tr>
<td>universal concept</td>
<td>$\top$</td>
<td></td>
</tr>
<tr>
<td>bottom concept</td>
<td>$\bot$</td>
<td></td>
</tr>
<tr>
<td>conjunction</td>
<td>$\land \lor \neg$</td>
<td></td>
</tr>
<tr>
<td>universal quantification</td>
<td>$\forall \exists$</td>
<td></td>
</tr>
<tr>
<td>existential quantification(E)</td>
<td>$\exists$</td>
<td></td>
</tr>
<tr>
<td>existential number restrictions(N)</td>
<td>$\exists^{m} \exists^{M}$</td>
<td></td>
</tr>
<tr>
<td>inverse role(1)</td>
<td>$^{-1}$</td>
<td></td>
</tr>
</tbody>
</table>

Concepts are interpreted as subset of a domain and roles as binary relations over that domain. $\mathcal{C} \cap \mathcal{D}$ represents the conjunction of two concepts and is interpreted as set intersection. Consequently, $\top$ represents the whole domain, and $\bot$ the empty set. $\forall \mathcal{R} \mathcal{C}$ is called universal quantification over roles and is used to denote those elements of the interpretation domain that are connected through role $\mathcal{R}$ only to instances of the concept $\mathcal{C}$. $\exists^{m} \exists^{M} \mathcal{R} \mathcal{C}$ and $\exists^{M} \mathcal{R} \mathcal{C}$ are called existential number restrictions, and impose in their instances restrictions on the minimum and maximum number of objects in concept $\mathcal{C}$ they are connected to through role $\mathcal{R}$, which are mainly different to the description logic $\text{ALCHI}$, where $\mathcal{R}$ and $\mathcal{C}$ are role name and concept description, respectively. More formally, an interpretation $I=(\Delta, \cdot, ^{\top})$ consists of an interpretation domain $\Delta$ and an interpretation function $^{\top}$ that maps every concept $\mathcal{C}$ to a subset $\Delta^\mathcal{C}$ of $\Delta$ and every role $\mathcal{R}$ to a subset $\Delta^\mathcal{R}$ of $\Delta \times \Delta$ according to the following semantic rules:

<table>
<thead>
<tr>
<th>syntax</th>
<th>semantics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\emptyset$</td>
<td>$\emptyset^\mathcal{C} \subseteq \Delta$</td>
</tr>
<tr>
<td>$\top$</td>
<td>$\Delta$</td>
</tr>
<tr>
<td>$\bot$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$\land \lor \neg$</td>
<td>$\mathcal{C} \land \mathcal{D}$</td>
</tr>
<tr>
<td>$\forall \mathcal{R} \mathcal{C}$</td>
<td>${x \in \Delta</td>
</tr>
<tr>
<td>$\exists \mathcal{R} \mathcal{C}$</td>
<td>${x \in \Delta</td>
</tr>
<tr>
<td>$\exists^{m} \exists^{M} \mathcal{R} \mathcal{C}$</td>
<td>${x \in \Delta</td>
</tr>
<tr>
<td>$\exists^{M} \mathcal{R} \mathcal{C}$</td>
<td>${x \in \Delta</td>
</tr>
<tr>
<td>$^{\top}$</td>
<td>${(x,y) \mid (x,y) \in \mathcal{R}}$</td>
</tr>
</tbody>
</table>

Similar to other description logics, an $\text{ALENT}_+$ knowledge base also consists of $\text{TBox}$ and $\text{ABox}$\cite{9-11}. $\text{TBox}$ is a set of the following statements: $\mathcal{C} \subseteq \mathcal{D}$, where $\mathcal{C}, \mathcal{D}$ are concepts. $\text{ABox}$ is a set of the following statements: $\mathcal{C}(\mathcal{a})$ or $\mathcal{R}(\mathcal{e}_1, \mathcal{e}_2)$, where $\mathcal{a}, \mathcal{e}_1, \mathcal{e}_2$ are constant symbols, $\mathcal{C}$ is a concept. Given a knowledge base $KB = (\text{TBox}, \text{ABox})$, for any statement $\mathcal{C} \subseteq \mathcal{D}$ in $\text{TBox}$, an interpretation $I$ satisfies the statement $\mathcal{C} \subseteq \mathcal{D}$ if $\mathcal{C} \subseteq \mathcal{D}$, denoted by $I \models \mathcal{C} \subseteq \mathcal{D}$. An interpretation $I$ is a model for a $\text{TBox}$ if $I$ satisfies all the statements in the $\text{TBox}$. For any statement $\mathcal{C}(\mathcal{a})$ or $\mathcal{R}(\mathcal{e}_1, \mathcal{e}_2)$ in $\text{ABox}$, an interpretation $I$ satisfies the statement $\mathcal{C}(\mathcal{a})$ if $\mathcal{a} \in \mathcal{C}$. An interpretation $I$ satisfies the statement $\mathcal{R}(\mathcal{e}_1, \mathcal{e}_2)$ if $(\mathcal{e}_1, \mathcal{e}_2) \in \mathcal{R}$. An interpretation $I$ is a model for a $\text{ABox}$ if $I$ satisfies all the statements in the $\text{ABox}$. An interpretation $I$ is a model for a $KB$ if it is both a model for $\text{TBox}$ and $\text{ABox}$.

### III. ER SCHEMAS/ER MODELS TAKEN AS LOGICAL THEORIES/MODELS

In this section, we can logically represent the connection between ER models and ER schemas by taking ER schemas as logical theories and ER models as the models for ER schemas, and then provide a faithful translation from ER schemas into $\text{ALENT}_+$ knowledge bases. Generally, setting up a translation from one formalism to another formalism is usually taken into account the following logic properties: the soundness and the completeness. However, the two properties do not immediately lead to the preservation of the unsatisfiability. In order to preserve the satisfiability and the unsatisfiability, we define the faithfulness of the translation, which implies that the translation is sound and complete.

#### A. ER schemas

An ER schema $S$ is constructed starting from pairwise disjoint of entity name symbols, relationship name symbols, ER-role name symbols, attribute symbols, and domain symbols. Formally, an ER schema $S$ is a septuple $(E, R, A, \rho, k, U, \text{isa})$, where

- $E$ is a set of entity set names, where its elements are denoted by $E_1, \ldots, E_m$;
- $R$ is a set of relationship set names, where its elements are denoted by $R(E_1, \ldots, E_n)$, where $1 \leq i \leq n$;
- $A$ is a set of the attributes, such that for each attribute $a \in A$ there is a non-empty domain $D_a$;
- $\rho$ is a function such that for any $E \in E$, $\rho(E) \subseteq A$; and for any $R(E_1, \ldots, E_n)$,
  
  \[ \rho(R(E_1, \ldots, E_n)) \subseteq A, \quad \text{and} \quad \rho(R(E_1, \ldots, E_n)) \supseteq \rho(E_1) \cup \ldots \cup \rho(E_n); \]
- $k$ is a function such that for any $E \in E$, $k(E) \subseteq \rho(E)$; and for any $R(E_1, \ldots, E_n)$,
  
  \[ k(R(E_1, \ldots, E_n)) \subseteq \rho(R(E_1, \ldots, E_n)), \quad \text{and} \]
  
  \[ k(R(E_1, \ldots, E_n)) \supseteq k(E_1) \cup \ldots \cup k(E_n); \]
- $U$ is a set of participation constraints of the form $(E, m, M, R)$; and
- $\text{isa}$ is a binary relation on $E$, that is, $\text{isa} \subseteq E \times E$, which is irreflexive, antisymmetric and transitive.

Example 1 \cite{12}. An ER schema of some college database is shown in the following Figure 1. The ER schema uses the notions of entity, relationship and attribute. Entities can be described as distinct objects that need to be represented in the database; relationships reflect interactions between entities, and properties of entities and relationships are described by attributes. For example, the set of entities $\text{Students}$ of the college database has the attributes student identification number ($\text{stno}$), student name ($\text{name}$), street address ($\text{addr}$), city/city($\text{city}$), state of residence ($\text{state}$) and zip code ($\text{zip}$).

The ER schema can be formalized as follows:
An ER schema \( S \) is a set of statements in the language \( L \). Formally,
\[
S = \{ \phi : \phi \text{ is a statement in the language } L \}.
\]

C. The ER models taken as models for logic theories

An ER model \( M \) is a quadruple \( (\Sigma, A, \{ D_a : a \in A \}, I) \) such that \( \Sigma \) is a non-empty universe; \( A \) is a set of attributes, such that for each \( a \in A \), there is a non-empty attribute domain \( D_a \); \( I \) is an interpretation such that
\[
\begin{align*}
&\diamond \text{For each entity set name } E, I(E) \subseteq \Sigma; \\
&\diamond \text{For each relationship set name } R, I(R) \subseteq \Sigma^n; \\
&\diamond \text{For each attribute } a \in A, \\
&
I(a) \subseteq (\Sigma \cup \bigcup_{i \in \omega} \Sigma^i) \times \bigcup_{a \in A} D_a,
\end{align*}
\]
where \( \omega \) is a set of some natural numbers; and
\[
\circ \text{ } I(\text{isa}) = \subseteq, \text{ which means that isa is a subconcept-superconcept relation; and there is a function } i \text{ such that for each attribute } a \in \rho(E) \subseteq A 	ext{ and } e \in I(E) \subseteq \Sigma,
\]
\[
i(e, a) = v \in D_a;
\]
and for each attribute \( a \in \rho(R) \) and \( e_1, \ldots, e_n \in \Sigma,
\]
\[
i((e_1, \ldots, e_n), a) = v \in D_a.
\]
A statement \( \phi \) is satisfied in the ER model \( M \), denoted by \( M \models \phi \), if

Case 1: if \( \phi = E \text{ isa } E' \), then \( I(E) \subseteq I(E') \);

Case 2: if \( \phi = a \in \rho(E) \), then \( a \in I(E) \);

Case 3: if \( \phi = a \in \rho(R(E_1, \ldots, E_n)) \),

then \( a \in I(R(E_1, \ldots, E_n)) \);

Case 4: if \( \phi = a \in k(E) \), then \( a \in k(I(E)) \);

Case 5: if \( \phi = a \in k(R(E_1, \ldots, E_n)) \),

then \( a \in k(I(R(E_1, \ldots, E_n))) \);

Case 6: if \( \phi = R(E_1, \ldots, E_n) \),

\[
\begin{align*}
\forall e_1, \ldots, e_n \in \Sigma((e_1, \ldots, e_n) & \in I(R) \Rightarrow \\
e_1 \in I(E_1) & \land \cdots \land e_n \in I(E_n));
\end{align*}
\]

Case 7: if \( \phi = (E, m, M, R) \), where \( E = E_i \) for some \( i \) in \( R(E_1, \ldots, E_n) \),

then \( \forall e \in I(E)(m \leq |\Phi| \leq M) \), where \( \Phi = \{t : t = (e_1, \ldots, e_{i-1}, e, e_{i+1}, \ldots, e_n) \in I(R)\} \).

An ER model \( M \) satisfies the ER schema \( S \), denoted by \( M \models S \), if for each statement \( \phi \) in \( S, M \models \phi \).
D. The translation $\sigma$ from the ER schemas into $\text{ALENI}_+$ knowledge bases

Let $\sigma$ be the translation translating statements in an ER schema into statements in $\text{ALENI}_+$. Then, $\sigma$ is defined as follows:

Let $L$ be the logical language for the description logic, which contains the following symbols:

- $\text{E} \in \{E_1, \ldots, E_n\}$; $\sigma(E) = E, \text{atomic concept name}$
- $\sigma(R) = R, \text{atomic role name}$
- $\sigma(a) = a, \text{atomic role name}$
- $\sigma(D_a) = D_a, \text{atomic concept name}$
- $\sigma(r_i) = r_i, \text{atomic role name}$
- $\sigma(\text{isa}) = \sqsubseteq$

At the syntactical level, $\sigma$ translates entity set names and relationship set names into concept names, attributes into role names, attributes domains into concept names; and $\text{isa}$ into $\sqsubseteq$. Precisely, for any entity set name $E$, relationship set name $R$, any attribute $a$, any ER-role $r_i$, $\sigma(E) = E, \text{atomic concept name}$ $\sigma(R) = R, \text{atomic role name}$ $\sigma(a) = a, \text{atomic role name}$ $\sigma(D_a) = D_a, \text{atomic concept name}$ $\sigma(r_i) = r_i, \text{atomic role name}$ $\sigma(\text{isa}) = \sqsubseteq$. $\sigma$ is defined as follows:

\[
\begin{align*}
\sigma(E \text{ isa } E') &= E \sqsubseteq E'; \\
\sigma(E \in \rho(E')) &= E \sqsubseteq \exists a.D_a; \\
\sigma(E \in \rho(R(E_1, \ldots, E_n))) &= R(E_1, \ldots, E_n) \sqsubseteq \exists a.D_a; \\
\sigma(E \in \rho(R(E_1, \ldots, E_n))) &= R(E_1, \ldots, E_n) \sqsubseteq \exists a.D_a; \\
\sigma(E \in \rho(R(E_1, \ldots, E_n))) &= R(E_1, \ldots, E_n) \sqsubseteq \exists a.D_a; \\
\sigma((E, m, M, R)) &= E \sqsubseteq \exists^{\geq 1} r_i.R \sqcap \exists^{k} r_i.R'. \\
\end{align*}
\]

where $E$ is some entity set.

Example 2. Let $KB = (ABox, TBox) = \sigma(S)$, where $TBox$ is the set of statements, and $ABox = \emptyset$. For example, by translating the relation presented above to the ER schema shown in Example 1, we obtain the following $\text{ALENI}_+$ knowledge base $KB = (TBox, ABox) = \sigma(S)$, where $\sigma(S)$ contains the following statements:

\[
\begin{align*}
E_1 &\sqsubseteq \exists^{\geq 1} r_{11}.E_1 \sqcap \exists^{\geq 1} r_{11}.R_i \sqcap \exists^{\leq 1} r_{11}.R_i; \\
E_2 &\sqsubseteq \exists^{\geq 2} r_{12}.R_i \sqcap \exists^{\leq 2} r_{12}.R_i; \\
E_3 &\sqsubseteq \exists^{\geq 1} r_{21}.R_i \sqcap \exists^{\leq 1} r_{21}.R_i; \\
E_4 &\sqsubseteq \exists^{\geq 1} r_{22}.R_i \sqcap \exists^{\leq 1} r_{22}.R_i; \\
E_5 &\sqsubseteq \exists^{\geq 1} r_{23}.R_i \sqcap \exists^{\leq 1} r_{23}.R_i; \\
E_6 &\sqsubseteq E_1. \\
\end{align*}
\]

Given an ER model $M = (\Sigma, A, \{D_a : a \in A\}, I)$, we define the translated model $\sigma(M) = (\Delta, I')$ of $M$ as follows: $\Delta = \Sigma \cup \bigcup_{a \in A} D_a$, and $I'$ is an interpretation such that

\[
\begin{align*}
\sigma(E) &= E, \text{isa } E', \text{for any entity set name } E, I'(E) = I(E); \\
\sigma(R) &= R, \text{isa } I'(R) = I(I(R)); \\
\sigma(a) &= a, \text{isa } I'(a) = I(a); \\
\sigma(r_i) &= r_i, \text{isa } I'(r_i) = I(t_i \in I(R)) \land r_i(t_i) = e, e \in \{1, 2, \ldots, n\}. \\
\end{align*}
\]

E. The Complexity of the Translation

In order to describe the complexity of the translation, we only take into the syntactical-level translations consideration. Let $S$ be an ER schema, based on the septuple $(E, R, A, \rho, k, U, \text{isa})$. We distinguish the following four cases: for the statements with the form $E \text{ isa } E'$, the total time of transformations is the number of elements in $\text{isa}$, that is, $|\text{isa}|$; for the statements with the form $R(E_1, \ldots, E_n)$, the total time is the number of elements in $R$; for the statements with the form $(E, m, M, R)$, the total time is the number of elements in $U$; and for other statements, the total time is at most the number of elements in $A$. Thus, the complexity of the translation is $|\text{isa}| + |R| + |U| + |A|$, which means that the algorithm is linear.

F. The Faithfulness of the Translation

In this section, we firstly define the faithfulness, which preserves the satisfiability and the unsatisfiability, and then show that our translation is faithful, which implies that the translation is sound and complete.

Definition 1. Let $\sigma$ be a translation from an ER schema $S$ into an $\text{ALENI}_+$ knowledge base $\sigma(S)$. For any ER model $M$, if $\sigma$ satisfies the following condition: $M$ satisfies $S$ if and only if $\sigma(M)$ satisfies $\sigma(S)$, that is, $M \models S$ if and only if $\sigma(M) \models \sigma(S)$, then $\sigma$ is faithful.

Proposition 1. Let $\sigma$ be a faithful translation from an ER schema $S$ into an $\text{ALENI}_+$ knowledge base. Then for any statement $\phi \in S$, $\phi$ is satisfiable in $S$ if and only if $\sigma(\phi)$ is satisfiable in $\sigma(S)$; that is, there is a ER model $M$ such that $M \models \phi$ if and only if $\sigma(M) \models \sigma(\phi)$.

Proposition 2. For any ER schema $S = \{\phi : \phi$ is a statement in $L\}$ and ER model $M = (\Sigma, A, \{D_a : a \in A\}, I)$, $M \models S$ if and only if $\sigma(M) \models \sigma(S)$. 

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Proof: \((\Rightarrow)\) Assume that \(M \models S\). We show that \(\sigma(M) \models \sigma(S)\). By the construction of \(\sigma(S)\), we distinguish seven cases to prove this proposition as follows:

**Case 1:** If \(E\) is a \(E'\), then \(I' \models E \subseteq E'\). By the definition of \(I'\) and \(M = S\), \(I'(E) = I(E) \subseteq I'(E')\).

**Case 2:** If \(a \in \rho(E)\), then \(I' \models E \subseteq \exists a.D_a\). By the definition of \(I'\), \(I'(E) = I(E)\). By \(M = S\), \(a \in \rho(I(E))\). For any element \(e \in I(E)\), because \(M\) is an ER model, \(i(e, a) = v \in D_a\), and further \((e, v) \in I(a) = I'(a)\), \(v \in D_a\), that is, \(I'(E) \subseteq \exists a.D_a\).

**Case 3:** If \(a \in \rho(R(E_1, ..., E_n))\), then

\[
I' \models R(E_1, ..., E_n) \subseteq \exists a.D_a.
\]

By the definition of \(I', I'(\vec{r}) = I(\vec{r})\), and by \(M \models S\), \(a \in \rho(I(R(E_1, ..., E_n)))\). For any element \(t = (e_1, ..., e_n) \in I(R(E_1, ..., E_n))\), because \(M\) is an ER model, \(i(t, a) = v \in D_a\), and further \((t, v) \in I(a) = I'(a)\), \(v \in D_a\), that is, \(I'(R(E_1, ..., E_n)) \subseteq \exists a.D_a\).

**Case 4:** If \(a \in k(E)\), then \(I' \models E \subseteq \exists a.D_a\). The process of proof is similar to the proof of Case 3.

**Case 5:** If \(a \in \rho(R(E_1, ..., E_n))\), then

\[
I' \models R(E_1, ..., E_n) \subseteq \exists a.D_a.
\]

which follows immediately from Case 2.

**Case 6:** Assume that \(r_i\) is the ER-role name of \(E_i\) in \(R(E_1, ..., E_n)\), \(i \in \{1, 2, ..., n\}\). For any \(t = (e_1, ..., e_n) \in I'(R(E_1, ..., E_n))\), by \(M \models S\) and the definition of \(I'\), \(e_i \in I(E_i) = I'(E_i)\), \(i \in \{1, 2, ..., n\}\). That is, \(I(t, e_i) \in I'(r_i)\), \(e_i \in I'(E_i)\), and hence

\[
I' \models R \subseteq \forall r_i.E_i \land \forall r_i.E_i.
\]

By Case 3, \(I' \models R \subseteq \forall e \in \rho(R(E_1, ..., E_n)) \exists a.D_a\). Hence,

\[
I' \models \forall r_1.E_1 \land \forall r_n.E_n \land \forall e \in \rho(R(E_1, ..., E_n)) \exists a.D_a.
\]

**Case 7:** Given \(E, m, M, R\), assume that \(R(E_1, ..., E_n)\), and \(E = E_i\), and \(r_i\) is the ER-role name of \(E_i\) in \(R(E_1, ..., E_n)\). By \(M = S\),

\[
\forall e \in I(E)(m \leq |F| \leq M),
\]

where \(\Phi = \{(e_1, ..., e_n) \subseteq I(\vec{r})\}\). Hence, there are at least \(m\) and at most \(M\) elements \(t \in I'(r)\) such that \(r_i(t) = e\). In other words, there exist at least \(m\) and at most \(M\) pairs in \(I'(r)\) that have \(e\) as their second component, and moreover all the first components are elements in \(I'(r)\). Therefore,

\[
I' \models E \subseteq \exists^m r_i \subseteq R \land \exists^M r_i \subseteq R.
\]

\((\Leftarrow)\) Let \(\sigma(M) \models \sigma(S)\). For any statement \(\phi\) in \(S\), we have to show that \(M \models \phi\).

**Case 1:** If \(E\) is \(E'\). By \(\sigma(M) \models \sigma(S)\) and the definition of \(I'\), \(I'(E) = I'(\sigma(E)) \subseteq I'(\sigma(E')) = I'(E')\), that is, \(M \models \phi\).

**Case 2:** If \(a \in \rho(E)\). By \(\sigma(M) \models \sigma(S)\) and the definition of \(I'\), \(I'(E) = I'(\sigma(E)) \subseteq I'(\exists a.D_a)\). For any \(e \in I(E)\), there exists an element \(v \in D_a\) such that \((e, v) \in I(a)\), that is, all elements in \(I(E)\) have values on the attribute \(a\), and hence \(a \in \rho(I(E))\).

**Case 3:** If \(\phi = a \in \rho(R(E_1, ..., E_n))\), then \(M \models a \in \rho(I(R(E_1, ..., E_n)))\), which follows directly from Case 2.

**Case 4:** If \(\phi = a \in k(E)\), then \(M \models a \in \rho(I(E))\).

The result follows directly from Case 2.

**Case 5:** If \(\phi = \sigma(a) \in k(R(E_1, ..., E_n))\), then \(M \models a \in \rho(I(R(E_1, ..., E_n)))\), which follows directly from Case 3.

**Case 6:** \(\phi = R(E_1, ..., E_n)\). For any \(x_1, ..., x_n \in \Sigma\), let \(t = (x_1, ..., x_n) \in I(R) = I'(E)\). Because \(I' \models R \subseteq \forall r_1.E_1 \land \forall r_n.E_n\), \(x_i \in I'(E_i) = I(E_i)\), \(i \in \{1, 2, ..., n\}\).

**Case 7:** \(\phi = (E, m, M, R)\), let \(E = E_i\). For any \(e \in I(E)\), \(E' = I'(E)\), by \(I' \models E \subseteq \exists^m r_i \subseteq R \land \exists^M r_i \subseteq R\)

and the definition of \(I'(r_i)\),

\[
m \leq |F| \leq M,
\]

where \(\phi = \{t = (e_1, ..., e_{i-1}, e, e_{i+1}, ..., e_n) : r_i(t) = e \land t \in I(\vec{r})\}\).

Hence, we have the following commutative diagram:

\[
\begin{array}{ccc}
\text{ER schema} & \xrightarrow{\sigma} & KB \\
\text{interpretation} & & \text{interpretation} \\
\text{model for } KB & \xrightarrow{\sigma} & \text{model for } KB
\end{array}
\]

IV. CONCLUSION

This paper proposed a faithful translation from ER schemas into \(\mathcal{ALE}\) knowledge bases, which allows us to reduce reasoning on ER schemas to finite models reasoning on \(\mathcal{ALE}\) KBs. However, several problems remain unsolved. One unsolved problem is how to transform ER schemas and ER models with imprecise information. Future works focus on these questions.

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