

A Contour Representation and Retrieval Algorithm based on Polar-histogram

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Abstract—This paper presents a novel method for contour-based shape retrieval. First, they come from contour points which can represent the shape very well as the object contour points got by contour tracing and denoising. Corner points, sampled points and two vector sets are obtained by leading vectors from each point to the contour centroid point. Then, the polar histogram of directions and distance in the vector sets are used to describe the shape. Finally, it combines the similarity measurement criterion of two contours with the polar histogram of the two feature points. The experiment result shows that our algorithm is efficient and is with better performance in the robustness to the scaling, rotation and translation, compared with the traditional algorithms.

Index Terms—image retrieval; shape representation; polar histogram; feature vector sets

I. INTRODUCTION

Generally researchers consider the color, texture, shape and spatial relationship of image as retrieval features in content-based image retrieval (CBIR). Feature selection, feature extraction and representation are related to the effectiveness and efficiency of image retrieval system. It is often linked to the target for shape feature. It does not change with surrounding environment, so the shape feature is more important^[1]. Shape representation handles or calculates the objective shape in some way. Its methods can be mainly divided into edge-based methods and region-based methods^[2]. Generally edge-based methods are better than other shape representation, because the profile is more stability; extraction is faster and contains more information of the target. So it is a preferred method^[3, 4, 5].

Distance histogram^[6] is an effective edge-based shape representation method. But distance histogram only describes the statistical characteristics of the distances. It ignores the spatial information, so two different shapes of graphics may have the same histogram method, leading to the higher false detection rate. It is proposed to resolve this problem as a shape vector called distance coherence, based on the distances histogram method [7]. The basic idea of distance coherence is to put pixels in each interval which divide into coherent pixels and incoherent pixels. [8] applies corners as the contour index, on the basis of

extracting contour corners. There are many methods to describe contour, such as literature [9] applies shape context as the contour descriptor; [10] introduced edge direction histogram as the shape descriptor. Article [11] applies Bezier parameters of the fitting contour curves points as the shape descriptor. [12-13] introduced based on the histogram. However, above methods have two problems in describing contour feature: first, does not consider that the shape impact by different contour points is different, thus ignores change information of the contour shape; second, only use the contour distant information, thus ignores the spatial information and makes the retrieval results inaccurate.

Therefore, this paper introduces a contour representation and retrieval algorithm based on polar-histogram. First, corner points and sampled points which reflect the contour features are extracted from all contour points, and two vector sets are obtained by leading vectors from each point to the contour centroid point. Then, the polar histogram of the direction and distance in the vector sets is applied to describe the shape. Finally, we analyze our experiment results. This paper also gives the concept of polar histogram which applied to image retrieval algorithm. The polar histogram is a two-dimensional one, which contains the distance and direction information of the image. It can describe the contour shape information better, compared with the traditional distance histogram.

II. FORMATION OF THE SHAPE DESCRIPTOR

Target shape can be obtained by way of contour extraction or image segmentation, such as contour tracing, based on contour image retrieval technologies. This paper firstly, gets a smooth curve by smooth operation of the tracked contours; then extract homogeneous sampling points and corner points as contour feature points and get feature vector set by leading vector to contour centroid; finally, obtain polar histogram by statistic distance and direction of feature vector and describe the target shape according to this histogram in order to form a robust shape descriptor.

A. Curve Smoothness

It assumes obtained object contour line P by edge detection and contour tracking, fig.1 (a) shows the map of Shanxi province, fig.1 (b) shows contour line of Shanxi

after edge detection and contour tracking. This paper shows the closed contour line with two one-dimensional detailed curve $x(i)$ and $y(i)$, $p = \{p(i) = (x(i), y(i)), i = 1, 2, 3, \dots, m\}$, m is pixel number in the contour.

$x(i)$ and $y(i)$ are not smooth because of the digital image errors and noise influence, such as some irregular serrated points in the curve, which can not only indicate the contour shape, but also can result in extra cusp, so $x(i)$ and $y(i)$ should be with curve-smoothing before extracted feature points. Gaussian smoothing is a common smoothing method, however, to reduce digital error and noise, it would also produce contraction error. This paper uses an adaptive smoothing method for the image contour, weight size changes with the different continuity of signal.

$S(x)$ is one-dimensional detailed signal before smoothing, $S^{t+1}(x)$ is the smoothed signal after $t + 1$ iterations.

$S^{t+1}(x)$ is expressed as:

$$S^{t+1}(x) = \frac{1}{W} \sum_{i=-N}^N S^t(x+i) \omega^t(x+i) \quad (1)$$

$$W = \sum_{i=-N}^N \omega^t(x+i) \quad (2)$$

$$\omega^t(x) = e^{-\frac{|S^{(t)}(x)|^2}{2k^2}} \quad (3)$$

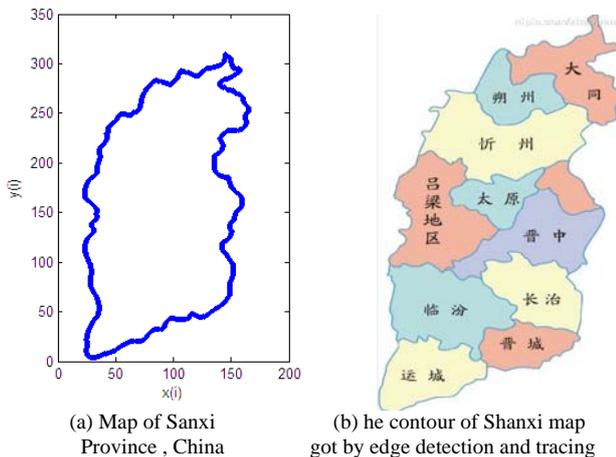


Fig.1. Map of Shanxi and the contour of Shanxi map got by Edge-detection and tracing

Where $\omega^t(x+1) \geq 0 (-N \leq i \leq N)$, it is convolution weights. For adaptive smoothing, if $N = 1$, $\omega^t(x)$ decreases with signal discontinuity at point x increasing. $S^{t+1}(x)$ is the derivative of signal $S^t(x)$ in equation (3), $S^{t+1}(x)$ is given in the form of discretization in equation (4); k is the contour smoothness, the greater k is, the stronger the smoothing effect, and all

unsmooth point will disappear, make curve passivated. Conversely, the smaller k is, the weaker smoothing effect. The selection of k depends on the contour complexity and noise interference, the right k is acquired by experiment and $k = 3$ in this paper.

$$S^{t+1}(x) = \frac{S^t(x+1) - S^t(x-1)}{2} \quad (4)$$

B. Feature Points Extraction

Definition 1: corner points are the local curvature maxima ones in the contour.

Definition 2: sampled points are got by equal portions sampling the contour lines.

Definition 3: feature points are the collection of corner points and sampled points.

Corner point is an important feature in shape recognition; it can describe the contour shape very well. But other contour points also contain more information which also have a very important role on describing contour shape. So this paper extracts corner points and sampled points as feature points on describing the contour shape.

a. Sampled points extraction

We should determine starting point (x_q, y_q) of the contour before extract sampled points. The biggest curvature pixel in the contour is defined as starting point of the sampling uniformity. If there is more than one curvature maximum pixel of the contour, we get the point which has bigger curvature adjacent pixel as starting point of sampling uniformity. If the starting point of sampling uniformity is $((x_d, y_d))$, then

$$(x_q, y_q) = (x(d), y(d)) \quad (5)$$

According to the equality principle, if pixel number is m , the equality number is n , we take a sampling point every m/n point from the starting point.

$E = (e_1, e_2, \dots, e_j, \dots, e_n)$ is the sequence of sampling points, $j = (1, 2, \dots, n)$, n is the number of sampling points. Then

$$E = \{(x_q, y_q), (x((d+jm/n) \oplus m), y((d+jm/n) \oplus m)), \dots, (x((d+m) \oplus m), y((d+m) \oplus m))\} \quad (6)$$

$\oplus m$ is modular arithmetic.

Fig.2 shows the sampled points of Shanxi contour line when $n = 20$.

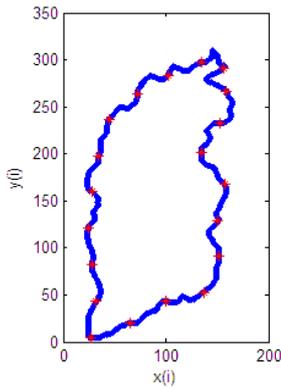


Fig.2. Results of uniform sampled points on the contour of Fig.1 (a) b. Corner points extraction

Coordinate $p(i)$ depends on $x(i)$ and $y(i)$, so we can get the corner points of p in contour line by combining corner points of $x(i)$ and $y(i)$.

Firstly, we calculate absolute curvature $q(i)$ in coordinate $x(i)$ and $y(i)$, the specific calculation as follows:

$$x''(i) \approx \frac{1}{h^2} [x(i-h) - 2x(i) + x(i+h)] \quad (7)$$

$x = x(i), i = 1, 2, \dots, n$ is discrete function, h is step, the number of adjacent pixels.

The second derivative of the curve is approximately equal to the curvature, if ρ is the curvature of $x(i)$ in point i , then:

$$\rho \approx \frac{1}{h^2} [x(i-h) - 2x(i) + x(i+h)] \quad (8)$$

If $q = |h^2 \rho|$, then:

$$q(i) = |x(i-h) - 2x(i) + x(i+h)| \quad (9)$$

The size of step h directly affects the accuracy of curvature calculation, the value of h is too large or too small that will produce many points which have same curvature in the contour line. It is found that the effect is more ideal when $h = 5$.

After got curvature absolute points on the curve, we found out the points that have maximum value of q , denoted by $qm(i)$, is the location of curvature extreme point on the curve. Setting the valve value τ , if $qm(i) \geq \tau$, marked the i point for the first corner point, conversely, it is not corner point. $a(j)$ is the location of corner point $x(i)$ $a(j) = i, j = 1, 2, \dots, n_x$, n_x is the number of corner points $x(i)$.

We can get the number of corner points $y(i)$ by the same method. If $b(j)$ is the location of corner point $y(i)$ $b(j) = i, j = 1, 2, \dots, n_y$, n_y is the number of

corner points $y(i)$.

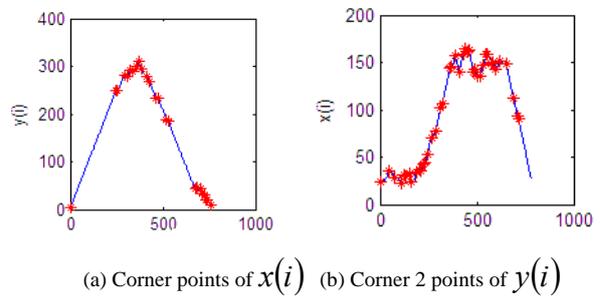
$U = (u_1, u_2, \dots, u_{n_{xy}})$ is the point sequence on the contour line, $j = 1, 2, \dots, n_{xy}$, n_x is the number of corner points on the contour line.

If $A = \{a(1), a(2), \dots, a(n_x)\}, B = \{b(1), b(2), \dots, b(n_y)\}$,

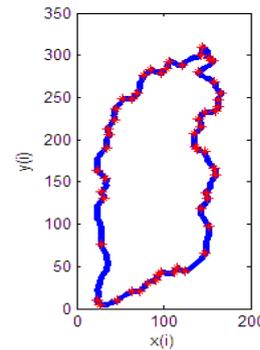
$$U = A \cup B$$

$$\{a(1), a(2), \dots, a(n_x)\} \cup \{b(1), b(2), \dots, b(n_y)\} \quad (10)$$

Fig.3 shows the corner point extraction results on the contour line of Shanxi province map, (a), (b) and (c) in the Fig.3 show respectively the corner points of $x(i)$, the corner points of $y(i)$, the corner points of whole contours on Fig.1 (b).



(a) Corner points of $x(i)$ (b) Corner 2 points of $y(i)$



(c) Corner points of whole contours

Fig.3. Results of the critical points on the contour of Fig.1 (a)

C. Feature Vector Sets

Firstly, we calculated centroid of contour line $c(x_0, y_0)$, to calculate feature vector sets.

$$\begin{cases} x_0 = \frac{1}{m} \sum_{i=0}^{m-1} x(i) \\ y_0 = \frac{1}{m} \sum_{i=0}^{m-1} y(i) \end{cases} \quad (11)$$

$(x(i), y(i))$ is the coordinate of contour pixels, m is the number of contour pixels.

Definition 4: Feature vectors are the vectors whose direction is from the feature points to the centroid of contour. The sets of all the feature vectors are called feature vector sets.

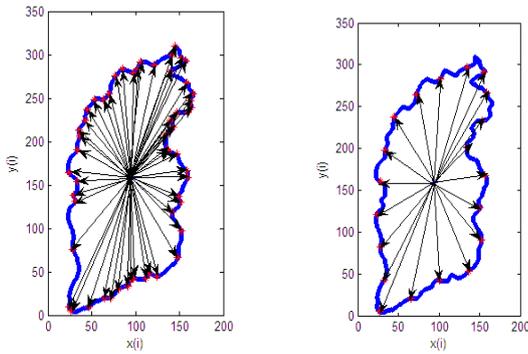
The feature vector sets which calculate from uniform sampled points are

$$\vec{V}e = (\vec{e}_1c, \vec{e}_2c, \dots, \vec{e}_ic, \dots, \vec{e}_nc), i = 1, 2, \dots, n \quad (12)$$

The feature vector sets which calculates form critical points are

$$\vec{V}u = (\vec{u}_1c, \vec{u}_2c, \dots, \vec{u}_ic, \dots, \vec{u}_{n_{xy}}c), i = 1, 2, \dots, n_{xy} \quad (13)$$

Fig.4 shows the schematic of the feature vector sets which calculates from critical points and uniform sampled points respectively.



(a) Feature vector sets with uniform sampled points (b) Feature vector sets with critical points

Fig.4. Vector sets of the two types feature points of the contour of Fig. 1(a)

D. Polar Histogram

The Polar-histogram is a two-dimensional histogram. It is fit to describe the contour than the one-dimensional histogram, as it not only concerns the distance information, but also the direction.

a. Polar transformation

The location of pixels in an image can be described by Cartesian coordinates (x, y) and Polar coordinates (r, θ) . The formula of transformation from Cartesian coordinates to Polar is followed, with placing the origin point of Polar at the centroid point:

$$\gamma = \sqrt{(x - x_0)^2 + (y - y_0)^2} \quad (14)$$

$$\theta = \arctan \left(\frac{y - y_0}{x - x_0} \right) \quad (15)$$

As it only needs to translate the coordinate of the feature points, the (x, y) in the formula is the coordinate of the feature point on the contour, (x_0, y_0) is the coordinate of the coordinate of the centroid of the contour, (γ, θ) is the variable in the Polar coordinate.

b. Polar histogram

The polar plane should be quantified to strike the polar histogram. The size of the quantitative scale would affect the accuracy of description and the calculation speed. The better size of the quantitative scale it is, and the higher description accuracy it is, and the calculation requires the more cost. In contrast, the coarser size of the quantitative scale it is, the lower description accuracy it is and the less cost the calculation requires. According to the test which the paper takes, both the description accuracy and the computational speed are ideal at the case that divides the

γ_{max} into 5 equal portions (γ_{max} is the maximum value of γ) and θ into 12 equal portions. The 5 equal portions of the γ_{max} divide the Polar coordinate into 5 rings, and the 12 equal portions of θ divide the polar plane into 12 sectors with 30° , and the polar plane is divided into 60 zones with the rings and sectors (Fig.5). Each zone corresponds to a component of the histogram, and takes the zone of $0^\circ < \theta \leq 30^\circ, 0 < \gamma \leq \frac{1}{5}\gamma_{max}$ as the first component, and $30^\circ < \theta \leq 60^\circ, 0 < \gamma \leq \frac{1}{5}\gamma_{max}$ as the second component, and so as to the sixtieth component ($330^\circ < \theta \leq 360^\circ, \frac{4}{5}\gamma_{max} < \gamma \leq \gamma_{max}$). The polar histogram is obtained with counting the number of the feature points which correspond to the feature vectors in each zone. The polar-histograms of critical points and uniform sampled points in the contour of Fig.1 (a) is shown in Fig.6.

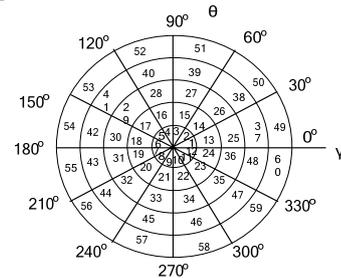
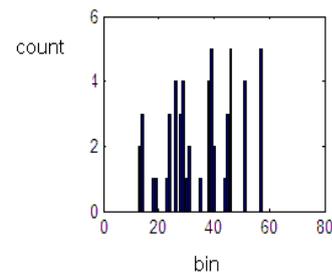
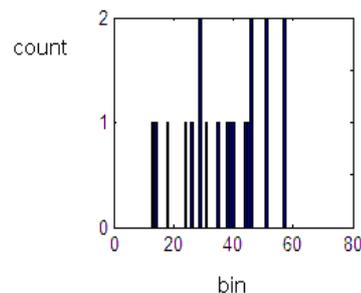


Fig.5. Diagram of quantitative polar plane



(a) Polar histogram with critical points of Shanxi Province Map



(b) Polar histogram with uniform sampled points of Shanxi Province Map

Fig.6. The polar-histograms of critical points and uniform sampled points in the contour of Fig.1 (a)

III. SIMILARITY MEASURE

The similarity of images can be measured with the distance between the corresponding histograms. The general characteristics of the contour can be described as $\{h(k), g(k)\}$, and the $h(k)$ and $g(k)$ respectively corresponds to polar histogram with critical points and polar histogram with uniform sampled points. The distance between the corresponding histograms, D_1 and D_2 , are defined as following:

$$D_1 = \frac{1}{2} \sum_{k=1}^k (h_{c1}(k) - h_{c2}(k))^2 \quad (16)$$

$$D_2 = \frac{1}{2} \sum_{k=1}^k (g_{c1}(k) - g_{c2}(k))^2 \quad (17)$$

The $c1$, $c2$ are the contour curve of any two images, and D_1 , D_2 are the distance of histogram with respectively corresponding to critical points and uniform sampled points. To obtain rotation invariance, the paper employs the method of minimum cyclic shift subtraction to calculate the distance of histogram.

The similarity of contours is defined as following:

$$D = \omega D_1 + (1 - \omega) D_2 \quad (18)$$

The ω is the weight of the feature vector. As the contour expression with critical points is more efficient than with uniform sampled points, the value of ω would be taken in the range of $\omega > 0.5$ (it takes 0.6 in the paper). It can be obtained from equation (18) that the low the value of D is, the high the similarity is.

IV. EXPERIMENT RESULTS AND ANALYSIS

A. Invariance Analysis of Feature Vector

The similarity matching algorithm of image retrieval should be as far as possible to the similarity judgment of human eyes. It mainly concerns to the 5 elements as the basis of people distinguishing two images of the same object: translation, rotation, scale, color and brightness change. This should have the invariance of the 5 characteristics as a fine image retrieval system. The color change and brightness are not concerned as the object of the image retrieval system in the paper is binary image. As the applied algorithm describing the object with feature points and centroid point, it has the translation invariance inherently.

a. Scale invariance

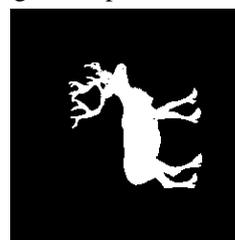
As the polar histogram includes the distance information of feature vector, and the distance information corresponds to the scale of the object, some treatments must be taken to the distance information to get scale invariance. The algorithm bases on the maximum distance, and quantifies the maximum distance into 5 equal portions, during the processing of polar plane quantifying. When the images scale change, the ratio of the feature vector model does not change, and feature points in special zones would be located in the same zones after scale changing. E.g. a vector of the feature vector sets \vec{Z} , the feature points corresponds to \vec{Z}

would be located in the i zone, in the case of $\frac{i-1}{5} < \frac{|\vec{Z}|}{\gamma_{\max}} \leq \frac{i}{5}$. After enlarging a times, the

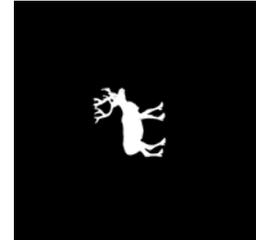
model of the feature vector changes. After changing, let the model of \vec{A} as $|\vec{Z}'| = a|\vec{Z}|$, $\gamma'_{\max} = a\gamma_{\max}$,

$$\frac{i-1}{5} < \frac{|\vec{Z}'|}{\gamma'_{\max}} = \frac{|\vec{Z}|}{a\gamma_{\max}} = \frac{|\vec{Z}|}{\gamma_{\max}} \leq \frac{i}{5}$$

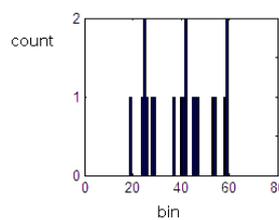
, and the feature points correspond to \vec{Z} are still located in the i zone after scale changing. Fig.7 shows the polar histogram with critical points and uniform sampled points respectively as an example of scale transformation effect. It can be seamed from Fig.7 (c), (e), (d) and (f) that the corresponding polar histogram doesn't change after scale transforming. It can get the result which the addressed algorithm possesses the scale invariance.



(a) Retrieval image

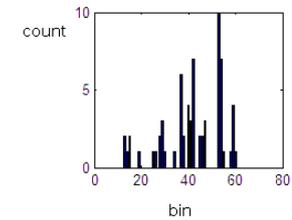


(b) the image after shrinking a times



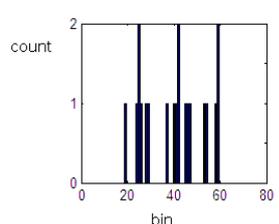
(c)

(c) Polar histogram of critical points of (a)

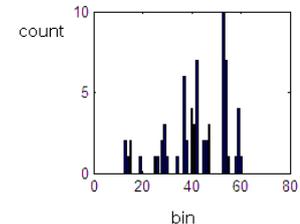


(d)

(d) Polar histogram of uniform sampled points of (a)



(e) Polar histogram of critical points of (b)



(f) Polar histogram of uniform sampled points of (b)

Fig.7. the effect of scale transform on polar-histograms

b. Rotation invariance

As the polar histogram possesses rotation invariance inherently, the applied algorithm adopts the minimum cyclic shift subtraction method to compute the histogram distance. It effectively eliminates the affection of image rotation. Fig.8 shows the polar histogram before and after rotation with critical points and uniform sampled points respectively. It can be seen in the figure that it is different

between the polar histograms of before and after rotation with critical points. It almost has no differences with uniform sampled points. The result is caused by the case of setting starting point while extracting the sampled points.

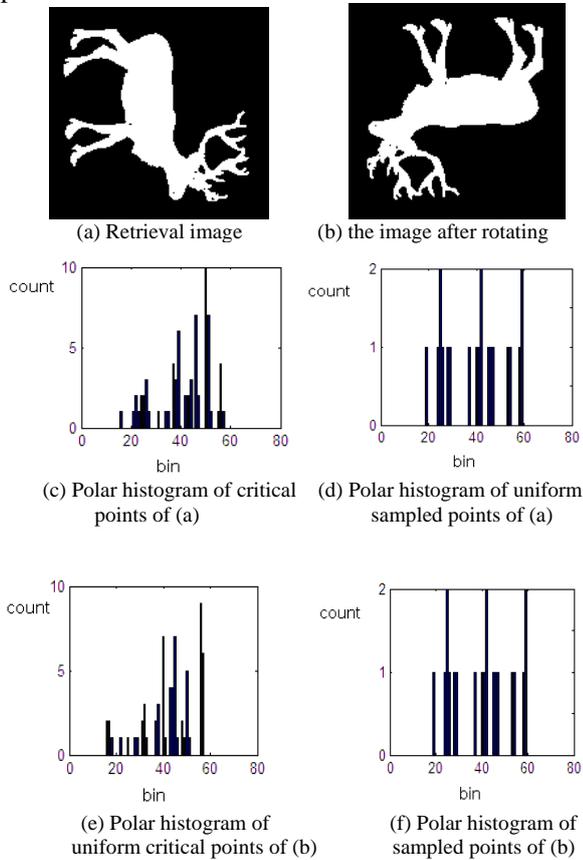


Fig.8. the effect of orientation transform on polar-histograms

B. Experiment Results

It is composed of 20 kinds and 400 images which are selected from the widely used MPEG-7 test image of test image database of the experiment. Each kind of images in test image database includes plenty of similarity information images with characteristics of scale transformation, rotation transformation and translation. Some images of the test image database are shown in Fig.9.

Image No.	1	2	3	4	5
Image.Type					
Image Number	20	20	20	20	20

Fig.9. some query images

The algorithm performance is evaluated with two indicators in test; one is precision and the other for recall. It is defined as following:

$$\text{Precision} = \frac{N_{correct}}{N_{retrieve}} \tag{18}$$

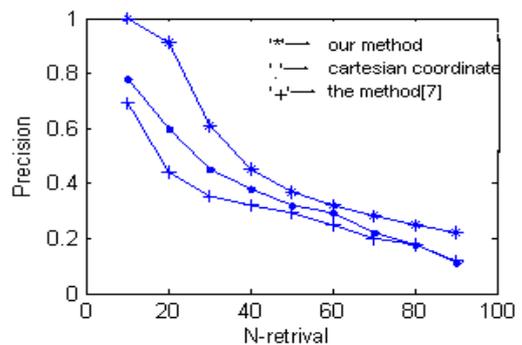
$$\text{Recall} = \frac{N_{correct}}{N_{corrlate}} \tag{19}$$

The $N_{correct}$ indicates the number of retrieved corresponding images, and $N_{retrieve}$ indicates the number of retrieved images, and $N_{corrlate}$ indicates the number of corresponding images in the test image database.

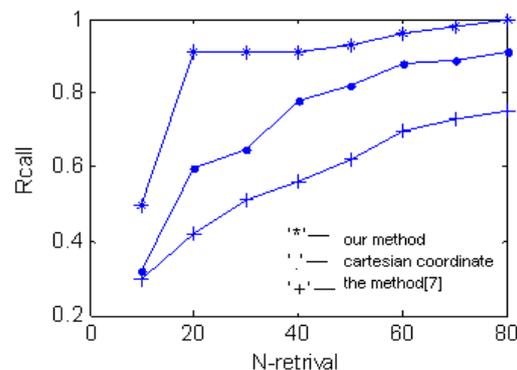
To further illustrate the superiority of Polar coordinate than the Cartesian coordinate, the curve of precision and recall under Cartesian coordinate are given respectively, and also the comparison of precision and recall with [7] (Fig.10). The applied three algorithms are tested in the same test image database. The property parameters in Fig.10 (a) is the average value of the selected 10 types of images that each kind selects one image for computation.

To further prove the superiority of the algorithm, the retrieval effects of 3 algorithms with 4 kind's images are also given in the paper (Fig.11, Fig.12, and Fig.13). The left image of each figure (Fig.11, Fig.12 and Fig.13) is the image, waiting for retrieved. It also exists in the test image database.

It is seen from the figures t at the performance of the algorithm which the paper addressed. It is better than the [7] addressed. It increases the precision and recall ratio. The results can be obtained that the applied algorithm possesses good invariance of translation, rotation and scale, as the similar images in each kind was retrieved.



(a) Precision curve comparison



(b) Recall curve comparison

Fig.10 Precision curves and Recall curves comparison in our algorithm and reference [7], in the system of rectangular coordinates

