A New Fractional Order Chaotic System and Its Compound Structure

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Abstract—Chaos may be degenerated because of the finite precision effect, hence, in this work, for given a new fractional order three-dimensional chaotic attractors, numerical investigations on the dynamics of this system have been carried out. The stability of equilibrium for the system is analyzed according to the qualitative theory. Furthermore, a new chaotic control technique is designed, the special compound structure of the new fractional order chaotic attractor is investigated, and some numerical simulations are proposed. The results show that the new fractional order chaotic system can generate complex compound structure under the control of the constant control parameter. This evolving procedure reveals the forming mechanisms of compound nature and finds some law which is very meaningful in investigating some complex chaotic dynamical phenomena.

Index Terms—fractional order, chaotic system, compound structure

I. INTRODUCTION

The dynamics of fractional-order systems have attracted increasing attention in recent years [1]. It has been shown that many systems can be described by fractional differential equations, and demonstrates chaotic behavior [2-3]. Fractional-order systems possess memory and display much more sophisticated dynamics compared to its integral-order counterpart [4], which is of great meaning in secure communication [5]. Extensive numerical work has been carried out in order to understand chaos in fractional order dynamical systems [6].

More recently, by utilizing fractional calculus technique, many investigations were devoted to the chaotic behaviors and chaotic control of dynamical systems involved the fractional derivative, called fractional-order chaotic system among the physicists and engineers [7-10]. It is known that the chaotic systems depend on parameters and initial conditions sensitively [11]. In this paper, taking a new fractional order chaotic system for illustration, experience of dynamical behavior is studied. By constant controller method, the existence of the compound nature is investigated. That is, as the amplitude of the control gains, the chaotic dynamic is confined from two simple attractors to the whole butterfly. The study shows that the dynamical behavior of the compound structure is closely related to the scope of the order and the amplitude of the constant controller, which maybe have applications in such fields as secure and digital communications, and so on.

The structure of this paper is as follow: section 2 briefly introduces the definition of fractional differential and the predictor corrector algorithm. In section 3, a necessary condition for double scroll attractor existence in fractional-order systems is introduced, and the scope of order which chaos phenomena appears in the new fractional chaotic system is discussed; In section 4, by means of Lyapunov exponents and the stability method, the complex dynamical behaviors of a new fractional-order chaotic system are investigated. In section 5, by the numerical simulation, the dynamic behavior of compound structure is discussed for the two 2-scroll attractor of the fractional order new chaotic system, and a general law of the relation between the amplitude of the constant controller and the scope of the order of the fractional order chaotic system is summed up; finally, some concluding remarks are given in section 6.

II. THE DEFINITIONS OF FRACTIONAL DERIVATIVES

As the fractional-order derivative is taken as the extension of the integer-order derivatives [12], the general fundamental operator is defined as follows:

$$D_q^a x = \begin{cases} \frac{d^q}{dt^q} x & q > 0 \\ 1 & q = 0 \\ \int_0^t (d\tau)^{-q} x & q < 0 \end{cases}$$

(1)
where $q$ is the order of the fractional integral derivatives; $a$ and $t$ are the limits of the operator.

During the development process of fractional calculus theory, there are several different basic definitions in fractional calculus; the most widely used definitions are Riemann-Liouville definition and Caputo definition [13]. We would prefer Caputo derivative to the former one, since the latter is more popular in real applications [14]. The mathematical expression of Caputo definition is as follows [15]:

$$D^q f(t) = \frac{1}{\Gamma(n-q)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{q+1-n}} d\tau \quad n-1 < q < n$$

(2)

where $n$ is the first integer larger than $q$, and $I'$ is the function of Gamma.

Firstly, we present in this section some basic definitions and properties.

A. Basic Concepts

Definition 1. A real function $f(x)$, $x>0$ is said to be in space $C_0$, $a \in \mathbb{R}$ if there exists a real number $p>a$, such that $f(x)=x^p f_1(x)$ where $f_1(x) \in C[a, \infty]$.

Definition 2. A real function $f(x)$, $x>0$ is said to be in space $C_{\alpha}$, $m \in \mathbb{N} \cup \{0\}$ if $f^{(m)} \in C_{\alpha}$.

Definition 3. Let $f \in C_0$ and $a \gg 1$, then the (left-sided) Riemann-Liouville integral of order $\mu$, $\mu>0$ is given by

$$I^\mu f(t) = \frac{1}{\Gamma(\mu)} \int_0^t (t-\tau)^{\mu-1} f(\tau) d\tau, t>0$$

(3)

Definition 4. The (left sided) Caputo fractional derivative of $f$, $f \in m \in \mathbb{N} \cup \{0\}$, is defined as:

$$I^{m-\mu} f(t) = \frac{\partial}{\partial t} I^{m-\mu} f(t) = I^{m-\mu} f(t)$$

(4)

$m-1<\mu< m, m \in \mathbb{N}$

Note that for $m-1<\mu \leq m, m \in \mathbb{N}$,

$$I^\mu D^\mu f(t) = f(t) - \sum_{k=0}^{m-1} \frac{f^{(k)}(0)}{\Gamma(m-k)} t^{m-k-1}$$

(5)

$$I^\mu I^\nu f(t) = \frac{\Gamma(\nu+1)}{\Gamma(\mu+\nu+1)} t^{\mu+\nu}$$

(6)

B. Numerical Methods for Solving Fractional Differential Equations

At present, there are many methods for solving fractional calculus operators. The commonly used algorithms are improved version of approximation algorithms of Bode plots and predictor-corrector algorithm [16], which is the generalization of Adams-Bashforth-Moulton algorithm [17]. Considering the following differential equation:

$$\begin{cases}
\frac{d^q y(t)}{dt^q} = f(t, y(t)), 0 \leq t \leq T \\
y^{(k)}(0) = y^{(k)}_0, k = 0, 1, \ldots, [q]-1
\end{cases}$$

(7)

Equation (7) is equivalent to the Volterra integral equation (4).

Let $h = \frac{2}{n}$, $t_n = nh, n = 0, 1, \ldots, N \in \mathbb{Z}^+$, the equation of (8) can be discrete as the equation of (9):

$$y_{n+1}(x) = \sum_{k=0}^{[q]} \frac{x^k}{k!} y^{(k)}(0) + \frac{1}{h^q} \int_0^t (t-\tau)^{-q-1} f(\tau, y(\tau)) d\tau$$

(8)

y(t) = \sum_{k=0}^{[q]} \frac{t^k}{k!} y^{(k)}(0) + \frac{1}{h^q} \int_0^t (t-\tau)^{-q-1} f(\tau, y(\tau)) d\tau

(9)

The estimation error of this approximation is described as follows:

$$\max_{j=0,1,\ldots,q} |y(t_j) - y_n(t_j)| = O(h^p)$$

(10)

where $p = \min(2, 1+q)$. Then, the above method can be applied to numerical solution of a fractional order system.

III. STABILITY AND CHAOS IN FRACTIONAL-ORDER SYSTEMS

At first, we draw to analyze the stability of fractional-order systems. It is known that the stable scope of fractional-order differential equations is more wide than the integer order one [18], because systems with memory are typically more stable than their memory-less counterpart [19], we can find it by comparing the scope of two types of system. Considering the following fractional order system:

$$D^q x = f(x), 0 < q \leq 1, x \in \mathbb{R}^n$$

(13)

Lemma 1: let the equilibrium points of the system of (13) is $x^*$ (the solution of the equation), if the eigenvalues of Jacobian matrix (13) which is in the equilibrium point satisfy the following condition [20]:

$$arg(eig(A)) > q \pi/2$$

(14)

Figure 1. Stability region of linear fractional order system with order $q$. The dynamics behavior of system (13) is stable. The necessary condition for fractional-order system (13) exhibits an $n$-scroll chaotic attractor to keep the eigenvalues $\lambda \in \Lambda$ in the unstable region, just as follows:
\[ q > \frac{2}{5} \tan^{-1} \left( \frac{\ln(\alpha)}{\ln(\gamma)} \right), \forall \lambda \in \Lambda \] (15)

From Fig 1, we can obtain the stability and unstable region of saddle points, and the minimum value of \( q \) for the fractional order system (13) to remain chaotic can be obtained.

IV. A NEW FRACTIONAL-ORDER CHAOTIC SYSTEM

In this Letter, a new chaotic attractor is introduced and its dynamical behaviour is studied [21], which is described by the following nonlinear differential equations and denoted as system (16).

\[
\begin{align*}
\dot{x} &= -\frac{a}{5b} x - yz \\
\dot{y} &= xz + ay \\
\dot{z} &= xy + bz
\end{align*}
(16)
\]

Here, \( x, y, z \) are the state variables, and \( a, b \) are constant parameters of the system. Here, some basic properties of the system (12) are analyzed in the following.

A. Dissipativity and Existence of Attractor

Let us find the general condition of dissipative region of the system:

\[
\text{div} \bar{F}(x, y, z) = \frac{\partial \bar{z}}{\partial x} + \frac{\partial \bar{y}}{\partial y} + \frac{\partial \bar{z}}{\partial z} = -\frac{ab}{a+b} + a + b
\]

\[ = \frac{(a+b)/2 + 3b^2/4}{a+b} < 0 \] (17)

Therefore, the system (16) is dissipative for the parameters satisfying \( a+b<0 \). So, the system (16) converges to a subset of measure zero volume exponentially:

\[
dV/dt = e^{-\lambda t} < 0 \] (18)

which means that for an initial volume \( v_0 \), the volume will become (18) as \( t \to \infty \) at an exponential rate, through the flow generated by the system. So, the system displays chaotic behavior.

B. Symmetry and Invariance

By the symmetry and invariance theory, we know that the chaotic system (16) is symmetrical about the three coordinate axes \( x, y, z \) respectively. This chaotic system is invariant under three coordinate transforms.

\[
\begin{align*}
(x, y, z) &\to (x, -y, -z) \\
(x, y, z) &\to (-x, y, z) \\
(x, y, z) &\to (-x, -y, z)
\end{align*}
(19)
\]

The system (16) is symmetrical about the coordinate axis \( x \) or \( z \) and is not symmetrical about the coordinate axis \( y \).

C. Equilibria

For the dynamics characteristic of the system (16) is determined by the equilibria greatly. So, we shall determine positions of singular points of the system (12), points of phase space by solving the following algebraic equations simultaneously.

\[
\begin{align*}
-\frac{a}{5b} x - yz &= 0 \\
xz + ay &= 0 \\
xy + bz &= 0
\end{align*}
(20)
\]

It is knowing that \( o_1 (0, 0, 0) \) is equilibrium. Thus, according to (16), we get that for \( ab>0 \) and \( a+b<0 \), the system has five singular point.

\[ x = +\sqrt{ab}, y = \pm \sqrt{bz}/a \] (21)

One can easily verified that the system has five equilibria, introducing the following new parameter

\[ p = \sqrt{ab}, q = \sqrt{ab/(a+b)} \] (22)

So the five equilibrium points denoted as:

\[
\begin{align*}
O(0, 0, 0) \\
E_1(p, q\sqrt{b}, -q\sqrt{a}) \\
E_2(p, -q\sqrt{b}, q\sqrt{a}) \\
E_3(-p, -q\sqrt{b}, -q\sqrt{a}) \\
E_4(-p, q\sqrt{b}, q\sqrt{a})
\end{align*}
(23)
\]

Assuming \( a=-10, b=-4 \), one can obtain

\[
\begin{align*}
O(0, 0, 0) \\
E_1(2\sqrt{10}, 4\sqrt{35}/7, -10\sqrt{14}/7) \\
E_2(2\sqrt{10}, -4\sqrt{35}/7, 10\sqrt{14}/7) \\
E_3(-2\sqrt{10}, -4\sqrt{35}/7, -10\sqrt{14}/7) \\
E_4(-2\sqrt{10}, 4\sqrt{35}/7, 10\sqrt{14}/7)
\end{align*}
(24)
\]

Then, the stability of the equilibrium points is discussed. The Jacobian matrix of system evaluated at equilibrium point \((x', y', z')\) is given by

\[
J = \begin{pmatrix}
-ab/(a+b) & -z' & -y' \\
z' & a & x' \\
y' & x' & b
\end{pmatrix}
(25)
\]

At the point \( O \), one can obtains the Jacobian

\[
J = \begin{pmatrix}
-ab/(a+b) & 0 & 0 \\
0 & a & 0 \\
0 & 0 & b
\end{pmatrix}
(26)
\]

So, the eigenvalues of the liberalized systems are obtained as follows.

\[
|\lambda I - J_0| = 0 \Rightarrow \lambda_1 = -ab/(a+b), \lambda_2 = a, \lambda_3 = b \] (27)

Set \( a=-10, b=-4 \), one can found that all the eigenvalues so the equilibrium \( O \) is unstable saddle point for the system (16)
Then, we calculate the Jacobin matrix for $E_0$, $E_2$, $E_3$, and $E_4$. The real equilibrium points and the eigenvalues of the corresponding Jacobian matrix are given in Table 1. Evaluation of Jacobian matrix at the equilibrium points yields the following eigenvalues.

<table>
<thead>
<tr>
<th>Equilibrium point</th>
<th>Eigenvalues $\lambda &lt; 0, i=1,2,3$</th>
<th>Nature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O$</td>
<td>-10.0000, -4.0000, 2.8571</td>
<td>Saddle point</td>
</tr>
<tr>
<td>$E_1$</td>
<td>-13.6106, 1.2339±5.6626i</td>
<td>Saddle point</td>
</tr>
<tr>
<td>$E_2$</td>
<td>-13.6106, 1.2339±5.6626i</td>
<td>Saddle point</td>
</tr>
<tr>
<td>$E_3$</td>
<td>-13.6106, 1.2339±5.6626i</td>
<td>Saddle point</td>
</tr>
<tr>
<td>$E_4$</td>
<td>-13.6106, 1.2339±5.6626i</td>
<td>Saddle point</td>
</tr>
</tbody>
</table>

For the nonzero equilibria, $\lambda_1$ is a negative real number, $\lambda_2$ and $\lambda_3$ become a pair of complex conjugate eigenvalues with positive parts. Moreover, any two nonzero equilibria are symmetric about one of the axes.

- $E_1$ and $E_2$, $E_3$ and $E_4$ are symmetric with respect to the $x$-axis.
- $E_1$ and $E_3$, $E_2$ and $E_4$ are symmetric with respect to the $y$-axis.
- $E_1$ and $E_4$, $E_2$ and $E_3$ are symmetric with respect to the $z$-axis.

It is clear shown that the above nonzero equilibria are all saddle points.

The corresponding Lyapunov exponents’ diagram of system (16) is shown in Figure 2. The three Lyapunov exponents are: $Le_1=1.15725$, $Le_2=0$, $Le_3=-12.2981$ and the Lyapunov dimension are $D=2.0939$.

![Figure 2. The Lyapunov exponents of new chaos system $a=-10, b=4$](image)

where $a=-10, b=4$ and initial conditions $X(0) = (1,2,-1)$ yield chaotic trajectory.

![Figure 3. Phase portraits of the stable attractors $a=-10, b=4$](image)

(a) $x$-y, (b) $y$-z

Here, we consider the corresponding fractional order system with commensurate order system:

\[
\begin{align*}
\frac{dx}{dt^q} &= -\frac{a}{b} x - yz \\
\frac{dy}{dt^q} &= xz + ay \\
\frac{dz}{dt^q} &= xy + bz
\end{align*}
\]

where $0 < q < 1$ is the fractional-order.

Based on the necessary condition in fractional-order systems to remain chaotic, we can get the following inequality in order to determine the stability condition:

\[\arg\{1.2339 \pm 5.6626i\} > q\pi / 2 \rightarrow q < 0.8634\]

We can obtain the maximum scope of the order $q$ for which the new systems with commensurate fractional-order and above parameters demonstrate chaos is $q \geq 0.8634$.

![Figure 4. Bifurcation diagram of $q, x_{z\max}$ ($a=-10, b=4$)](image)

This is supported by the values of bifurcation diagram of Figure 4. The corresponding largest Lyapunov exponent is: $Le=1.15725$ and the Lyapunov dimension is $D=2.0939$.

Chaotic attractors appear in this system with a wide parameter range and display some complex dynamical behaviors. By the predictor-corrector algorithm, the state trajectories of the system for different values of $q$ are illustrated in Figure 4 and 5.
When the initial value is \( X(0) = (x(0), y(0), z(0)) \), if \( z(0) > 0 \), the system displays a chaotic attractor above the plane \( z = 0 \); if \( z(0) < 0 \), the system shows another chaotic attractor below the plane \( z = 0 \), which is displayed in Figure 6.

When the initial value is \( X(0) = (x(0), y(0), z(0)) \), if \( z(0) > 0 \), the system displays a chaotic attractor above the plane \( z = 0 \); if \( z(0) < 0 \), the system shows another chaotic attractor below the plane \( z = 0 \), which is displayed in Figure 6.

V. COMPOUND STRUCTURE OF A NEW FRACTIONAL-ORDER CHAOTIC SYSTEM

In order to study whether the compound structure of fractional-order new system is the same as the corresponding integer one, we adopt the similar method as the paper [22], we add a constant controller on the right of the first equation of the new fractional-order equation, and then study the compound structure of the new fractional-order chaotic system, the dynamic equation of the controlled system can be expressed as follows:

\[
\begin{align*}
\frac{d^q x}{dt^q} &= -\frac{a_0}{\alpha^q} x - yz + m \\
\frac{d^q y}{dt^q} &= xz + ay \\
\frac{d^q z}{dt^q} &= xy + bz 
\end{align*}
\]  

(30)

Since the parameters \( a, b \) are fixed, here only two cases will be considered as follows.

A. Fix \( q \)

Let \( m \) vary, the system (30) is equated to the integer-order system. Which is calculated numerically with \(-40 \leq m \leq 40\), and the increment of \( m \) equals to 0.01. Figure 7 shows the bifurcation diagram of the second state component versus constant controller.

When increasing the control parameter \( m \) this chaotic attractor modifies its features and to the screw-type form before its disappearance.

When \( q = 1 \) and \( 0 \leq m \leq 40 \), three dimensional phase diagrams of the system are shown in Figure 8.

From Figure 7 and 8, it is clear that with an increase in the value of parameter \( m \), a period doubling phenomenon is observed, Figure 6 shows that the systems are limit cycle, period-doubling bifurcations, four periodic, the left or right half cycle attractor, periodic window and complete attractor, respectively. When \( m = 5 \), the system has a complete attractor, Phase diagram of \( m = 5 \) is plotted in Figure 6 (a). The variation rule of the bifurcation diagram is:

When \( m \geq 32.9 \), the system is limit cycle;
When \( 21.3 \leq m < 32.9 \), the system is period-doubling bifurcations;
When \( 19.2 \leq m < 21.3 \), the system is four periodic;
When \( 14.8 \leq m < 19.6 \), the system is reduced to the left or right half cycle attractor;
When $12.7 \leq m \leq 14.8$, there is a periodic window; when $m \leq 12.7$, the system has a complete attractor. It is known that Lyapunov exponents are the average exponential rates of divergence or convergence of nearby trajectories in the phase space. To demonstrate the chaotic dynamics, the largest Lyapunov exponent is the first thing to be considered. Any system containing one or more positive Lyapunov exponents is defined to be chaotic [23]. The corresponding largest Lyapunov exponents (Le) diagrams of system (15) versus $m$ are shown in Figure 9. The result is consistent with the bifurcation diagram of Figure 5.

\[ \begin{align*}
\text{Figure 9. The Largest Lyapunov Exponent of } m
\end{align*} \]

**B. Fix $m$**

Let $m$ vary, the bifurcation diagrams of new chaotic system are shown in Figure 10, where $q=0.98$, $q=0.95$, $q=0.93$, $q=0.90$, $q=0.88$, $q=0.85$, respectively.

\[ \begin{align*}
\text{Figure 10. Bifurcation diagram of new chaotic system } (a) q=0.98; (b) q=0.95; (c) q=0.93; (d) q=0.90; (e) q=0.88; (f) q=0.85
\end{align*} \]

Figure 7 and 10 shows that the dynamical behaviors of system (30) is closely related to the order and the amplitude of the constant controller, that is: when the order of system is certain, the period-doubling bifurcation point is related to the amplitude of the controller, the higher the amplitude of the controller is, more earlier the period-doubling bifurcation point appears; similarly, when the amplitude of the controller is certain, the lower the order is, more earlier the period-doubling bifurcation point appears.

Form Figure 6 and Figure 11, we can draw a conclusion that the new chaotic system which has both upper-attractor and lower-attractor has compound structure, by controlling the amplitude of the controller, the new chaotic system can emerge from chaotic attractor to simple limit cycle.

\[ \begin{align*}
\text{Figure 11. 3-D phase diagrams of the upper-attractor (initial value } (x_0, y_0, z_0) z_0>0) \text{ and lower-attractor (initial value } (x_0, y_0, z_0) z_0<0) \text{ when } q=0.95 (a) m=5; (b) m=11; (c) m=14; (d) m=35
\end{align*} \]

Similarly, we add a constant controller on the right of the second equation of the new fractional order equation and then study the compound structure of the new fractional order chaotic system, the dynamic equation of the controlled system can be expressed as follows:

\[ \begin{align*}
\frac{d^q x}{dt^q} &= -\frac{a_0}{z_0} x - yz \\
\frac{d^q y}{dt^q} &= xz + ay + m \\
\frac{d^q z}{dt^q} &= xy + bz
\end{align*} \]  \hspace{1cm} (31)

Here, let $a=10$, $b=4$, by tuning the constant controller, the multiply scroll chaotic attractor can be confined from 2-scroller attractor to a similar simple limit cycle.

Systems (31) have the similar compound structure as system (30). The bifurcation diagram of new chaotic system (31) is shown in Figure 12 and Figure 13.

\[ \begin{align*}
\text{Figure 12. Bifurcation diagram of system (31), } q=0.95 \text{ (a) } a=10, b=4
\end{align*} \]
Based on the necessary condition in fractional-order systems to remain chaotic, we can get the maximum scope of the order q for which the new systems with commensurate fractional-order and above parameters demonstrate chaos.

The corresponding Lyapunov exponents (Le) and Lyapunov dimension (D) of system (31) are shown in below, \( q=1, a=-10, b=-4 \).

When \( m=5 \), \( Le_1=0.999038 \), \( Le_2=0.00431694 \), \( Le_3=-12.1362 \), \( D=2.0819 \);
When \( m=14 \), \( Le_1=0.75897 \), \( Le_2=0.00281377 \), \( Le_3=-11.8999 \), \( D=2.0635 \);
When \( m=25 \), \( Le_1=0.00938412 \), \( Le_2=-0.246004 \), \( Le_3=-10.9361 \), \( D=1.0381 \);
When \( m=28 \), \( Le_1=-0.10235 \), \( Le_2=-0.00403814 \), \( Le_3=-10.9361 \), \( D=1.002 \).

Similarly, we add a constant controller on the right of the third equation of the new fractional order equation and then study the compound structure of the new fractional order chaotic system, the dynamic equation of the controlled system can be expressed as follows:

\[
\begin{align*}
\frac{d^qx}{dt^q} &= -\frac{a}{m}x - yz \\
\frac{d^qy}{dt^q} &= xz + ay \\
\frac{d^qz}{dt^q} &= xy + bz + m
\end{align*}
\] (32)

Systems (32) have the similar compound structure as system (30). Based on the necessary condition in fractional-order systems to remain chaotic, we can get the maximum scope of the order q for which the new systems with commensurate fractional-order.

When \( q=1, a=-10, b=-4 \) the bifurcation diagram of new chaotic system (32) is shown in Figure 14 and 15.

Figure 14. Bifurcation diagram of system (32), \( q_{x_{max}} (a=-10, b=-4, q=1) \).

When \( q=1, a=-10, b=-4 \), the corresponding Lyapunov exponents (Le) and Lyapunov dimension (D) of system (32) are shown in below.

When \( m=5 \), \( Le_1=-0.415647 \), \( Le_2=-0.00652155 \), \( Le_3=-11.5536 \), \( D=2.0355 \);
When \( m=9 \), \( Le_1=-0.00403814 \), \( Le_2=-0.307191 \), \( Le_3=-10.8397 \), \( D=1.0131 \);
When \( m=14 \), \( Le_1=-0.187556 \), \( Le_2=-0.189969 \), \( Le_3=-10.7653 \), \( D=0 \);
When \( m=26 \), \( Le_1=-1.9648 \), \( Le_2=-1.9648 \), \( Le_3=-9.1784 \), \( D=1.002 \).

According to Figure 10, Figure 13 and Figure 15, when the simple variable constant controller m gains, the whole chaotic attractor could be obtained by merging two simple attractors together, which are derived from some simple limit cycles. The period-bifurcation point of controlled system which leads to the compound structure is controlled by amplitude of the constant controller and the order of system. The results for understanding of fractional order chaotic system’s compound structure will undoubtedly have some reference value.

VI. CONCLUSIONS

In this paper, a new fractional order chaotic system is extensively investigated numerically. It is found that the system displays comprehensive dynamical behavior. The results of this study have been validated by the existence of a positive Lyapunov exponent. The stability of equilibrium for the system is analyzed according to the qualitative theory. Moreover, the compound structure is investigated in the fractional-order system. There are abundant and complex dynamical behaviors. This will be quite helpful to understand not merely chaotic dynamics behaviors but also chaos control and synchronization in engineering applications. Therefore, it deserves further investigation in the future.

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