Research on the Expected Synchronization and Anticipating Synchronization of Time-delay Systems

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***Abstract*—It is a hard problem to anticipate the dynamic characteristics of non-linear chaotic system. In this paper, based on a stability theorem proved for linear chaotic systems, a scheme for anticipating synchronization of chaotic system is proposed. This paper investigate the synchronization between the receiver system and the future state of a transmitter system for an arbitrarily long anticipation time, both master and slave systems are considered to be involved with time-delay by the proposed method. The synchronization scheme guarantees one to predict the dynamics of chaotic transmitter at any point of time. Where synchronization error will be forced and then kept inside a ball around the origin. So the synchronization can be done with any desired accuracy. The proposed method can be easily extended to synchronize other chaotic systems. Numerical simulation results are used to verify the theoretical analysis using different values of parameter.**

***Index Terms*—time-delay system; chaotic system; anticipation synchronization**

I. INTRODUCTION

Recently, control and synchronization of chaotic motion are key problems in chaotic application field [1]. In 2000, Voss proposed a new real-time delay synchronization systems method, known as the expected synchronization [2]. In this method, the response system can keep synchronization with the future state of drive system. That was the response system predicting the future state of the drive system. In 2002, E. M. Shahverdiev group proposed a new theory on the expected synchronization of Reversed-Phase (RP) [3], to realize the response system giving the future state of the drive system a reverse forecast. After E. M. Shahverdiev group and K. Pyragas etc. many times did further analysis and research on the expected synchronization and RP expected synchronization of time-delay systems, such as with a coupling term, the two coupling terms the expected synchronization of time-delay systems [5,6] , anti-expected synchronization[3], with multiple coupling terms the expected synchronization of time-delay systems [7-8].

The theoretical analysis and simulation results provide a theoretical basis and examples for further research on long-term prediction of time-delay systems. We also found that the expected synchronization of time-delay system, more is given deductions and analysis of special form in current literatures, and theoretical analysis of the general form is not detailed enough; for multiple coupling terms the *RP* expected synchronization of time-delay systems , in-depth theoretical analysis and simulation example aren't given. Therefore, using the expected synchronization and *RP* expected synchronization proposed by E. M. Shahverdiev etc. for reference, we will further improve the theoretical analysis on the expected synchronization and RP expected synchronization with multiple coupling teams in this paper; on the other hand, according to the classification of different forms of the coupling term of delay system, based on the support of the theory, we will give the theory and the simulation on expected synchronization and *RP* expected synchronization separately.

II. THE THEORY ON EXPECTED SYNCHRONIZATION OF TIME-DELAY SYSTEMS

Before the concrete theory introduction, firstly, we introduce a lemma [9, 10] to support the theoretical derivation and numerical simulation of this paper. Consider the following form of time-delay systems, namely:

$$dx / dt = Ax + f(x) \tag{1}$$

where $x(t) \in R^n$, $X\tau=x(x-\tau)$, Let D is the domain, initial value $x(t_0, \ )$ given, and then the equation (1) has a unique solution, denoted as $x(t_0, \ )$ [9]. Suppose the Function F is continuous and derivable, and then define a continuous Lyapunov function $x(t_0, \ )$, denoted as: $V{:}D \rightarrow R^+$ .Along

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the equation (1) path to the function $V(t)$ derivation, we can obtain:

Lemma 1 [11]: Suppose u and v is *K-class* function, and there exists a function $V(t,)$ satisfying the following conditions:

(1) $\forall t \geq 0, \exists \phi \in C, s.t.\ \ u(|\phi(0)|) \leq V(t,\phi) \leq v(|\phi|)$;

(2) $\forall t_0 \geq 0, \exists \phi \in C, s.t.\ \ \dot{V}(t, x_t(t_0,\phi)) \leq 0, (t \geq t_0)$;

Then the zero solution of the equation (1) is (consistent) stable.

III. The Expected Synchronization Between Time-delay Systems Coupled With Delay

This section will classify the different forms under time-delay systems with coupling term, introduce separately the expected synchronization theory of different forms under time-delay systems, and give the derivation of the expected synchronization with multiple coupling terms theory in detail.

First consider the simplest case that is the expected synchronization of time-delay systems coupled with a delay. Drive system and response system [12] are as follows:

$$dx(t)/dt = -ax(t) + f(x_\tau) \tag{2}$$

$$dy(t)/dt = -ay(t) + f(x) \tag{3}$$

where $x_\tau=(x\text{-}\tau)$, define the error e (t) =x (t)-y_τ, and then the error system is:

$$de(t)/dt = -ax(t) + f(x_\tau) + ay_\tau - f(x_\tau) = -ae(t) \tag{4}$$

Make Lyapunov function unction $V(t) = e(t)^2/2$,

Then along the equation (4) track, deriving to $V(t)$ us can obtain:

$$\dot{V}(t) = \dot{e}(t)e(t) = -ae^2(t) \tag{5}$$

According to lemma, the zero solution of equation (4) is consistent and stable. When a>0, and the origin of error system is asymptotically stable, the expected synchronization is reached between system (1-2).

The expected synchronization between time-delay systems coupled with two different delays

Consider the expected synchronization between time-delay systems coupled with two different delays. Drive system and response system are set up as follows:

$$dx/dt = -ax(t) + f(x_{\tau_1}) \tag{6}$$

$$dy/dt = -ay(t) + f(x_{\tau_2}) \tag{7}$$

where $x_\tau=x\ (t\text{-}\tau_i),\ (i=1,\ 2)$, Assume $\tau_1>\tau_2$, and define the error $e\ (t) = x\ (t) - y_{\tau 1\text{-}\tau 2}$,

Then the error system can be obtained:

$$\begin{aligned} de(t)/dt &= -ax(t) + f(x_{\tau_1}) + ay_{\tau_1-\tau_2} \\ &- f(x_{\tau_1}) = -ae(t) \end{aligned} \tag{8}$$

Select the Lyapunov function $V(t) = e(t)^2/2$, calculate derivative of $V(t)$ along the equation (8) path, one get:

$$V(t) = e(t)\ e(t) = -ae^2(t)$$

For $\tau_1>\tau_2$ case, the error can be defined as

$$e(t) = x(t) - y_{\tau 1\text{-}\tau 2}$$

The error system and its stability analysis are similar with $\tau_1>\tau_2$, this section is no longer analysis.

The expected synchronization between time-delay systems coupled with multiple delays

Consider the expected synchronization of delay system of the general case in the section, that is the expected synchronization of time-delay systems coupled with multiple delays, and the first two cases can be considered as a special case of time-delay systems coupled with multiple delay. Then, this section gives a sufficient condition to achieve the expected synchronization between the drive system and response system and this proof for the sufficient condition based on the Krasovskii-Lyapunov stability theory.

Consider the drive system and response system [13]

$$dx(t)/dt = -ax(t) + m_{1x} f(x_{\tau_1}) + \cdots + m_{nx} f(x_{\tau_n}) \tag{9}$$

$$\begin{aligned} dy(t)/dt = -ay(t) + m_{1y} f(y_{\tau_1}) +, \cdots, + \\ m_{ny} f(y_{\tau_n}) - kf(x_{\tau_k}) \end{aligned} \tag{10}$$

where $x_{\tau i}=(x\text{-}\tau_i), y_{\tau i}=y(t\text{-}\tau_i), l=1,2,\cdots,n,\ \tau_k>0,$,m_{ix} and m_{iy} are the feedback gain coefficients, and coefficient k is the expression of coupling strength.

Assume that $\tau_i>\tau_k$ defines the error

$$e(t) = x(t) - y_{\tau 1\text{-}\tau 2}, i=1,2,3\cdots,n$$

If $m_{ix}=m_{iy}$, and $f(x)$ is continuous and differentiable, according to the mean value theorem, we can get

$$de/dt = -ae + \sum\nolimits_{j=1, j\neq i}^{n} m_{jy} f'(\xi_j) e_{\tau_j} + m_{iy} f'(\xi_i) e_{\tau_i} \tag{11}$$

Theorem 1[14]: For arbitrary $t>0$, if the inequality always holds:

$$a > \sum\nolimits_{j=1, j\neq i} \left| m_{jy} f'(\xi_j) | + | m_{iy} f'(\xi_i) \right|$$

When $t\rightarrow\infty$, the error $e(t)\rightarrow 0$, namely, the error system (11) is asymptotically stable at the origin, Therefore, the sufficient condition for the inequality V, expected synchronization between the drive system (9) and the response system (10) can always achiev.

The inequality $V(e(t)) < 0$ always holds.

According to Lemma, easy to select to meet Lemma's two conditions $u(x)$ and $v(x)$, the zero solution is uniformly stable. Here, because the function $f(x)$ and the reciprocal $f(x)$ are uncertain, we will ignore the selection of the function *k-class*. The proof is over.

IV. The Theory on the RP Expected Synchronization of Time-delay Systems

Similar to the way of the discussions with the expected synchronization, this section will classify the different forms under time-delay systems with coupling term, introduce separately the *RP* expected synchronization theory of different cases, and give the derivation of the *RP* expected synchronization with multiple coupling terms theory in detail.

The *RP* expected synchronization between time-delay systems coupled with a delay

First consider the case with a delay. Drive system and response system are defined as follows:

$$dx/dt = -ax(t) + f(x_\tau) \tag{12}$$

$$dy/dt = -ay(t) + f(x) \tag{13}$$

where $x_\tau = x(t-\tau)$,

Define the error,

$$e(t)=x(t)-(-y_\tau)=x(t)+y_\tau,$$

Then the error system is:

$$de(t)/dt = ax(t) + f(x_\tau) - ay_\tau - f(x_\tau) = -ae(t) \tag{14}$$

Select Lyapunov function $V(t) = e(t)^2/2$, then along the equation (14) track, deriving to $V(t)$ us can obtain:

$$V(t) = e(t)\, e(t) = -ae^2(t).$$

Obviously,

If $a>0$, $V(t)<0$

Select $u(s)=s^2/2$,

$V=as^2$,

$us \in K$,

Then $V(t) \leqslant -av^2(t)$ holds

We may know according to lemma that the zero solution of the equation (14) is consistent and stable. When $a<0$, and the origin of error system is asymptotically stable, the RP expected synchronization between the systems (12) and (13) can always achieve.

The *RP* expected synchronization between time-delay systems coupled with two different delays.

Consider the *RP* expected synchronization with two different delays. Drive system and response system are set up as follows:

$$dx(t)/dt = -ax(t) + f(x_{\tau_1}) \tag{15}$$

$$dy(t)/dt = -ay(t) - f(x_{\tau_2}) \tag{16}$$

where $x_{\tau i}=x(t-\tau_i),(i=1,2)$, Assume $\tau_1>\tau_2$, and define the error $e(t)=x(t)-(-y_{\tau 1-\tau 2})=x(t)+y_{\tau 1-\tau 2}$, then the error system can be obtained:

$$\begin{aligned} de(t)/dt &= -ax(t) + f(x_{\tau_1}) - ay_{\tau_1-\tau_2} - f(x_{\tau_1}) \\ &= -ae(t) \end{aligned} \tag{17}$$

Select the Lyapunov function $V(t) = e(t)^2/2$, calculate derivative of $V(t)$ along the equation (17)'s path, we may know according to lemma that the zero solution of equation (17) is consistent and stable.

The RP expected synchronization between time-delay systems coupled with multiple delays

Consider the RP expected synchronization of delay system of the general case in the section, that is the RP expected synchronization of time-delay systems coupled with multiple delays, and the first two cases can be considered as a special case of time-delay systems coupled with multiple delays. Then, this section gives a sufficient condition to achieve the RP expected synchronization between the drive system and response system.

Define the drive system and response system:

$$dx(t)/dt = -ax(t) + m_{1x} f(x_{\tau_1}) +, \cdots, + m_{nx} f(x_{\tau_n}) \tag{18}$$

$$\begin{aligned} dy(t)/dt = -ay(t) - m_{1y} f(y_{\tau_1}) -, \cdots, - \\ m_{ny} f(y_{\tau_n}) + kf(x_{\tau_k}) \end{aligned} \tag{19}$$

where $x_{\tau l}=x(t-\tau_l), y_{\tau l}=y(t-\tau_l), l=1,2,\cdots,n$, $\tau_k>0$, m_{lx} and m_{ly} are the feedback gain coefficients, and coefficient k is the expression of coupling strength. Assume that $\tau_i>\tau_k$ defines the error

$e(t) = x(t) - (-y_{\tau 1-\tau k}) = x(t) + y_{\tau 1-\tau k}, (i \in 1,2,\cdots,n)$

Then the error system is

$$\begin{aligned} de(t)/dt = -ae + \sum\nolimits_{j=1, j\neq i} m_{jy} [f(x_{\tau_j}) - f(y_{\tau_j+\tau_i-\tau_k})] + \\ m_{iy} [f(x_{\tau_i}) - f(y_{2\tau_i-\tau_k})] \end{aligned}$$

If $f(x)$ is continuous and differentiable, according to the mean value theorem, we can get:

$$\begin{aligned} de(t)/dt = -ae + \sum\nolimits_{j=1, j\neq i} (m_{jy} f'(\xi_j) e_{\tau_j} + \\ m_{iy} f'(\xi_i) e_{\tau_i}) \end{aligned} \tag{20}$$

where

$$\xi_j \in (\eta_{1j}, \eta_{2j}),\ \xi_i \in (\theta_1, \theta_2),$$

$$\eta_{1j} = \min\{x_{\tau_j}, y_{\tau_j+\tau_i-\tau_k}\},\ \eta_{2j} = \max\{x_{\tau_j}, y_{\tau_j+\tau_i-\tau_k}\}$$

$$\theta_1 = \min\{x_{\tau_i}, y_{2\tau_i-\tau_k}\},\ \theta_2 = \max\{x_{\tau_i}, y_{2\tau_i-\tau_k}\}.$$

Theorem [16]: For arbitrary t>0, if the inequality

$$a > \sum\nolimits_{j=1, j\neq i} \left| m_{jy} f'(\xi_j) \right| + \left| m_{iy} f'(\xi_i) \right|$$

Always holds, the error system (20) is asymptotically stable at the origin, and the RP expected synchronization between the drive system (18) and the response system (19) can always achieve.

The proof of the theorem can refer to the proof of Theorem 1, so here no proof is given in detail.

V. Expected Synchronization of Time-delay Systems on Numerical Simulation of The Mackey-Glass

In the previous two sections on the basis of theoretical analysis, this section will analyze the application of the expected synchronization and *RP* expected synchronization in the specific time-delay system, and give the numerical simulation results.

The classical Mackey-Glass system [17] as the research object, the equation is as follows [18]:

$$dx(t)/dt = -cx(t) + ax_{t-\tau}/(1 + x_{t-\tau}^b) \tag{21}$$

where $x(t)$ means that on the time, t, a, b, c are equation parameters, and τ is time delay parameter. Usually taking, τ is increasing constantly, and as τ increases, the dimension of chaotic dynamics in time-delay system is growing constantly. JD Farmer conducted in-depth study on the complex dynamics of Mackey-Glass system [84]:

When $\tau<4.35$, the system is stable at a fixed point; $4.35<\tau<13.1$

The system is in the limit cycle state; $13.1<\tau<16.8$, the system presents periodic states; $\tau>16.8$, the system appears the chaotic attractor.

We will discuss the expected synchronization and RP expected synchronization of the Mackey-Glass system. Referring to of the three cases of the expected synchronization theory in Section 3, in example analysis and value simulation stage, we will also have sequential analysis for three cases.

A. The Expected Synchronization Coupled with A Delay

Drive system and response system are as follows:

$$\frac{dx(t)}{dt}=-cx(t)+\frac{ax_\tau}{1+x_\tau^b} \tag{22}$$

$$\frac{dy(t)}{dt}=-cy(t)+\frac{ax(t)}{1+x(t)^b} \tag{23}$$

where $x_\tau=x_{i-\tau}$.

Define the error $e(t)=x(t)-y_\tau$, According to the theoretical analysis in section 3, when $c>0$, the expected synchronization between the drive system and the response system can always achieve.

Selecting the system parameters $a=0.2,\ b=10,\ c=0.1$, Numerical simulation is shown in Figure 1-2.

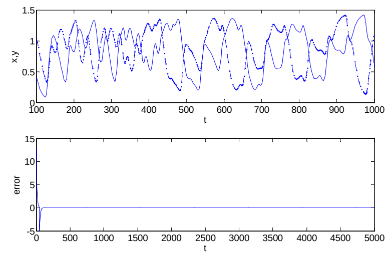

Figure 1.Response system variable $y(t)$ (dashed lines) predicting the expected synchronization and error $e(t)$ of drive system variable $x(t)$ (solid line), $\tau=30$

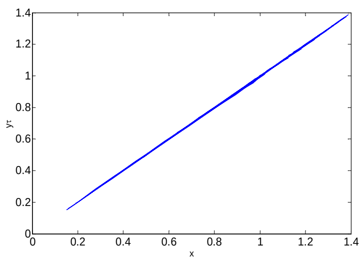

Figure 2.The phase diagram of state variables $x(t)$ and $y(t-(\tau_1-\tau_2)$, $\tau=30$

B. The Expected Synchronization Coupled with Two Different Delays

Drive system and response system coupled with two different delays are as follows:

$$\frac{dx(t)}{dt}=-cx(t)+\frac{ax_{\tau_1}}{1+x_{\tau_1}^b} \tag{24}$$

$$\frac{dy(t)}{dt}=-cy(t)-\frac{ax_{\tau_2}}{1+x_{\tau_2}^b} \tag{25}$$

Define the error

$$e(t)=x(t)-y_{\tau_1-\tau_2},\ \tau_1>\tau_2$$

According to the theoretical analysis, when $c>0$, the expected synchronization between the drive system and the response system can always achieve. And when $t\to\infty, e(t)\to 0$, selecting the system parameters $a=0.2,\ b=10,\ c=0.1$, numerical simulation is shown in Figure 3-4.

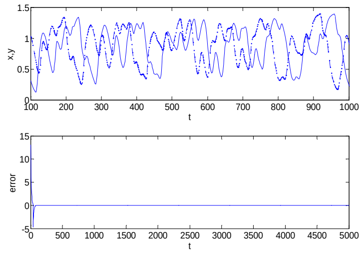

Figure 3. Response system variable $y(t)$ (dashed lines) predicting the expected synchronization and error $e(t)$ of drive system variable $x(t)$ (solid line), $\tau_1=45,\ \tau_2=5$

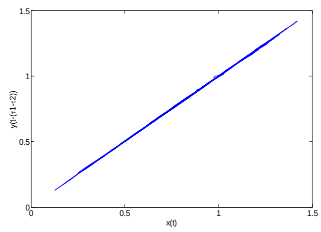

Figure 4. The phase diagram of state variables $x(t)$ versus $y(t-(\tau_1-\tau_2)$, $\tau_1=45,\ \tau_2=5$

C. The Expected Synchronization between Time-delay Systems Coupled with Multiple Delays

In the last section, we give the general form of time-delay systems coupled with multiple delays. Given different values for parameter n, the corresponding numerical simulations between coupled systems are similar So, in this section we will only discuss the expected synchronization taking the Mackey-Glass system as an example in the case of $n=1$ and $n=2$.

The nonlinear part of the Mackey-Glass system is expressed as:

When $n=1$, drive system and response system are as follows:

$$dx(t)/dt=-cx(t)+m_{1x}f(x_{\tau_1}) \tag{26}$$

$$dy(t)/dt=-cy(t)+m_{1y}f(y_{\tau_1})-kf(x_{\tau_k}) \tag{27}$$

Define the error $e(t)=x(t)-y_{\tau_1-\tau_k}$ $(\tau_1>\tau_k)$, then when $m_{1x}+k=m_{1y}$, the error system is:

$$de(t)/dt=-ce(t)+m_{1y}f'(\xi)e_{\tau_1} \tag{28}$$

Where

$\xi\in(\theta_1,\theta_2)$, $\theta_1=\min\{x_{\tau_1}, y_{2\tau_1-\tau_k}\}$, $\theta_2=\max\{x_{\tau_1}, y_{2\tau_1-\tau_k}\}$

According to the theoretical analysis is easy to know that when $c > |m_{1y} f'(\xi)|$, the expected synchronization can always achieve. In [15, 18],

$$x= ((b+1)/ (b-1))^{1/b}$$

Takes $|f'(x_\tau)|$ the maximum value, and maximum value is $a\,(b-1)^2/4b$.

Therefore, other appropriate parameters are easily selected to achieve the expected synchronization between the drive system and response system.

Selecting the parameters:

$$a=0.2,\ b=10,\ c=0.1,$$
$$\tau_1=50,\ \tau_k=20,$$
$$m_{1x}=1,\ m_{1y}=0.1,\ k=-0.9$$

Numerical simulation is shown in Figure 5-6.

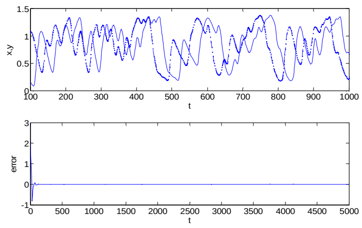

Figure 5. Response system variable $y\ (t)$ (dashed lines) predicting the expected synchronization and error $e\ (t)$ of drive system variable $x\ (t)$ (solid line), $\tau_1=50,\ \tau_2=20$

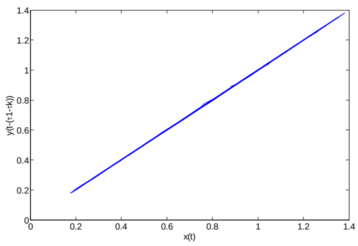

Figure 6.The phase diagram of state variables $x\ (t)$ and $y\ (t-(\tau_1-\tau_2),\ \tau_1=5,\ \tau_2=20$

When $n = 2$, the corresponding drive system and response system are as follows:

$$dx(t)/dt = -cx(t) + m_{1x} f(x_{\tau_1}) + m_{2x} f(x_{\tau_2}) \qquad (29)$$

$$dy(t)/dt = -cy(t) + m_{1y} f(y_{\tau_1}) + m_{2y} f(y_{\tau 2}) - kf(x_{\tau_k}) \quad (30)$$

Here we discuss the systems and simulation results dividing two kinds of situations under two different errors first, we discuss the first case defining error:

$$e(t) = x(t) - y_{\tau_1-\tau_k}\ (\tau_1 > \tau_k).$$

When $m_{1x} + k = m_{1y}$, $m_{2x} = m_{2y}$ the error system is:

$$de(t)/dt = -ce(t) + m_{1y} f'(\xi_1) e_{\tau_1} + m_{2y} f'(\xi_2) e_{\tau_2} \qquad (31)$$

The expected synchronization between the drive system and the response system can always achieve. Selecting the parameters:

$$a=0.2,\ b=10,\ c=0.1$$
$$\tau_1=25,\ \tau_2=30,\ \tau_k=10$$
$$m_{1x}=1,\ m_{1y}=-0.1,\ m_{2x}=m_{2y}=0.1,\ k=-1.1$$

Numerical simulation is shown in Figure 7-8.

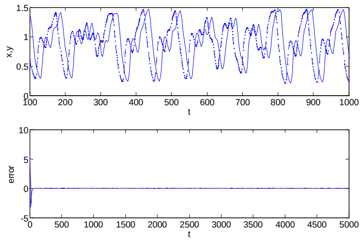

Figure 7. Response system variable $y\ (t)$ (dashed lines) predicting the expected synchronization and error (t) of drive system variable $x\ (t)$ (solid line), $\tau_1=25,\ \tau_2=30,\ \tau_k=10$

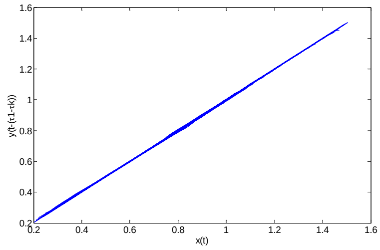

Figure 8.The phase diagram of state variable x (t) and τ1=25, τ2=25, τk=10, y (t-(τ1-τ2)

Next, we discuss the second case defining the error

$$e\ (t) = x\ (t) - y_{\tau 1-\tau k},\ \tau_1 > \tau_k.$$

when $m_{1x} = m_{1y}$, $m_{2x}+k = m_{2y}$ he error system is

$$de(t)/dt = -ce(t) + m_{1y} f'(\xi_1) e_{\tau_1} + m_{2y} f'(\xi_2) e_{\tau_2} \qquad (32)$$

The expected synchronization between the drive system and the response system can always achieve.

Selecting the parameter:

$$a=0.2,\ b=10,\ c=0.1$$
$$\tau_1=50,\ \tau_2=35,\ \tau_k=20$$
$$m_{1x}= m_{1y}=-0.05,\ m_{2y}=0.1,\ m_{2x}=0.9,\ k=-0.8$$

Numerical simulation is shown in Figure 9-10.

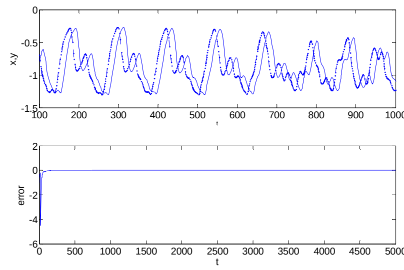

Figure 9. Response system variable $y\ (t)$ (dashed lines) predicting the expected synchronization and error $e(t)$ of drive system variable $x\ (t)$ (solid line), $\tau_1=50,\ \tau_2=35,\ \tau_k=20$

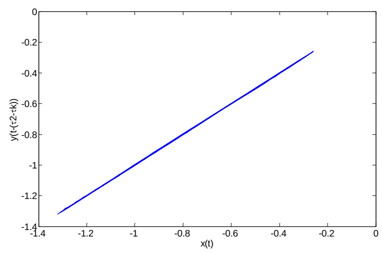

Figure 10. The phase diagram of state variables $x\ (t)$ and $y\ (t-(\tau_1-\tau_2)$, $\tau_1=50,\ \tau_2=35,\ \tau_k=20$

VI. The RP Expected Synchronization of Mackey-Glass Time-delay Systems

Referring to of the three cases of the RP expected synchronization theory in Section 4.4, we will conduct example analysis and numerical simulation for the RP expected synchronization of Mackey-Glass time-delay systems followed by the three cases.

A. The RP Expected Synchronization Coupled with A Delay

Drive system and response system are as follows:

$$\frac{dx(t)}{dt} = -cx(t) + \frac{ax_\tau}{1 + x_\tau^b} \tag{33}$$

$$\frac{dy(t)}{dt} = -cy(t) - \frac{ax(t)}{1 + x(t)^b} \tag{34}$$

According to the theoretical analysis, the RP expected synchronization between the drive system and the response system can be achieved. Selecting the system parameters $a=0.2,\ b=10,\ c=0.1$, numerical simulation is shown in Figure 11-12.

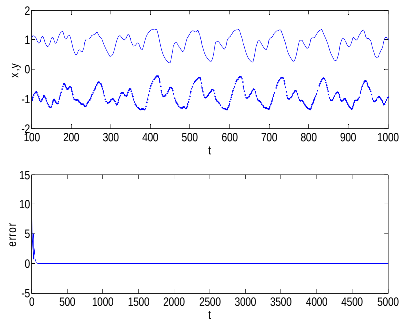

Figure 11. Drive system variable $x\ (t)$ (solid line), response system variable $y\ (t)$ (dashed lines), and error $e\ (t) = x\ (t) + y\ (t\text{-}\tau)$, $\tau_1=30$

Simulation results show that: when achieving synchronization, the response variables $y\ (t)$ struck and driving variables $x\ (t)$, strack are opposite, and the whole translates τ time units toward left.

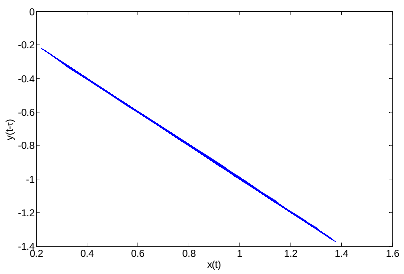

Figure 12.The phase diagram of state variables $x\ (t)$ and $y\ (t\text{-}\tau)$, $\tau=30$

B. The RP Expected Synchronization Coupled with Two Different Delays

Drive system and response system coupled with two different delays are as follows:

$$\frac{dx(t)}{dt} = -cx(t) + \frac{ax_{\tau_1}}{1 + x_{\tau_1}^b} \tag{35}$$

$$\frac{dy(t)}{dt} = -cy(t) - \frac{ax_{\tau_2}}{1 + x_{\tau_2}^b} \tag{36}$$

According to the theoretical analysis in section 4.2, when $c<0$, the *RP* expected synchronization between the system (35) and (36) can always achieve. And Selecting the system parameters $a=0.2,\ b=10,\ c=0.1$, numerical simulation is shown in Figure 13-14.

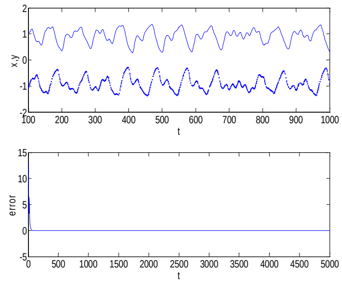

Figure 13. Drive system variable $x\ (t)$ (solid line), response system variable $y\ (t)$ (dashed lines), and error $e\ (t) = x\ (t) + y\ (t\text{-}(\tau_1\text{-}\tau_2))$, $\tau_1=25,\ \tau_2=11$

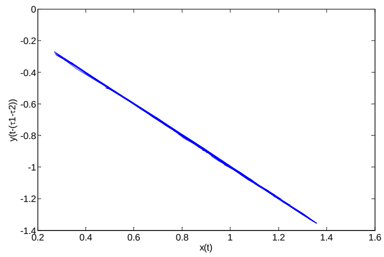

Figure 14. The phase diagram of state variables $x\ (t)$ and $y\ (t\text{-}(\tau_1\text{-}\tau_2))$, $\tau_1=25,\ \tau_2=11$

C. The RP Expected Synchronization Coupled with Multiple Delays

In the section 5, we give the general form of time-delay systems coupled with multiple delays. Given different values for parameter n, the corresponding numerical simulations are similar so, in this section we will only discuss the RP expected synchronization of the Mackey-Glass system in the case of $n=1$ and $n=2$ [21], and denote it as

$$f(x(t-\tau)) = ax_{t-\tau} / (1 + x_{t-\tau}^b)$$

When $n=1$, drive system and response system are as follows:

$$dx(t)/dt = -cx(t) + m_{1x} f(x_{\tau_1}) \tag{37}$$

$$dy(t)/dt = -cy(t) - m_{1y} f(y_{\tau_1}) + kf(x_{\tau_k}) \tag{38}$$

Define the error, $e\ (t) = x\ (t) - y_{\tau_1-\tau_k},\ \tau_1>\tau_k$

The error system is:

$$de(t)/dt = -ce(t) + m_{1y} f'(\xi) e_{\tau_1} \tag{39}$$

where $\theta_2 = \max\{x_{\tau_1}, y_{2\tau_1-\tau_k}\}$, $\theta_1 = \min\{x_{\tau_1}, y_{2\tau_1-\tau_k}\}$ $\xi \in (\theta_1, \theta_2)$

When $c > |m_{1y} f'(\xi)|$, the RP expected synchronization between the drive system and the response system can always achieve. Selecting the parameters:

a=0.2, b=10, c=0.1

$\tau_1=30,\ \tau_k=2$

$m_{1x}=1,\ m_{1y}=0.01,\ k=-0.99$

Numerical simulation is shown in Figure 15-16.

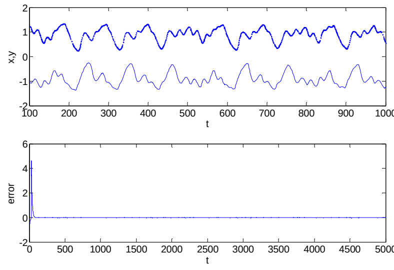

Figure 15. Drive system variable $x(t)$ (solid line), response system variable $y(t)$ (dashed lines), and error $e(t) = x(t) + y(t-(\tau_1\text{-}\tau_2))$, $\tau_1=30,\ \tau_2=2$

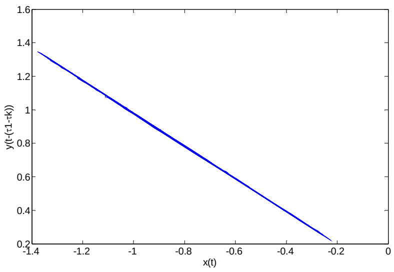

Figure 16. The phase diagram of state variables $x(t)$ and $y(t-(\tau_1\text{-}\tau_2))$, $\tau_1=30,\ \tau_2=2$

When $n=2$, the corresponding drive system and response system are as follows:

$$dx(t)/dt = -cx(t) + m_{1x} f(x_{\tau_1}) + m_{2x} f(x_{\tau_2}) \qquad (40)$$

$$dy(t)/dt = -cy(t) - m_{1y} f(y_{\tau_1}) - m_{2y} f(y_{\tau 2}) + kf(x_{\tau_k}) \qquad (41)$$

Here we discuss the systems and simulation results dividing two kinds of situations under two different errors first, we discuss the first case defining error:

$$e(t) = x(t) - y_{\tau 1\text{-}\tau k},\ \tau_1 > \tau_k.$$

When $m_{1x}+k=m_{1y}$, $m_{2x}=m_{2y}$, the error system is:

$$de(t)/dt = -ce(t) + m_{1y} f'(\xi_1) e_{\tau_1} + m_{2y} f'(\xi_2) e_{\tau_2} \qquad (42)$$

Selecting the parameters:

$a=0.2,\ b=10,\ c=0.1$

$\tau_1=30,\ \tau_2=20,\ \tau_k=15$

$m_{1x}=0.99,\ m_{1y}=-0.01,\ m_{2x}=m_{2y}=0.01,\ k=-1$

Numerical simulation is shown in Figure 17-18.

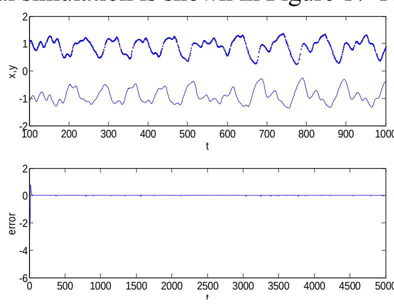

Figure 17 Drive system variable $x(t)$ (solid line), response system variable $y(t)$ (dashed lines), and error $e(t) = x(t) + y(t-(\tau_1\text{-}\tau_2))$, $\tau_1=30,\ \tau_2=20,\ \tau_k=15$

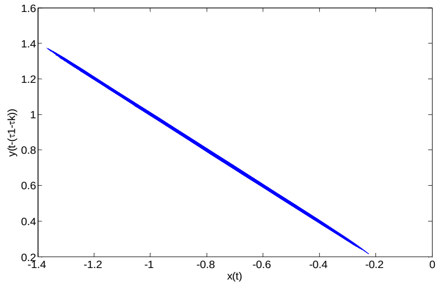

Figure 18. The phase diagram of state variables $x(t)$ and $y(t-(\tau_1\text{-}\tau_2))$, $\tau_1=30,\ \tau_2=20,\ \tau_k=15$

The *RP* expected synchronization between the drive system and the response system can always achieve.

Selecting the system parameters:

$a=0.2,\ b=10,\ c=0.1$

$\tau_1=25,\ \tau_2=35,\ \tau_k=5$

$m_{1x}= m_{1y}=-0.01,\ m_{2x}=0.79,\ m_{2y}=0.01,\ k=-0.8$

The corresponding numerical simulation is shown in Figure 19-20.

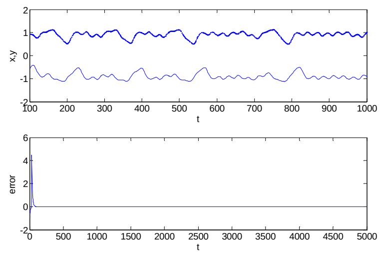

Figure 19. Drive system variable $x(t)$ (solid line), response system variable $y(t)$ (dashed lines), and error $e(t) = x(t) + y(t-(\tau_1\text{-}\tau_2))$, $\tau_1=25,\ \tau_2=35,\ \tau_k=5$

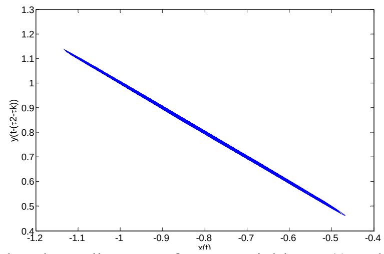

Figure 20 the phase diagram of state variables $x(t)$ and $y(t-(\tau_1\text{-}\tau_2))$, $\tau_1=25,\ \tau_2=35,\ \tau_k=5$

VII. CONCLUSIONS

In this paper, with a time delay, two different delays and multiple delays between the two coupled time-delay systems the basic theories of the expected synchronization and RP expected synchronization are respectively conducted a detailed derivation. And sufficient conditions, which the special form and general form between coupling time-delay systems have to meet to achieve the synchronization and the certification process are given. On the basis of theoretical derivation, to the Mackey-Glass system for instance, the general case and special case of the expected synchronization and RP expected synchronization are conducted numerical simulation and the simulation results further verify the correctness of theoretical analysis

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