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Abstract—The dynamic behavior of fractional order systems have received increasing attention in recent years, In this paper the reliable phase synchronization problem between two coupled chaotic fractional order system with varying time is constructed, An active delay expected synchronization between two coupled chaotic fractional order systems and hyperchaotic fractional order systems is analyzed by utilizing Laplace transform. Furthermore, we investigated the necessary conditions for fractional order Rossler systems to exhibit chaotic attractor. Then, based on the stability results of fractional order systems, sufficient conditions for phase synchronization of the fractional models of Rossler systems and hyperchaotic system are derived. The synchronization scheme that is simple and global enabled synchronization of fractional order chaotic systems to be achieved, Theory analysis and corresponding numerical simulations results show that the chaos in such fractional order system with varying time delay can be synchronized and the method is effective and feasible.

Index Terms—autonomous system, the expected synchronization, fractional order, high dimension

I. INTRODUCTION

In recent years, chaos control and synchronization have become a hot topic in the study of nonlinear systems. It has been applied in many areas, for example, in secure communication, model of brain waves and image recognition, etc. [1-2]. Up to now, researchers have proposed several types of synchronization, such as the classic complete synchronization [3], effectively complete synchronization [4], the reverse synchronization [5-6], generalized synchronization [7], phase synchronization [1], projective synchronization [4], the expected synchronization and delay synchronization [8-10].

In the chaos control and synchronization, the unique nature of the chaotic system itself, such as extreme sensitivity to initial values and long-term unpredictability receive long-term attention by people. In 2000, Voss proposed a new method, known as the expected synchronization [11]. In this method, the response system can keep synchronization with the future state of drive system, which was the response system predicting the future state of the drive system. The method was affirmed by the people, but in further study, the researchers found that there were some limitations in this method [12-13], in which the expected synchronization of Voss method is only suitable for continuous chaos systems with delay, and has some constraints on the expected time with the coupling coefficiency. To solve the limitations of Voss, the 12th literature proposed a method of Coupled bidirectional delay, and a sufficient condition theoretical framework based on Krasovskill-Lyapunov for the independence of delay, by which the expected synchronization of the system was researched in numerical simulation, a fractional order Rossler system as an example.

This paper is structured as follows, Section I describes the theoretical analysis of expected synchronization of autonomous systems, Section II describes the expected synchronization of three-dimensional autonomous system, Section III presents the expected synchronization of high dimensional autonomous system, and the last part contains conclusions.

II. THE EXPECTED SYNCHRONIZATION THEORY OF AUTONOMOUS SYSTEMS

Consider the following form of the nonlinear systems, namely

$$dx / dt = Ax + f(x) \tag{1}$$

Where xR^{n} is n-dimensional state vector, $A \in R^{n \times n}$ is Constant coefficient matrix, and f (x) is nonlinear vector function of the system (1)

$$f(x) = (f_1(x), f_2(x), \cdots, f_n(x))^T, \mathbb{R}^n \to \mathbb{R}$$

Is nonlinear vector function of the system. Assumption 1: Assume that the state of chaotic systems is bounded in this chapter, that is

 \mathbf{x}_i (t) $\leq M_i$, where $M_i \in R$, (*i=1, 2, …, n*).

Assumption 2: Assume that nonlinear function f(x) satisfies the Lipschitz condition

$$\left|f_{i}(x) - f_{j}(x)\right| \leq l_{i} \max_{i \leq j \leq n} \left|x_{i} - y_{j}\right|$$

$$\tag{2}$$

Where $y_{\tau} = y(t-\tau)$, For every initial value and expected time, if the equation $\lim_{x\to\infty} ||x(t) - y(t-\tau)|| = 0$ holds, the response system (3) is able to predict the states of the drive system (1) in the interval (t, t+ τ).

Define the state error equation as follows:

$$E(t) = x(t) - y(t - \tau)$$
(3)

Equation (3) minus equation (1), then the dynamic system expression for the error equation is (3), where $\Omega \in \mathbb{R}^{n}$ belongs to the boundary of all attractors in the system (1), and l_{i} is a positive number, often called the Lipschitz constant.

Note 1: Most of the chaotic systems existing almost meet the assumptions 1 and assumption 2. See [14], in which describes boundaries of the most chaotic systems.

If the partial derivative $|\partial f_i / \partial x_j|, i, j = 1, \dots, n$ is bounded in the right end of the system (1), assumption 2 is established. In general, most of the chaotic systems meet these conditions [12].

Consider the following two intercoupling systems:

$$dx / dt = Ax + f(x) + K_1(x - y_{\tau})$$
(4)

$$dy / dt = Ay + f(y) + K_2(y_{\tau} - x)$$
(5)

 $de / dt = Ae(t) + F(e(t)) + K_1 e(t) + K_2 e(t - \tau)$ (6) where F(e(t)) = f(e(t)) - f(x(t) - e(t))

We know that if the origin of the error system (6) is asymptotically stable, the equation

 $\lim ||x(t) - y(t - \tau)|| = 0 \quad \text{must set up.}$

Based on the premise theory, similar to the Theorem 1 in [12], we can derive the sufficient condition, which can ensure that the origin of error system (6) is asymptotically stable. It must be pointed out that, there are little errors for the proof process of the sufficient condition. Once again, we give the sufficient condition, and give the proof process corrected.

Theorem 1: If there are two constant matrices k_1 , k_2 to ensure that the following matrix M is negative definite, the origin of error system (6) is asymptotically stable.

Where $l = \{l_1, l_2, l_3 \dots l_n\}$, *n* is the number of non-zero l_i , *I* is a unit matrix and constant matrix $B = (2ml+1)I = I \cdot b_1$.

Proof: First, construct the Lyapunov Function

$$V(t) = e^{T}(t)e(t) + \int_{t-\tau}^{t} e^{T}(s)e(s)ds$$
(7)

Along the track of the error system (7), we can get the derivative of Lyapunov function, that is:

$$\dot{V}(t) = e^{T}(t)\dot{e}(t) + e^{T}(t)e(t) - e^{T}(t-\tau)e(t-\tau)$$

= $e^{T}(t)(A + K_{1} + A^{T} + K_{1}^{T}I)e(t) + e^{T}(t)K_{2}e(t-\tau)$
+ $e^{T}(t)F(e) + F^{T}(e)e(t) + e^{T}(t-\tau)K_{2}^{T}e(t) - e^{T}(t-\tau)e(t-\tau)$

Satisfy conditions, the expected synchronization can be achieved between the drive system (3) and the response system (4). Assuming the coefficient matrix A of power system (1) 2n is n-order Square Matrix; the Square Matrix is the order of. And, for the control matrixes K_1 , K_2 , there exists $2n^2$ unknown parameters, to be identified to get the concrete matrix M. Therefore, if the matrix M is low order matrix, we can easily select the appropriate control matrix K_1 , K_2 , conversely, if the matrix MLipschitz inequality described by application assumptions 2, we get:

$$V(t) \leq e^{T}(t)(A + K_{1} + A^{T} + K_{1}^{T} + (2m+l))e(t) + e^{T}(t)K_{2}e(t-\tau) + e^{T}(t)K_{2}e(t-\tau)K_{2}^{t}$$
(9)
= $(e^{T}(t), e^{T}(t-\tau))M\begin{pmatrix} e(t) \\ e^{T}(t-\tau) \end{pmatrix}$

Therefore, if the matrix

.

$$M = \begin{bmatrix} A + K_1 + A^T + K_1^T & K_2 \\ K_2^T & -I \end{bmatrix}$$
(10)

Is negative definite, $V(t) \le 0$ always holds, and when e(t)=0, V(t)=0.

Proof process is over

From Theorem 5.1 we know that if the coefficient matrixes K_1 , K_2 are able to be found to be higher order matrix, selecting the appropriate control matrix K_1 , K_2 , and satisfying the matrix M negative definite will become more complicated. To simplify the process, it considered only a special case in the actual numerical analysis, namely, the control matrix K_1 , K_2 , are diagonal matrixes and the matrix M is diagonally dominant matrix.

III. THE EXPECTED SYNCHRONIZATION OF THREE-DIMENSIONAL AUTONOMOUS SYSTEM

In it takes integer-order three-dimensional autonomous Rossler system as simulation object in some numerical simulations, after the establishment of the form (1-2) of the drive system and response system, achieves the expected synchronization between the drive system and response system. At present, the dynamics analysis of the fractionalorder chaotic system and control synchronization have also drawn common concern of the researchers. Just think: Can the expected synchronization theory described in the previous section achieve expected synchronization of the fractional- order chaotic system. In this section, we're going to explore the issue.

First, taking integer-order Liu chaotic system of three-dimensional autonomous system as the analysis object [15], we discuss the expected synchronization on integer-order Liu chaotic system and the corresponding fractional-order Liu chaotic system.

Corresponding to (1-2)'s drive system and response system are as follows:

$$\begin{cases} dx_1 / dt = a(y_1 - x_1) + k_{11}(x_1 - x_{2,\tau}) \\ dy_1 / dt = bx_1 - cx_1z_1 + k_{12}(y_1 - y_{2,\tau}) \\ dz_1 / dt = -dz_1 + hx_1^2 + k_{13}(z_1 - z_{2,\tau}) \end{cases}$$
(11)

$$\begin{cases} dx_2 / dt = a(y_2 - x_2) + k_{21}(x_1 - x_{2,\tau}) \\ dy_2 / dt = bx_2 - cx_2z_2 + k_{22}(y_1 - y_{2,\tau}) \\ dz_2 / dt = -dz_2 + hx_2^2 + k_{23}(z_1 - z_{2,\tau}) \end{cases}$$
(12)

where system parameters take

a=10, b=40

The integer-order Liu chaotic system shows chaotic behavior. According to theorem 1, if we can find suitable control matrix K_1 , K_2 , to make the matrix M negative definite, the expected synchronization between the drive system and response system can achieve for any initial value and the expected time τ . Here we analyze how to select the appropriate control matrix K_1 , K_2 , and achieve the expected synchronization of the drive system (11) and the response system (12) through specific numerical simulation. In section 1, we analyzed the complex impact that the difference of the matrix M order made the sufficient condition of the matrix M negative definite achieved. For specific examples of integer-order Liu chaotic system, the order of Matrix M is 6, so we can assume that the matrix M is diagonally dominant matrix, and control matrix K_1 , K_2 , is the diagonal matrix to simplify the analysis process.

Assuming control matrixes

 $K_{1} = diag (k_{11}, k_{12}, k_{13})$ $K_{2} = diag (k_{21}, k_{22}, k_{23})$ C constant matrix and unit matrix $B = diag (b_{1}, b_{2}, b_{3}, b_{4}),$ I = diag (1, 1, 1)

And the coefficient matrix of integer-order Liu chaotic system

A = [-a, a, 0; b, 0, 0; 0, 0, -d]

Then, according to Theorem 1, the system coefficient matrix A, control matrix K_1 , K_2 , and unit matrix I substituted into matrix M, we can get the expression of matrix M:

$$M = \begin{bmatrix} A + K_1 + A^T + K_1^T & K_2 \\ K_2^T & -I \end{bmatrix}$$
(13)

If diagonally dominant matrix M is negative definite, the following inequality must be true.

That is when (13) holds, the matrix M is negative definite. We carry out specific numerical simulation here. Observing inequalities (14), we found that if the value of the control matrix K_2 was determined, selecting the control matrix K_1 will become very simple. In addition, parameters can be determined through the Lipschitz condition, so m=2, $l\approx 150.993$, $b_1=2ml+1\approx 605$.

$$\begin{cases} -2a + 2k_{11} + b_1 < -|a+b| - |k_{21}| \\ 2k_{12} + b_1 < -|a+b| - |k_{22}| \\ -2d + 2k_{13} + b_1 < -|k_{23}| \\ |k_{21}|, |k_{22}|, |k_{23}| < 1 \end{cases}$$
(14)

For the special case, when control matrix K_2 is zero matrixes, and substituting the system parameters and b_1 into the inequalities (13), we can get the range of control matrix K_1 each parameter, as follows: $K_{12}=-320, K_{12}=-305, K_{12}=-305.$

$$K_{11} = -320, K_{12} = -330, K_{13} = -305, K_{21} = K_{22} = K_{23} = 0$$

Then meeting the conditions of all control parameters above, the expected synchronization between the drive system (11) and the response system (12) can be always achieved, for every initial value and the expected time, and as time goes to infinity, total error (15) tends to zero.

$$E(t) = \sqrt{(x_1 - x_{2,\tau})^2 + (y_1 - y_{2,\tau})^2 + (z_1 - z_{2,\tau})^2}$$
(15)

Selecting the specific control parameters

$$K_{11}$$
=-317.5, K_{12} =-327.5, K_{13} =-300
 K_{21} = K_{22} = K_{23}

Initial values: $X_1 = [1, 2, 3]$, $X^2 = [-3, -2, -1]$ and the expected time $\tau = 20$ then the corresponding simulation results are shown in Figure 1-2.

$$\begin{array}{c} & & \\ & &$$

Figure 1. Response system variable $X_2(t)$ (dashed lines) predicting drive system variable $X_1(t)$ (solid line)



Figure 2. The total error E(t).

Considering the general case that element of the control matrix is not all zero, if selecting the control parameters and substituting the system parameters and constant b_1 into the inequalities (15),

$$K_{21}$$
=-0.5, K_{22} =0.5, K_{23} =0.8

The range of control matrix K_{I} , each parameter is as follows: K_{II} , K_{I2} , K_{I3} satisfying the inequality (14), the expected synchronization between the drive system (11) and the response system (12) can be always achieved, and as time goes to infinity, total error E(t) tends to zero.

Selecting the initial values $X_1 = [-1, 2, 5], X_2 = [3, -2, 4]$ and the expected time

$$K_{11}$$
=-318, K_{12} =-330, K_{13} =-301,
 K_{21} =-0.5, K_{22} =0.5, K_{23} =0.8,
 τ =15

Then the corresponding simulation results are shown in Figure 3-4.



Figure 3. Response system variable X_l (*t*) (dashed lines) predicting drive system variable X_l (*t*) (solid line)



Figure 4. The total error E(t)

Selecting the above two sets of specific control matrixes K_1 , K_2 , the simulation results are achieved for every initial value and every expected time τ , on the expected synchronization between drive system (11) and the response system (12). Similarly, selecting another group to satisfy inequalities (15) of the control matrix parameters, drive systems and response systems can also achieve the expected synchronization.

The expected synchronization of the corresponding fractional-order Liu chaotic system

Taking Integer-order Liu system corresponding to the homogeneous fractional-order Liu system as the simulation object, we discuss the expected synchronization of fractional-order chaotic system. Seeing fractional-order chaotic system diagram of the stable region, we know that the stable region of fractional-order chaotic systems is obviously larger than of the integer-order chaotic systems. Therefore, according to Theorem 1, it is easy to find conditions to meet the expected synchronization of fractional-order chaotic systems. We omit the related theoretical analysis, only taking 0.9 order of the homogeneous fractional-order Liu chaotic system as the simulation example, to study the expected synchronization by the numerical analysis and simulation.

Drive system and response system are as follows:

$$\begin{cases} dx_1^{0.9} / dt^{0.9} = a(y_1 - x_1) + k_{11}(x_1 - x_{2,\tau}) \\ dy_1^{0.9} / dt^{0.9} = bx_1 - cx_1z_1 + k_{12}(y_1 - y_{2,\tau}) \\ dz_1^{0.9} / dt^{0.9} = -dz_1 + hx_1^2 + k_{13}(z_1 - z_{2,\tau}) \end{cases}$$
(16)

$$\begin{cases} dx_2^{0.9} / dt^{0.9} = a(y_2 - x_2) + k_{21}(x_1 - x_{2,\tau}) \\ dy_2^{0.9} / dt^{0.9} = bx_2 - cx_2z_2 + k_{22}(y_1 - y_{2,\tau}) \\ dz_2^{0.9} / dt^{0.9} = -dz_2 + hx_2^2 + k_{23}(z_1 - z_{2,\tau}) \end{cases}$$
(17)

So F(e(t))=f(e(t))-f(x(t)-e(t)), then the error dynamic system is:

$$d^{q}x(t) / dt^{q} = Ae(t) + F(e(t)) + K_{1}e(t) + K_{2}e(t-\tau)$$
(18)

We analyzed the expression of matrix M in Theorem 1, easily found that the system parameter values in the matrix M and symbolic representation of the control matrixes K_1 , K_2 keep consistent for the corresponding integer and fractional-order Liu system, but the constant b_1 determined by the Lipschitz conditions may be different. Therefore, for the fractional-order Liu system, the inequality systems (16) also enables diagonally dominant matrix M to be negative definite. As about the fractional-order Liu system, according to Lipschitz condition, we can determine the parameters m=2, $l\approx$ 120.3976, $b_1=2ml+1\approx482.6$ the system parameters are substituted into (14) get:

$$\begin{cases} k_{11} < (-512.6 - |k_{21}|) / 2 \\ k_{12} < (-532.6 - |k_{22}|) / 2 \\ k_{13} < (-477.6 - |k_{23}|) / 2 \\ |k_{21}|, |k_{22}|, |k_{23}| < 1 \end{cases}$$

The following was the specific numerical simulation. Simulations of two cases are discussed, in special case, when the control matrix K_1 is zero matrixes, the control matrix K_1 , K_2 diagonal elements of the range is:

 $k_{11} < -256.3, k_{12} < -266.3, k_{13} < -239.8$

Selecting the initial $X_1 = [-1, 2, 3]$, $X_2 = [5, -6, 1]$, the expected time and the control parameters:

$$K_{11} = -260, K_{12} = -270, K_{13} < -240$$
$$K_{11} = K_{12} = K_{13} = 0$$
$$\tau = 10$$

The simulation results are shown in Figure 5-6:



Figure 5. Response system variable X_l (*t*) (dashed lines) predicting drive system variable X_l (*t*) (solid line)



Figure 6. The total error E(t)

When the control matrix K_1 is non-zero matrix, and we select K_1 =diag (0.4, 0.6, -0.5), the control matrix diagonal elements of the range is:

$$K_{11}$$
=-257, K_{12} =-267, K_{13} =-240
 K_{11} =0.4, K_{12} =0.6, K_{13} =-0.5

Selecting the initial expected time $X_1 = [-5, 2, 1]$, $X_1 = [1, 6, 3]$, $\tau = 25$. The simulation results are shown in Figure 7-8.



If selecting control parameters, any other initial values, and any expected time to meet conditions can still achieve the expected synchronization of fractional-order Liu chaotic system, these numerical results reconfirm the feasibility of expected synchronization theory this section describes. We can still use the above idea to achieve the expected synchronization of other integer-order and fractional-order dynamic systems, such as integer-order Lorenz system [16], integer-order Chen system [17], integer-order Lü system [18], fractional-order Chua system [19], fractional-order Lorenz system family [20], Lu system [21], etc.

Above, we discussed the application on the expected synchronization theory in integer-order Liu chaotic system of three-dimensional autonomous system and the corresponding fractional-order Liu chaotic system , successfully achieved the expected synchronization for any initial value, any expected time, and confirmed that the theory can be applied in fractional-order chaotic systems.

IV. THE EXPECTED SYNCHRONIZATION OF HIGH DIMENSIONAL AUTONOMOUS SYSTEM

In recent years, dynamics analysis and control of high-dimensional chaotic system also received much concern. After achieving the expected synchronization on three-dimensional integer-order and the corresponding fractional-order chaotic systems, we further envisage whether the expected synchronization can be achieved for high-dimensional case. From the theoretical analysis point of view, the result is positive. In order to support the conclusion in this section more fully, we still discuss the problem taking four-dimensional hyperchaotic Chen system [22] as an example and give the specific simulation results and instructions in the section.

A. The Expected Synchronization of Integer-order Hyperchaotic Chen System

First of all, corresponding to equation (1, 3) of the drive system and response system equations are as follows:

$$\begin{cases} dx_{1} / dt = a(y_{1} - x_{1}) + w_{1} + k_{11}(x_{1} - x_{2,r}) \\ dy_{1} / dt = dx_{1} - x_{1}z_{1} + cy_{1} + k_{12}(y_{1} - y_{2,r}) \\ dz_{1} / dt = x_{1}y_{1} - bz_{1} + k_{13}(z_{1} - z_{2,r}) \\ dw_{1} / dt = y_{1}z_{1} + rw_{1} + k_{14}(w_{1} - w_{2,r}) \end{cases}$$
(19)

$$dx_{2} / dt = a(y_{2} - x_{2}) + w_{2} + k_{21}(x_{2,r} - x_{1})$$

$$dy_{2} / dt = dx_{2} - x_{2}z_{2} + cy_{2} + k_{22}(y_{2,r} - y_{1})$$

$$dz_{2} / dt = x_{2}y_{2} - bz_{2} + k_{23}(z_{2,r} - z_{1})$$

$$dw_{2} / dt = y_{2}z_{2} + rw_{2} + k_{24}(w_{2,r} - w_{1})$$
(20)

where, when the system parameters are: a=35, b=3, c=12, d=7, r=0.5. The integer-order hyperchaotic Chen system presents chaotic state.

Next, we determine the control parameter matrixes K_1 , K_2 to meet the Theorem 5.1. For the expected synchronization of high-dimensional chaotic system, we can assume that the matrix M is diagonally dominant matrix to find the appropriate control matrixes K_1 , K_2 , which has been discussed. When the coefficient matrix A of hyperchaotic Chen system will be substituted into the matrix M, the matrix M of the form can be obtained as

$$M = \begin{bmatrix} A + K_1 + A^T + 2m + I & K_2 \\ K_2^T & -I \end{bmatrix}$$
(21)

According to Theorem 5.1, if the matrix M is negative definite, the following inequalities systems hold as (22).

The parameters can *m* be determined through Lipschitz condition, so $m=3 \ l \approx 61.14444 \ b_1=2ml+1 \approx 368$. It can be found from the above inequality, as long as the value of taking control matrix K_2 is set, it is easy to obtain the range of control matrix K_1 .

$$\begin{cases} -2a + 2k_{11} + b_1 < -|a + d| - 1 - |k_{21}| \\ 2c + 2k_{12} + b_1 < -|d + a| - |k_{22}| \\ -2b + 2k_{13} + b_1 < -|k_{23}| \\ 2r + 2k_{14} + b_1 < -1 - |k_{24}| \\ |k_{21}|, |k_{22}|, |k_{23}|, |k_{24}| < 1 \end{cases}$$
(22)

First, considering the special case, assuming K_2 =diag (0.0.0.0), we make the system parameters of hyperchaotic Chen system substituted into the simplified inequality (22) equations, therefore, the range of control matrix K_1 , K_2 can be obtained.

 $K_{11} < -170.5, K_{12} < -217, K_{13} < -181, K_{14} < 185$

According to Theorem 5.1, for every initial value and expected time, the expected synchronization between the drive system (19) and the response system (20) can be always achieved. And for total error:

$$E(t) = \sqrt{(x_1 - x_{2,\tau})^2 + (y_1 - y_{2,\tau})^2 + (z_1 - z_{2,\tau})^2}$$

As time goes to infinity, the total error (t) tends to zero. Selecting the control matrix parameters

$$K_{11}$$
=-171, K_{12} =-220, K_{13} =-185, K_{14} <-190
 K_{21} = K_{22} = K_{23} = K_{24} =0

Initial value $X_2 = [-4, -3, -2, -1]$, $X_1(0) = [1, 2, 3, 4]$ the expected time $\tau = 10$, the corresponding simulation results are shown in Figure 9-10.



Figure 9. Response system variable X_l (*t*) (dashed lines) predicting drive system variable X_l (*t*) (solid line)



Figure 10. The total error E (t)

Secondly, considering the the control matrix is non-zero matrix. Assuming control matrix

 K_1 =diag (0.5, -0.5, 0.8, -0.2)

The control parameters of hyperchaotic Chen system substituted into the simplified inequality equations (22), the range of control matrix K_I is:

 K_{11} <-170.75, K_{12} <-217.25, K_{13} <-181.4, K_{14} <-185.1 Selecting the specific control parameters K_{11} <-180, K_{12} <-220, K_{13} <-190, K_{14} <-188 Initial values

 $X_1(0) = [1, 0, 3, 8], X_2(0) = [-14, -3, 12, 0]$ While the expected time is $\tau = 50$, the corresponding simulation results are shown in Figure 11, 12.



Figure 11. Response system variable X_I (*t*) (dashed lines) predicting drive system variable X_I (*t*) (solid line)



Figure 12. The total error E(t)

From the special cases to the general ones, the simulation results show that Theorem 1 can achieve the expected synchronization of integer-order hyperchaotic Chen system, and the overall error E(t) will tend to infinity as

Time tends to zero. Further, Theorem 1 is still able to

B. The Expected Synchronization of Fractional -order Hyperchaotic Chen System

Taking the corresponding fractional-order hyperchaotic Chen system of Integer- order hyperchaotic Chen system as specific simulation object, we discuss the expected synchronization of corresponding drive system and response system. Equations of fractional-order hyperchaotic Chen system are as follows:

$$\begin{cases} d^{q_1} x / dt^{q_1} = a(y-x) + w \\ d^{q_2} y / dt^{q_2} = dx - xz + cy \\ d^{q_3} z / dt^{q_3} = xy - bz \\ d^{q_4} w / dt^{q_4} = yz + rw \end{cases}$$
(23)

when the order is $q_{1=} q_{2=} q_{3=} q_{4-}$

According to the stability theory of fractional-order systems, we can obtain the necessary condition of presenting chaotic behavior of fractional hyperchaotic Chen system by calculating, which is q>0.9630. If the stable regions of corresponding fractional-order chaotic systems are larger than of integer-order chaotic system, Theorem 1 will apply equally to the expected synchronization of corresponding fractional chaotic system.

According to the necessary condition, we take the 0.98 order homogeneous hyperchaotic Chen system as the research object. In corresponding to the drive system (24) and response system of equations are as follow:

$$\begin{cases} d^{0.98} x_1 / dt^{0.98} = a(y_1 - x_1) + w_1 + k_{11}(x_1 - x_{2,\tau}) \\ d^{0.98} y_1 / dt^{0.98} = dx_1 - x_1 z_1 + cy_1 + k_{12}(y_1 - y_{2,\tau}) \\ d^{0.98} z_1 / dt^{0.98} = x_1 y_1 - bz_1 + k_{13}(z_1 - z_{2,\tau}) \\ d^{0.98} w_1 / dt^{0.98} = y_1 z_1 + r w_1 + k_{14}(w_1 - w_{2,\tau}) \end{cases}$$

$$\begin{cases} d^{0.98} x_2 / dt^{0.98} = a(y_2 - x_2) + w_2 + k_{21}(x_{2,\tau} - x_1) \\ d^{0.98} y_2 / dt^{0.98} = dx_2 - x_2 z_2 + cy_2 + k_{22}(y_{2,\tau} - y_1) \\ d^{0.98} z_2 / dt^{0.98} = x_2 y_2 - bz_2 + k_{22}(z_2 - z_1) \end{cases}$$
(24)

 $\begin{cases} d^{0.98}z_2 / dt^{0.98} = x_2 y_2 - bz_2 + k_{23}(z_{2,r} - z_1) \\ d^{0.98}w_2 / dt^{0.98} = y_2 z_2 + r w_2 + k_{24}(w_{2,r} - w_1) \end{cases}$

And the corresponding error system is:

 $d^{0.98}e(t) / dt^{0.98} = Ae(t) + F(e(t)) + K_1e(t) + K_2e(t-\tau)$ (26) where

$$e(t) = (e_1(t), e_2(t), e_3(t), e_4(t))^t$$

$$F(e(t)) = f(e(t)) - f(x(t) - e(t))$$

As Theorem *I* also applies to the corresponding fractional-order chaotic systems, similarly to the analysis process of the section, we can get the value conditions met by the control matrixes K_1 , K_2 . According to Theorem 1, if the matrix is negative definite,

$$M = \begin{bmatrix} A + K_1 + A^T + 2m + I & K_2 \\ K_2^T & -I \end{bmatrix}$$
(27)

The following inequality hold:

$$\begin{cases} -2a + 2k_{11} + b_1 < -|a+d| - 1 - |k_{21}| \\ 2c + 2k_{12} + b_1 < -|d+a| - |k_{22}| \\ -2b + 2k_{13} + b_1 < -|k_{23}| \\ 2r + 2k_{14} + b_1 < -1 - |k_{24}| \\ |k_{21}|, |k_{22}|, |k_{23}|, |k_{24}| < 1 \end{cases}$$

$$(28)$$

Where

A=[-a, a, 0, 1; d, c, 0, 0; 0, 0, -b, 0; 0, 0, 0, r], I=diag (1, 1, 1, 1) $B=diag (b_1, b_2, b_3, b_4),$ $K_1=diag (k_{11}, k_{12}, k_{13}, k_{14}),$ $K_2=diag (k_{21}, k_{22}, k_{23}, k_{24})$

By analyzing the 0.98 order hyperchaotic Chen system of equations, we can obtain

m=3, $l\approx69.4517$, $b_1=2ml+1\approx417.4$

We carry out specific numerical simulation here. Similarly, we first determine the specific value of the control matrix, next get the range of control matrix K_1 by control matrix K_2 , and then select the appropriate value of the numerical simulation.

Still first consider the special case that the control matrix is zero matrixes. Setting K_2 =diag (0.0.0.0) the system parameters of the fractional-order hyperchaotic Chen system are substituted into (28), and then we can get the range of the control matrix K_1 :

 $K_{11} < -195.35, K_{12} < -241.85, K_{13} < -205.85, K_{14} < -209.85$

For every initial value and expected time, the expected synchronization between the drive system (24) and the response system (25) can be always achieved, and as time goes to infinity, total error E (t) tends to zero.

Select the following specific parameter values

$$K_{11} <-200, K_{12} <-242, K_{13} <-206, K_{14} <-210$$

 $K_{21} = K_{22} = K_{23} = K_{24} = 0$

When the expected time is $\tau = 20$, the initial value is

 $X_1(0) = [1, 20, 3, 0], \quad X_2(0) = [-4, 3, -2, 6]$





Figure 13. Response system variable X_l (*t*) (dashed lines) predicting drive system variable X_l (*t*) (solid line)

Consider the general case that the control matrix K_2 is not zero matrixes. Setting

 $K_2 = diag (0, 0.9, -0.4, 0.2)$ The range of the control matrix is: $K_{11} < -195.35, K_{12} < -242.3, K_{13} < -206.05, K_{14} < -209.9$



Figure 14. The total error E(t)

Similarly, for every initial value and expected time τ , the expected synchronization between the drive system (24) and the response system (25) can be always achieved, and as time goes to infinity, total error E(t) tends to zero.

Select the parameter values of control matrix K_{11} <-200, K_{12} <-245, K_{13} <-207, K_{14} <210 K_{21} =0, K_{22} =0.9, K_{23} =-0.4, K_{24} =0.2

When the expected time is $\tau=30$, the initial value is

 $X_1(0) = [0, -5, 11, 4], \quad X_2(0) = [5, 3, -12, 6]$

The simulation results are shown in Figure 15, 16.



Figure 15. Response system variables $X_I(t)$ (dashed lines) predicting drive system variable $X_I(t)$ (solid line)



Figure 16. The total error E(t)

The results achieved expected above the synchronization of fractional-order hyperchaotic Chen system. We further implement the expected synchronization of integer-order and fractional-order four-dimensional hyperchaotic system, and we are also able to implement the expected synchronization of other integer-order and fractional-order hyperchaotic system by using the same method for every initial value and expected time, such as integer-order hyperchaotic Lü system, integer-order hyperchaotic Rossler system,

fractional-order hyperchaotic Rossler system, fractional-order hyperchaotic Lü system and so on.

V. CONCLUSION

This section describes the expected synchronization theory, respectively taking the three-dimensional integer-order and the corresponding fractional-order Liu system, and four-dimensional integer-order and the corresponding fractional-order hyperchaotic Chen system as the analysis object, analyze the sufficient condition satisfying Theorem 1 for different systems, and give specific simulation examples of which the simulation results have confirmed realization of the expected synchronization. But it is necessary to point out that: Theorem 1 is only a necessary condition but not sufficient condition, that there exist control parameters not satisfying the theorem, but still able to achieve the expected synchronization. Therefore, we need to improve the part designing more effective controller to get a more precise condition

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