The Central DOA Estimation Algorithm Based on Support Vector Regression for Coherently Distributed Source

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Abstract—**In this paper, the problem of estimating the central direction of arrival (DOA) of coherently distributed source impinging upon a uniform linear array is considered. An efficient method based on the support vector regression is proposed. After a training phase in which several known input/output mapping are used to determine the parameters of the support vector machines, among the outputs of the array and the central DOA of unknown plane waves is approximated by means of a family of support vector machines. So they perform well in response to input signals that have not been initially included in the training set. Furthermore, particle swarm optimization (PSO) algorithm is expressed for determination of the support vector machine parameters, which is very crucial for its learning results and generalization ability. Several numeral results are provided for the validation of the proposed approach.**

Index Terms—**coherently distributed source; the central DOA; angular spread; support vector machines; particle swarm optimization**

I. Introduction

Sensor array processing plays a prominent role in the propagation of plane waves through a medium. The problem of finding the directions impinging on an array antenna or sensor array, namely, direction finding or DOA estimation, has been of interest for several decades. This is because the direction is a useful parameter for several systems, such as wireless communications, radar, navigation, etc. In most DOA estimation algorithms, it is commonly assumed that the received signals originate from far-field point sources and give rise to perfectly planar wavefronts which impinge on the array from discrete and fixed DOAs. However, in many practical applications, such as radar, sonar and mobile communications, the sensor array often receives sources which have been reflected by a number of scatters. The scattered signals are received from a narrow angular region, an alternative signal model can be derived which is called distributed source model [1-4].

Note that depending on the relationship between the channel coherency time and the observation period, the sources can be viewed either as coherently distributed or incoherently distributed [5]. A source is called coherently distributed if the signal components arriving from different directions are replicas of the same signal, whereas in the incoherently distributed source case, all signals coming from different directions are assumed to be uncorrelated. Indeed, if the channel coherency time is much smaller than the observation period, then the incoherently distributed model is relevant. In the opposite case, the coherently distributed model or a partially coherent model can be used.

Several methods have been proposed for estimating parameters for these two types of distributed sources. Indeed, in coherently distributed source case, the rank of the noise-free covariance matrix is equal to the number of sources. On the other hand, for incoherently distributed sources, the rank of the noise-free covariance matrix increases as the angular spread increases. In particular, for a single-source case, the rank can reach the number of array sensor [6]. However, most of the signal energy is concentrated within the first few eigenvalues of the noise-free covariance matrix. The number of these eigenvalues is referred to as the effective dimension of the signal subspace. It is generally smaller than the number of sensors.

Typically, the statistics of a distributed source is parameterized by its central DOA and angular spread. A number of investigators have proposed distributed source modeling, and several parameter estimation techniques have been proposed in the literature [7-12].To begin with, attempts for coherently distributed source modeling and parameter estimation have been accomplished in [7], where the central DOAs and angular spreads are estimated by algorithms based on MUSIC using a uniform linear array. However, this algorithm needs two dimensional joint searching and assumes that the multiple

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sources must have identical and known angular signal intensity function. In contrast to some computationally complex approaches such as the maximum likelihood [8], the dispersed signal parameter estimator (DISPARE) [9], and the covariance fitting [10] have been provided. Subsequently, a classical localization algorithm has been used to estimate both virtual parameters and deduced the required ones. The focus in [11] has been on root-MUSIC which was shown to provide better accuracy with relatively low computational complexity compared to some other point-source localization algorithm [12-13].

Other robust techniques using the array geometry have recently been developed. A typical example is low-complexity parameter estimation with ESPRIT technique [14], which employs eigenvalue decomposition with two uniform linear arrays. The ESPRIT algorithm is still computationally extensive and time consuming especially when the number of antenna array elements is larger than the number of incident signals. An asymptotic maximum likelihood for joint estimation of the central DOA and angular spreads of multiple distributed sources is presented in [15]. Though it has best precision, the computationally load is high.

Recently, many low-complexity methods are proposed to reduce the computational burden of estimators [16-18]. For example, the decoupled COMET-EXIP [16] uses two successive one dimensional searches instead of a two dimensional search for parameter estimation of a single incoherently distributed source.

Furthermore, methods based on the use of neural networks and radial basis function networks have also been efficiently applied for point source DOA estimation [19-20]. In these works, the outputs of the array, properly preprocessed, are used as input data for a family of neural networks trained with a subset of the possible configurations of the impinging sources.

In this paper, an alternative algorithm is proposed, which is based on a support vector regression. In particular, the support vector regression approximates the unknown function that relates the received signals to the angles of incidence. The support vector regression is based on the theory of support vector machines, which are a nonlinear generalization of the generalized portrait algorithm. In the past few years, there has been a great interest in the development of support vector machines, mainly because they have yielded excellent generalization performances in applications [21-22]. And fast iterative algorithms based on the use of support vector machines and relatively simple to be implemented, have been developed [23-24].

The remainder of this paper is organized as follows. Section II presents the array configuration and system model. Section III proposes a central DOA estimation algorithm for coherently distributed source based on support vector regression. Section IV shows particle swarm optimization for parameter section of support vector regression. Section V gives simulation results. And Section VI concludes the paper.

II. PROBEM STATEMENT AND PRELIMINARIES

In this work, a uniform linear array composed M elements with interelement spacing d is considered. q electromagnetic narrowband plane waves impinge on the array from directions.

The complex envelope of the array output can be written as

$$\boldsymbol{X}(t)=\sum_{i=1}^{q}\boldsymbol{S}_i(t)+\boldsymbol{N}(t) \tag{1}$$

where $\boldsymbol{X}(t)$ is the array snapshot vector, $\boldsymbol{S}_i(t)$ is the vector that describes the contribution of the ith signal source to the array output, and the noise $\boldsymbol{N}(t)$ is zero-mean and spatially and temporally white and Gaussian,

$$E\left\{\boldsymbol{N}(t)\boldsymbol{N}^{\mathrm{H}}(t')\right\}=\sigma^2\boldsymbol{I}\delta_{tt'} \tag{2}$$

and

$$E\left\{\boldsymbol{N}(t)\boldsymbol{N}^{\mathrm{T}}(t')\right\}=0,\forall t,t' \tag{3}$$

where σ^2 is the noise variance, $\boldsymbol{I}$ denotes identity matrix and $\delta_{tt'}$ is the Kronecker delta function with $\delta_{tt'}=1$ for $t=t'$ and $\delta_{tt'}=0$ for $t\neq t'$. We also assume that the signal is uncorrelated with the noise.

In point source model, the baseband signal of the ith source is modeled as

$$\boldsymbol{S}_i(t)=s_i(t)\boldsymbol{a}(\theta_i) \tag{4}$$

where $s_i(t)$ is the complex envelope of the ith source, θ_i is its DOA, and $\boldsymbol{a}(\theta_i)=[1,e^{-j2\pi d/\lambda\sin\theta_i},\cdots,e^{-j2\pi(M-1)d/\lambda\sin\theta_i}]^{\mathrm{T}}$ is the corresponding steering vector, d is the distance between two adjacent sensors, λ is the wavelength of the impinging signal.

In many environments for modern radio communications, the transmitted signal is often obstructed by buildings, vehicles, trees, etc., and/or reflected by rough surfaces. Hence, the absence of a single Line-Of-Sight (LOS) ray will violate the classical point source assumption.

Assume a single narrow point source that contributes with a large number of wavefronts originating from multi-path reflections near the source and during transmission. If we observe the baseband signals received at the antenna array, it is possible to regard the source just as a spatially distributed source as Fig.1.

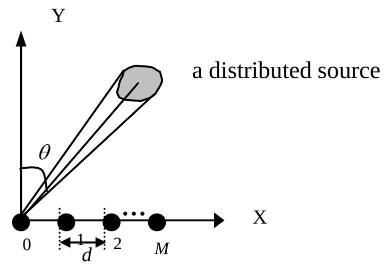

Fig.1 Distributed source model

In distributed signal model, the source energy is considered to be spread over some angular volume. Hence, $\boldsymbol{S}_i(t)$ is written as

$$\boldsymbol{S}_i(t)=\int_{\vartheta\in\Theta}\boldsymbol{a}(\vartheta)\varsigma(\vartheta,\boldsymbol{\psi}_i,t)\mathrm{d}\vartheta \tag{5}$$

where Θ is the set of the steering vector over some parameter space of interest, $\varsigma\left(\vartheta,\boldsymbol{\psi}_i,t\right)$ is a complex random angular-temporal signal intensity which can be expressed as

$$\varsigma\left(\vartheta,\boldsymbol{\psi}_i,t\right)=s\left(t\right)\ell\left(\vartheta;\boldsymbol{\psi}_i\right) \tag{6}$$

under the coherently distributed source assumptions, $\boldsymbol{\psi}_i$ is the location parameter. Examples of the parameter vector are the mean and standard deviation of a source with Gaussian angular signal intensity.

The steering vector of distributed source is defined as

$$\boldsymbol{b}\left(\boldsymbol{\psi}_i\right)=\int_{\vartheta\in\Theta}\boldsymbol{a}\left(\vartheta\right)\ell\left(\vartheta;\boldsymbol{\psi}_i\right)\mathrm{d}\vartheta \tag{7}$$

As a common example of the coherently distributed source, assume that the deterministic angular signal intensity $\ell\left(\vartheta;\boldsymbol{\psi}_i\right)$ has the Gaussian shape

$$\ell\left(\vartheta;\theta_i,\sigma_{\theta_i}\right)=\frac{1}{\sqrt{2\pi}\sigma_{\theta_i}}\exp\left(-\frac{\left(\vartheta-\theta_i\right)^2}{2\sigma_{\theta_i}^2}\right) \tag{8}$$

Here $\boldsymbol{\psi}_i=\left[\theta_i,\sigma_{\theta_i}\right]$, θ_i is the central DOA, σ_{θ_i} is angular spread.

Using the above definitions, the covariance matrix of the output signal vector can be written as

$$\boldsymbol{R}_{XX}=E\left[\boldsymbol{X}\left(t\right)\boldsymbol{X}^{\mathrm{H}}\left(t\right)\right] \tag{9}$$

In practical situations, the true covariance matrix of $\boldsymbol{X}\left(t\right)$ is unavailable but can be estimated. Therefore, the sample covariance matrix with N snapshots is defined as

$$\widehat{\boldsymbol{R}}_{XX}=\frac{1}{N}\sum_{t=1}^{N}\boldsymbol{X}\left(t\right)\boldsymbol{X}^{\mathrm{H}}\left(t\right) \tag{10}$$

III. The Central DOA Estimation Based on Support Vector Regression

The support vector regression is based on the theory of support vector machines, which are a nonlinear generalization of the generalized portrait algorithm developed by Vapnik [25]. In particular, support vector machines have a rigorous mathematical foundation, which is based on the learning theory.

Since the correlation matrix $\boldsymbol{R}_{XX}$ is symmetric, only the upper triangular part is considered. These matrix elements are organized in an array $\boldsymbol{V}$, given by

$$\boldsymbol{V}=\left[\boldsymbol{r}_{11},\boldsymbol{r}_{12},\cdots,\boldsymbol{r}_{1M},\boldsymbol{r}_{22},\boldsymbol{r}_{23},\cdots,\boldsymbol{r}_{2M},\cdots,\boldsymbol{r}_{mm},\cdots\boldsymbol{r}_{mM},\cdots,\boldsymbol{r}_{MM}\right] \tag{11}$$

where $r_{hk}=\left[\boldsymbol{R}\right]_{hk},h,k=1,\cdots,M$.

The array $\boldsymbol{V}$ is then normalized in order to obtain the input data $\boldsymbol{Z}$,

$$\boldsymbol{Z}=\frac{\boldsymbol{V}}{\left\|\boldsymbol{V}\right\|} \tag{12}$$

Since $\boldsymbol{Z}\in\Sigma,\Sigma\subset C^{M(M+1)/2}$, and $\boldsymbol{\theta}=\left[\theta_1\ \cdots\ \theta_q\right]\in\Theta$, $\Theta\subset R^q$, so a mapping $G:\Theta\to\Sigma$ exists. The problem of the central DOA estimation can be thought as the retrieval of $\boldsymbol{\theta}$, starting from the knowledge of the array $\boldsymbol{Z}$. To this end, the unknown inverse mapping $F:\Sigma\to\Theta$ has to be found. The components of F are estimated by using a regression approach, in which, starting from the knowledge of several input/output pairs $\left(\boldsymbol{Z},\theta\right)$, an approximation $\tilde{F}$ to F is constructed at the end of the training phase.

By using the support vector regression, $\tilde{F}$ is defined as

$$\tilde{F}(\boldsymbol{Z})=\left\langle\boldsymbol{w},\Phi\left(\boldsymbol{Z}\right)\right\rangle+b \tag{13}$$

where $\langle\cdot,\cdot\rangle$ denotes the scalar product, Φ is a nonlinear function that performs a transformation of the input array from the space Σ to a high dimensional space, and $\boldsymbol{w}$ and b are parameters which are obtained by minimizing the regression risk, defined as

$$R_{reg}=\frac{1}{2}\left\|\boldsymbol{w}\right\|^2+\tilde{C}_1\sum_{i=1}^{L}f\left(\boldsymbol{Z}_i,\theta_i\right) \tag{14}$$

where $\tilde{C}_1$ is a constant and $f\left(\boldsymbol{Z}_i,\theta_i\right)$ is the so-called ε insensitive loss function, given by

$$f\left(\boldsymbol{Z}_i,\theta_i\right)=\begin{cases}0, & \text{if } \left|\theta_i-F\left(\boldsymbol{Z}_i\right)\right|\le\varepsilon\\ \left|\theta_i-F\left(\boldsymbol{Z}_i\right)\right|-\varepsilon, & otherwise\end{cases} \tag{15}$$
$$i=1,2,\cdots,\mathrm{L}$$

The (15) can be rewritten as follows, considering the regression error.

$$f\left(\boldsymbol{Z}_i,\theta_i\right)=\begin{cases}0, & \text{if } \left|\theta_i-F\left(\boldsymbol{Z}_i\right)\right|\le\varepsilon\\ \theta_i-\boldsymbol{w}\cdot\Phi\left(\boldsymbol{Z}_i\right)-b\le\varepsilon+\xi_i & \\ \boldsymbol{w}\cdot\Phi\left(\boldsymbol{Z}_i\right)+b-\theta_i\le\varepsilon+\xi_i^{'} & \end{cases},\ otherwise \tag{16}$$
$$\xi_i,\xi_i^{'}\ge 0\quad i=1,2,\cdots L$$

where $\xi_i,\xi_i^{'}$ are slack variables. So the problem is equivalent to minimize

$$\min\quad \frac{1}{2}\left\|\boldsymbol{w}\right\|^2+\tilde{C}_1\left(\sum_{i=1}^{L}\xi_i+\sum_{i=1}^{L}\xi_i^{'}\right)$$
$$\begin{cases}\theta_i-\boldsymbol{w}\cdot\Phi\left(\boldsymbol{Z}_i\right)-b\le\varepsilon+\xi\\ \boldsymbol{w}\cdot\Phi\left(\boldsymbol{Z}_i\right)+b-\theta_i\le\varepsilon+\xi_i^{'}\\ \xi_i\ge 0,\xi_i^{'}\ge 0\end{cases} \tag{17}$$

$$L\left(\boldsymbol{w},\xi,\xi_i^{'}\right)=\frac{1}{2}\left\|\boldsymbol{w}\right\|^2+\tilde{C}_1\sum_{i=1}^{L}\left(\xi_i+\xi_i^{'}\right)-\sum_{i=1}^{L}\left(\lambda_i\xi_i+\lambda_i^{'}\xi_i^{'}\right)$$
$$+\sum_{i=1}^{L}\alpha_i\left[\theta_i-\left\langle\boldsymbol{w}^{\mathrm{T}}\cdot\Phi\left(\boldsymbol{Z}_i\right)\right\rangle-b-\varepsilon-\xi_i\right]$$
$$+\sum_{i=1}^{L}\alpha_i^{'}\left[\left\langle\boldsymbol{w}^{\mathrm{T}}\cdot\Phi\left(\boldsymbol{Z}_i\right)\right\rangle+b-\theta_i-\varepsilon-\xi_i^{'}\right] \tag{18}$$

Besides, the KKT(Karush-Kuhn-Tucker) conditions force $\frac{\partial L}{\partial\boldsymbol{w}}=0,\frac{\partial L}{\partial b}=0,\frac{\partial L}{\partial\xi_i}=0,\frac{\partial L}{\partial\xi_i^{'}}\lambda_i\xi_i=0,\lambda_i^{'}\xi_i^{'}=0$. Applying them, we obtain an optimal solution for support vector regression weights $\boldsymbol{w}=\sum_{i=1}^{L}\left(\alpha_i-\alpha_i^{'}\right)\Phi\left(\boldsymbol{Z}_i\right)$. So (18) can be expressed as

$$L\left(\alpha,\alpha^{'}\right)=-\frac{1}{2}\sum_{i=1}^{L}\sum_{j=1}^{L}\left(\alpha_i-\alpha_i^{'}\right)\left\langle\Phi\left(\boldsymbol{Z}_i\right)\cdot\Phi\left(\boldsymbol{Z}_j\right)\right\rangle\left(\alpha_j-\alpha_j^{'}\right)+\sum_{i=1}^{L}\theta_i\left(\alpha_i-\alpha_i^{'}\right)-\varepsilon\sum_{i=1}^{L}\left(\alpha_i+\alpha_i^{'}\right) \tag{19}$$

Subject to

$$\sum_{i=1}^{L}\left(\alpha_i-\alpha_i^{'}\right)=0 \qquad 0\le\alpha_i,\alpha_i^{'}\le\tilde{C}_1 \qquad i=1,2,\cdots,L \tag{20}$$

The dual variables $\alpha_i-\alpha_i^{'}$ and b are computed by using KKT conditions.

The regression function for tracking coherently distributed source is given

$$\tilde{F}\left(\boldsymbol{Z}\right)=\sum_{i=1}^{L}\left(\alpha-\alpha^{'}\right)\left\langle\Phi\left(\boldsymbol{Z}_i\right)\cdot\Phi\left(\boldsymbol{Z}\right)\right\rangle+b=\sum_{i=1}^{L}\left(\alpha-\alpha^{'}\right)K\left\langle\boldsymbol{Z}_i\cdot\boldsymbol{Z}\right\rangle+b \tag{21}$$

where $K\left\langle\boldsymbol{Z}_i\cdot\boldsymbol{Z}\right\rangle$ is the kernel function working on the original space Σ'.

Several kernel functions help the support vector regression in obtaining the optimal solution. The most frequently used such kernel functions are the polynomial, sigmoid and radial basis kernel function (RBF) as follows [26],

$$K\left(x,x_i\right)=\left[(x\cdot x_i)+1\right]^q \tag{22}$$

$$K\left(x,x_i\right)=\tanh\left(v(x\cdot x_i)+e\right) \tag{23}$$

$$K(x,x_i)=\exp\left\{-\frac{\left|x-x_i\right|^2}{\gamma^2}\right\} \tag{24}$$

The RBF is generally applied most frequently, because it can classify multi-dimensional data, unlike a linear kernel function. Additionally, the RBF has fewer parameters to set than a polynomial kernel. RBF and other kernel functions have similar overall performance. Consequently, RBF is an effective option for kernel function. Therefore, this study applies an RBF kernel function in the support vector regression to obtain optimal solution.

IV. Particle Swarm Optimization for Parameter Selection of Support Vector Regression Problem Statement and Preliminaries

The determination and selection for the parameters of the support vector machine is important in most applications.

Two major RBF parameters applied in support vector machine, $\tilde{C}_1$ and γ, must be set appropriately. Parameter $\tilde{C}_1$ represents the cost of the penalty. The choice of value for C influences on the classification outcome. If $\tilde{C}_1$ is too large, then the classification accuracy rate is very high in the training phase, but very low in the testing phase. If $\tilde{C}_1$ is too small, then the classification accuracy rate unsatisfactory, making the model useless. Parameter γ has a much greater influence on classification outcomes than $\tilde{C}_1$, because its value affects the partitioning outcome in the feature space. An excessively large value for parameter $\tilde{C}_1$ results in over-fitting, while a disproportionately small value leads to under-fitting.

Grid search is the most common method to determine appropriate values for $\tilde{C}_1$ and γ [27]. Values for parameters $\tilde{C}_1$ and γ that lead to the highest classification accuracy rate in this interval can be found by setting appropriate values for the upper and lower bounds (the search interval) and the jumping interval in the search. Nevertheless, this approach is a local search method, and vulnerable to local optima. Additionally, setting the search interval is a problem. Too large a search interval wastes computational resource, while too small a search interval might render a satisfactory outcome impossible.

In addition to the commonly used, grid search, other techniques are employed in support vector machine to improve the possibility of a correct choice of parameter values. Pai and Hong proposed an SA-based approach to obtain parameter values for support vector machine, and applied it in real data [28]; however, this approach does not address feature selection, and therefore may exclude the optimal result.

As well as the two parameters $\tilde{C}_1$ and γ, other factors, such as the quality of the feature's dataset, may influence the classification accuracy rate. For instance, the correlations between features influence the classification result. Accidental removal of important features might lower the classification accuracy rate. Additionally, some dataset features may have no influence at all, or may contain a high level of noise. Removing such features can improve the searching speed and accuracy rate.

Here the particle swarm optimization (PSO) algorithm is used to optimize the parameters of support vector machine. PSO is an emerging population-based meta-heuristic that simulates social behavior such as birds flocking to a promising position to achieve precise objectives in a multidimensional space [29]. Like evolutionary algorithms, PSO performs searches using a population (called swarm) of individuals (called particles) that are updated from iteration to iteration. To discover the optimal solution, each particle changes its searching direction according to two factors, its own best previous experience and the best experience of all other members.

Each particle represents a candidate position. A particle is considered as a point in a D-dimension space, and its status is characterized according to its position and velocity. The D-dimensional position for the particle i at iteration t can be represented as $\boldsymbol{x}_i^t=\left\{x_{i1}^t,\ x_{i2}^t,\ \cdots,\ x_{iD}^t\right\}$. Likewise, the velocity, i.e., distance change, which is also a D-dimension vector, for particle i at iteration t can be described as $\boldsymbol{t}_i^t=\left\{t_{i1}^t,\ t_{i2}^t,\ \cdots,\ t_{iD}^t\right\}$.

Let $\boldsymbol{p}_i^t=\left\{p_{i1}^t,\ p_{i2}^t,\ \cdots,\ p_{iD}^t\right\}$ represent the best solution that particle i at iteration t, and $\boldsymbol{p}_g^t=\left\{p_{g1}^t,\ p_{g2}^t,\ \cdots,\ p_{gD}^t\right\}$ denote the best solution obtained from $\boldsymbol{p}_i^t$ in the population at iteration t. To search for the optimal solution, each particle changes its velocity according to the cognition and social parts as follows,

$$V_{id}^{t} = V_{id}^{t-1} + c_1 r_1 \left(p_{id}^{t} - x_{id}^{t} \right) + c_2 r_2 \left(p_{gd}^{t} - x_{id}^{t} \right) \quad d = 1, 2, \cdots, D \tag{25}$$

where c_1 and c_2 are accelerating factors, and r_1 and r_2 are random numbers uniformly distributed in $(0,1)$. Each particle then moves to a new potential solution based on the following equation

$$X_{id}^{t+1} = X_{id}^{t} + V_{id}^{t}, d = 1, 2, \cdots, D \tag{26}$$

The generalization ability of support vector machine algorithms depends on a set of parameters, including the penalty actor $\tilde{C}_1$, the estimated accuracy ε and the RBF kernel parameter γ . Defining $\boldsymbol{U}_i = \left(\tilde{C}_i, \varepsilon_i, \gamma_i \right)$, the speed $\boldsymbol{V}_i = \left(v_{i1}, v_{i2}, v_{i3} \right)$, the history optimal location $\boldsymbol{P}_i = \left(p_{i1}, p_{i2}, p_{i3} \right)$,the global optimal location $\boldsymbol{G}_i = \left(g_{i1}, g_{i2}, g_{i3} \right)$ for ith particle. The update for location and speed are written as

$$\tilde{C}_i = \tilde{C}_i + v_{i1} \tag{27}$$

$$\begin{aligned} v_{ik} &= w_1 v_{i1} + c_1 \times rand(\ ) \times \left(p_{i1} - \tilde{C}_i \right) \\ &\quad + c_2 \times rand(\ ) \times \left(g_{i1} - \tilde{C}_i \right) \end{aligned} \tag{28}$$

where w_1 is inertia factor,

$$w_1(itc) = w_{\min} + \frac{w_{\max} - w_{\min}}{itc} \times \left(itc_{\max} - itc \right) \tag{29}$$

where itc is the number of iteration, $itc_{\max}$ is the maximal number of iteration, $w_{\max}$ and $w_{\min}$ are the maximal and minimum inertia factors, respectively. The fitness function for the proposed central DOA estimation algorithm is defined as

$$fits = \sqrt{\frac{1}{K} \sum_{i=1}^{K} \left(\theta_i - \hat{\theta}_i \right)^2} \tag{30}$$

The basic process of the particle swarm optimization for parameter selection of support vector regression is given as follows,

Step1,(Initialization) Randomly generate initial particles;

Step2, (Fitness) Measure the fitness of each particle in the population;

Step3, (Update) Compute the velocity of each particle with (28);

Step4, (Construction) For each particle, move to the next position according to (26);

Step5, (Termination) Stop the algorithm if termination criterion is satisfied; Return to Step 2 otherwise.

V. NUMBERICAL RESULTS

Several numerical simulations have been performed in order to validate the proposed approach. An array composed by 8 elements with interelement distance $d = 0.5\lambda$ is considered.

The kernel function used in this work is a radial kernel. We investigate the performances of the proposed fast DOA estimation of single coherently distributed source with Gaussian deterministic angular signal density.

Moreover, in the considered simulations, following a widely used approach, an estimate of the correlation matrix $\boldsymbol{R}_{XX}$ is simply computed by averaging the values of 50 snapshots of $\boldsymbol{X}$. In the first example, we numerically illustrate the proposed algorithm for selecting the parameter of support vector machine. Considering a single coherently distributed source with angular spread and SNR=10dB. In order to cover the whole region of interest, the range is $\left[-90^{\circ}, 90^{\circ} \right]$ for the central DOA in training sample set. After the training phase, the test phase is performed by considering different values of the central DOAs of the impinging waves. The support vector machine parameters are initialized as $C \in [30, 500]$, $\varepsilon \in [0, 0.02]$ and $\gamma \in [0.01, 2]$. The speed range are $[-500, 500]$, $[-0.02, 0.02]$ and $[-2, 2]$, respectively. The fitness function is initialized as 0. Fig.2 illustrates the iterative process. When the number of iteration is about 35, the fitness function is convergence. The optimal location of the particle is $(230.4331, 0, 0.5734)$,which is support vector regression parameter.

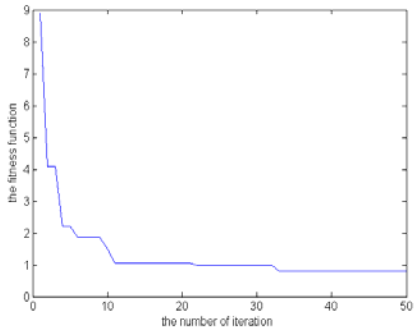

Fig.2 The iterative

The estimated DOA values are reported in Fig.3. In the abscissa, the indexes of the samples belonging to the test set are indicated. The corresponding actual and estimated values of the incident angles are reported. As can be seen, the proposed method is able to obtain quite good results for almost all of the considered DOAs.

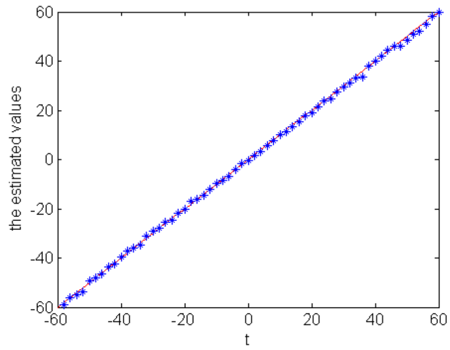

Fig.3 The actual and estimated values of the central DOA

The root-mean-squared errors (RMSEs) of the estimated central DOA by the proposed method are illustrated at different SNR in Fig. 4. As it can be seen, the proposed algorithm has a better estimation performance at low SNR.

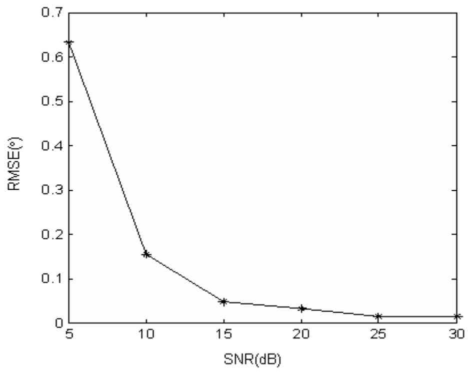

Fig.4 The RMSE of the estimated central DOA versus SNR

The influence of the number of snapshots is investigated in Fig.5 for SNR=10dB.It can be observed that the proposed algorithm presents effective performance even for a small number of snapshots.

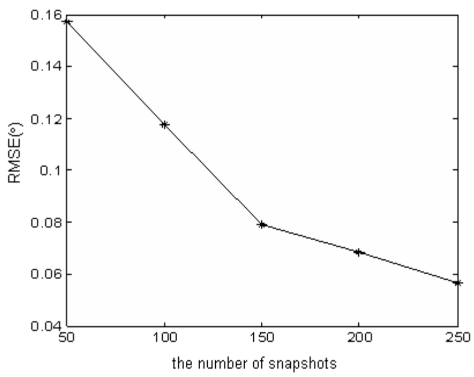

Fig.5 The RMSE of the estimated central DOA versus the number of snapshots

VI. CONCLUSION

A method for estimating the central DOA of coherently distributed source has been proposed. The developed method is based on the use of a support vector regression approach for the approximation of the unknown mapping that performs the transformation from the outputs of the smart array to the central DOA of coherently distributed source, which can be used when the sample set is small. Furthermore, the reported results show that the method is able to correctly produce outputs corresponding to accurate estimations even in large angular spread. The approach is able to reach this goal in a very short time and with good generalization capabilities.

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REFERENCES

[1] D. Asztely and B. Ottersten, "The effects of local scattering on direction of arrival estimation with MUSIC and ESPRIT," *Proceedings of ICASSP1998, Seattle, Washington*, pp. 3333-3336, 1998.
[2] A. Hassanien, S. Shahbazpanahi and A. B. Gershman, "A generalized Capon estimator for localization of multiple spread sources," *IEEE Trans. Signal Processing*, vol.52, pp. 280-283, January 2004.
[3] J. Lee, J. Joung and J. D. Kim, "A method for the direction-of-arrival estimation of incoherently distributed sources," *IEEE Trans. Vehicular Technology*, vol.57, pp. 2885-2893, May 2008.
[4] M. Souden, S. Affes and J. Benesty, "A two-stage approach to estimate the angles of arrival and the angular spreads of locally scatters sources," *IEEE Trans. Signal Processing*, vol.56, pp. 1968-1983, May 2008.
[5] R. Raich, J. Goldberg and H.Messor, "Bearing estimation for a distributed source: modeling, inherent accuracy limitations and algorithm," *IEEE Trans. Signal Processing*, vol.48, pp. 429-441, February 2000.
[6] A. Zoubir, Y. Wang and P. Charge, "Efficient subspace-based estimator for localization of multiple incoherently distributed source," *IEEE Trans. Signal Processing*, vol.56, pp. 532-542, February 2008.
[7] S.Valaee, B. Champagne and P. Kabal, "Parameter localization of distributed sources," *IEEE Trans. Signal Processing*, vol. 43, pp. 2144-2153, February 1995.
[8] T.Trump and B. Ottersten, "Estimation of nominal direction of arrival and angular spread using an array of sensors," *Signal Processing*, vol.50, pp.57-69, April 1996.
[9] Y.Meng, P.Stoica and K.Wong, "Estimation of the directions of arrival of spatially dispersed signals in array processing," *Inst.Elect.Eng.Proc.Radar,Sonar, Navigat.*, vol.143, pp.1-9, February 1996.
[10] S.Shahbazpanahi, S.Valaee and A. B. Gershman, "A covariance fitting approach to parametric localization of multiple incoherently distributed source, "*IEEE Trans. Signal Processing*, vol.52, pp.592-600, March 2004.
[11] M. Bengtsson and B. Ottersten, "Low-complexity estimators for distributed sources," *IEEE Trans.Signal Processing*, vol.48, pp.2185-2194, August 2000.
[12] P.Stoica and K.C.Sharman, "Novel eigenanalysis method for direction estimation," *IEE Proc.F Radar, Signal Processing*, vol.137, pp.19-26, February 1990.
[13] R.Roy and T.Kailath, "ESPRIT-estimation of signal parameters via rotational invariance techniques," *IEEE Trans.Acoust.,Speech,Signal Processing*, vol.37, pp.984-995, July 1989.
[14] S. Shahbazpanahi, S. Valaee and M. H. Bastani, "Distributed source localization using ESPRIT algorithm," *IEEE Trans. Signal Processing*, vol. 49, pp. 2169-2178, October 2001.
[15] B. T. Sieskul, " An asymptotic maximum likelihood for joint estimation of nominal angles and angular spreads of multiple spatially distributed sources," *IEEE Trans. Vehicular Technology*, vol.59, pp. 1534-1538, March 2010.

[16] O.Besson, P.Stoica, "Decoupled estimation of DOA and angular spread for a spatially distributed source," *IEEE Trans. Signal Processing*, vol.48, pp. 1872-1882, July 2000.
[17] A.Zoubir, Y.Wang, "Robust generalized Capon algorithm for estimating the angular parameters of multiple incoherently distributed sources," *IET Signal Process.*, pp. 163-168, February 2008.
[18] A. Monakov ,O. Besson, " Direction finding for an extended target with possibly non-symmetric spatial spectrum," *IEEE Trans. Signal Processing*, vol. 52, pp. 283-287, January 2004.
[19] A.H.EI Zooghby, C.G.Christodoulou and M. Georgiopoulos, "performance of radial-basis function network for direction arrival estimaion with antenna arrays," *IEEE Trans. Antennas Propag.*, vol.45, pp.1611-1617, November 1997.
[20] A.H.EI Zooghby, C.G.Christodoulou and M. Georgiopoulos, "A neural network-based smart antenna for multiple source tracking," *IEEE Trans. Antennas Propag.*, vol.48, pp.768-776, May 2000.
[21] D. Zhu, M. Tao and L. Sun, "Fuzzy support vector machine control for 6-DOF parallel robot, " *Journal of Computers*, vol.6, pp.1926-1934, June 2011.
[22] V. A. Sotiris, P.W. Tse and M.G. Pecht, "Anomaly detection through a Bayesian support vector machine," *IEEE Trans.Reliability*, vol.59, pp.277-286, Feb. 2010.
[23] C. Xie, C. Shao and D. Zhao, "Parameters optimization of least squares support vector machines and its application," *Journal of Computers*, vol.6, pp.1935-1941, September 2011.
[24] D. Geebelen, J. Suykens and J.Vandewalle, "Reducing the number of support vectors of SVM classifiers using the smoothed separable case approximation," *IEEE Trans. Neural Networks and Learning Systems*, vol.23, pp.682-688, April 2012.
[25] V.Vapnik and A.Lerner, "Pattern recognition using generalized portrait method," *Automation Remote Control*, vol. 24, pp. 774-780, June 1963.
[26] Y. Liao, S.C. Fang and H.L.W. Nuttle, "A neural network model with bounded-weights for pattern classification," *Computers and Operations Research*, vol.31, pp.1411-1426, 2004.
[27] T. Shon, Y.Kim, C. Lee and J.Moon, "A machine learning framework for network anomaly detection using SVM and GA," *In Proceedings of IEEE Workshop on Information Assurance and Security*, pp. 176-183, 2005.
[28] P.F. Pai and W.C.Hong, "Support vector machines with simulated annealing algorithm in electricity load forecasting,"*Energy Conversion and Management*, vol.46, pp.2669-2688, 2005.
[29] J. Kennedy and R.C. Eberhart, "Particles swarm optimation," *Proceedings of IEEE Conference on Neural Network*, vol.4, pp.1942-1948, 1995.

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