Application of Set-theoretic Granular Computing

Yanli Zhu
School of Information and Engineering, Henan Institute of Science and Technology, Xinxiang, China
Email: yanl.zhu@yahoo.com.cn

Quanrui Wang
School of Information and Engineering, Henan Institute of Science and Technology, Xinxiang, China
Email: ji_ai @163.com

Shiyong Li
School of Information and Engineering, Henan Institute of Science and Technology, Xinxiang, China
Email: nickel_641@163.com

***Abstract*—Uncertain knowledge management is a key issue in the expert system. By analysis of the reasons for knowledge's uncertainty in complete universe and uncompleted universe, two methods based on set-theoretic granular computing are presented to deal with the uncertainty knowledge which is the rough observable method and rough inclusion method respectively. The rough set can be approximately expressed as a union of some basic sets on the given universe using the concepts of rough membership function and rough observable set. Experiments on the information system show that the original indefinite sets can be described with its approximate sets, which have the certain knowledge. So the uncertainty sets have the certainty at some degree, and from which one can get more deterministic information.**

***Index Terms*—rough set; granular computing; rough observable set; set-theoretic model of granular computing; rough inclusion**

I. INTRODUCTION

The uncertainty of objective thing or phenomenon of the real world may lead people to have incomplete and inaccurate knowledge of each field. Therefore, most of the time, people analysis, reasoning, judging, forecasting and make a decision of things, and also with an effective way of thinking, which focus on a particular level of abstraction and ignore irrelevant details. The uncertainty of knowledge is mainly caused by two reasons: one comes from the boundary region of the rough set on the given domain; the other directly comes from the binary relation of the universe and the knowledge module it produces, namely the approximation space itself.

Rough set [1] is being used as an effective model to deal with imprecise knowledge. One of the main goals of the rough set analysis is to synthesize approximation of concepts from the acquired data [2]. The uncertainty of knowledge is mainly caused by two reasons: one comes from the boundary region of the rough set in the approximation space; the other directly comes from the binary relation of universe and the knowledge module it produces, namely the approximation space itself [3]. In rough set theory, the uncertainty of knowledge caused by the above two reasons can be described according to the methods of Shannon's entropy or the accuracy of approximation. However, the method about how to deal with the uncertainty knowledge has not yet been present. And in the book [4], Nguyen provides a broad snapshot of intelligent technologies for inconsistency resolution and offers an invaluable source of reference on the topic. Therefore, in the above-mentioned works, the indiscernible relation is replaced by a dominant relation to solve the multi-criteria sorting problem; and the data table is replaced by a pair wise comparison table to solve multi-criteria choice and ranking problems. The approach is called the dominance-based rough set approach (DRSA) [5]. So it is still an important direction, whether it is possible to find out an effective method to deal with the uncertainty of knowledge.

The capability of rough sets in dealing with the continuous attribute values in the information system is limited because the values are usually processed by discrediting them into the binary system in rough sets, and it may bring errors by using this method apparently for most of the continuous attributes are fuzzy attributes [6]. In many situations, it is impossible or unnecessary to distinguish individual objects or elements in a universe. For example, if a group of students is described by using several interests and hobbies, many students would share the same interests and hobbies, and hence they are indistinguishable. This forces us to think a subset of the students as one unit, instead of many individuals. In other words, one has to consider groups, classes, or clusters of elements. They are referred to as granules and can be either crisp or fuzzy. Elements in a granule may be drawn together by indistinguishability, similarity, proximity, or functionality. Such a clustering of elements leads to information or knowledge granulation, which form a basis of granular computing. When a problem involves incomplete, uncertain, or vague information, it may be difficult to differentiate distinct elements and one is forced to consider granules. Alternatively, although detailed information may be available, it may be sauciest to use granules in order to have an efficient solution.

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Very precise solutions may, in fact, not be required for many practical problems.

Rough Computing is the most powerful granular computing technique for an unknown object [7]. A granular structure provides structured description of a system or an application under consideration. The current research in granular computing is dominated by set-theoretic models such as rough sets and fuzzy sets [8]. In a specific application, individuals sharing the same properties can be put into the one granule. Basic granules represent the basic knowledge of human intelligence. They are the main focuses or basic observations of a problem in the real world. Once the basic granules are properly identified, one can investigate the contact between them and define the related computational operations based on them. In this way, structural knowledge is formed to help us to see the relationships between different parts of the problem, and hence understand the real-world problem more clearly.

The method of dealing with the uncertainty knowledge is based on complete domain in rough set theory. And often, people have the ignorant of a domain or known part of a domain. With this situation, a method of rough inclusion can be used to deal with the uncertainty knowledge based on the relation of rough sets and rough observable sets.

From the view of granular computing, the certainty and uncertainty have different forms at different granularity levels; they are not diametrically opposed, but can be transformed into each other at a certain level of granularity. The uncertainty knowledge may be other levels of certainty issue, and there may also have defined rules, which are hidden in some uncertainties phenomenon. From the perspective of natural phenomena, people divided it into deterministic phenomenon and the phenomenon of uncertainty. Uncertainty phenomenon is the complexity of the certainty phenomenon at different granularity levels, which can be divided into random phenomenon and fuzzy phenomena; and the deterministic phenomenon is the special circumstances of the uncertainty phenomenon at a granular level. From a mathematical point, the importance to establish the mathematical foundations of theoretical uncertainty is becoming increasingly, particularly the establishment of the axiomatic method of uncertainty theory, theory of the formation of a branch of mathematics of uncertainty, thereby contributing to the uncertainty theory of development. With the in-depth research of probability and fuzzy in mathematical theory, uncertainty has gradually been awareness of the phenomenon and takes advantage of real life.

So we hope to combine these two theories of the rough logic theory and the granular computing theory to make a new method of dealing with the uncertainty of knowledge. With the concepts of rough observable set and rough inclusion, the knowledge of incomplete domain can be seen as sets of granules. With the set-theoretic model of granular computing, it can get some partial knowledge of unknown sets with knowledge of certainty sets on the incomplete domain.

II. Basic Concepts of Rough Sets

A. Connotation of Knowledge

The various concept of knowledge may be understood, related and handled in different categories. In rough set theory, knowledge is regarded as an ability of categorizing the realistic or abstract items, namely knowledge may be understood as the partition of data.

Let U denote a finite and non-empty set called the universe. Let $\boldsymbol{R}$ be a family of all equivalence relations on U, thus a knowledge base can be defined as a relation system $K = (U, \boldsymbol{R})$. Let $\boldsymbol{P}$ be a non-empty set and $\boldsymbol{P} \subseteq \boldsymbol{R}$, therefore, $\cap \boldsymbol{P}$ is also an equivalence relation which denoted by IND ($\boldsymbol{P}$). The partition U/IND ($\boldsymbol{P}$) contains equivalence classes, which are respected to equivalence relation IND ($\boldsymbol{P}$). U/IND (**P**) is known as $\boldsymbol{P}$-basic knowledge, and the equivalence class in U/IND ($\boldsymbol{P}$) is known as the basic concept. If R is an equivalence relation of $\boldsymbol{P}$, then U/R is a partition of U with respect to R. For $x \in U$, the partition can be expressed $U/R = \cup [x]_R$, and $[x]_R$ is the equivalence class about the element x. The empty set $\varnothing$ and the elements of U/R are called elementary sets. A finite union of elementary sets, i.e., the union of one or more elementary sets, is called a composed set.

B. Lower and Upper Approximations

The indiscernible relation is a fundamental notion in the rough set analysis [9, 10].

Let U be the universe and R be an equivalence relation on U. The pair $A = (U, R)$ is called the approximation space. The equivalence relation R partitions the set U into disjoint subsets. For any element $x \in U$, the equivalence class of R determined by element x will be denoted by $[x]_R$.

Definition 1 Let $A = (U, R)$ be an approximation space and let X be a subset of U with respect to R. If X can be expressed as any finite union of elementary sets with respect to R, then X will be called an *R-discernible* set [1], otherwise X will be called an *R-indiscernible* set.

Definition 2 Let $A = (U, R)$ be an approximation space and let X be a subset of U with respect to R. The lower and upper approximations of X are defined respectively as follows:

$$R_*(X) = \bigcup \{[x]_R \mid [x]_R \subseteq X, x \in U\} \quad (1)$$

$$R^*(X) = \bigcup \{[x]_R \mid [x]_R \cap X \neq \varnothing, x \in U\} \quad (2)$$

The above shown that the R_* (X) is the greatest R-definable set, contained in X. The R^* (X) is the smallest R-definable set containing X.

C. Accuracy of Approximation

The uncertainty of knowledge in rough set theory is caused by lots of reasons; one of them is the boundary region of the rough set in the approximation space. So we can introduce the concept with accuracy of approximation in rough set theory.

Definition 3 Let X be a rough set on U with respect to R. The rough set X can be characterized numerically by the following coefficient.

$$\alpha_R(X)=\frac{|R_*(X)|}{|R^*(X)|} \tag{3}$$

Called rough accuracy, where X is a non-empty set, and $|X|$ denotes the cardinality of X.

The accuracy of approximation describes the degree of people mastering the present knowledge in rough set theory. Obviously, 0≤ $_R(X)$ ≤1. If $_R(X)$ =1, X is crisp with respect to R, and otherwise, if $_R(X)$ <1, X is rough with respect to R.

D. Rough Membership Function

Definition 4 Let A= (U, R) be an approximate space and let x be the element of U. $[x]_R$ denotes the equivalence class of x. So rough sets can be also defined using a rough membership function [11], defined as

$$\mu_X^R(x)=\frac{|X\cap[x]_R|}{|[x]_R|}, x\in U \tag{4}$$

And $|X|$ denotes the cardinality of X. Obviously $^R_X(x)$ $\in$[0, 1].

The rough membership function expresses conditional probability that x belongs to X with respect to R, and it can be interpreted as a degree of certainty which x belongs to X. According to this definition, the same element has the same rough membership. In a general way, the degree of uncertainty knowledge can be described by rough membership in rough set theory.

E. Set-theoretic Model of Granular Computing [12]

Granules are the building blocks to form a granular structure. The internal structure of a granule represents the characterization of the granule. Analyzing the intimate structure of a granule helps us to understand why individuals draw together.

In the study of formal concepts, every concept is understood as a unit of thoughts that consists of two parts, the extension and intension of the concept [13]. The extension of a concept is the set of objects or entities, which are instances of the concept. All objects in the extension have the same properties that characterize the concept. The intension (comprehension) of a concept consists of all properties or attributes that are valid for all those objects to which the concept applies. A concept is thus described jointly by its extension and intension, i.e., a set of objects and a set of properties. This formulation enables us to study formal concepts in a set-theoretic framework.

Consider a universe U. Each subset $A\subseteq U$ may be interpreted as the extension of certain concept. All elements of A are characterized by certain properties, namely, the intension of the concepts. In the following discussion, we only consider the extension of a concept without explicitly referring to its intension. One may say that elements in A are similar or indistinguishable based solely on properties in the intension. That is, we regard the elements of A to be equivalent, as they are all instances of the same concept.

Each granule represents the certain concept such that each element of the granule is an instance of the concept. Granules can be constructed through the use of information tables, as being done in rough set approach [14, 15]. We, therefore, concentrate mainly on operations on granules. The use of crisp sets (granules) is not as restrictive as it may appear. A fuzzy set (granule) can be equivalently expressed a family of crisp sets (granules) using its α-cuts. Operations on fuzzy granules can therefore be defined by operations on α-cuts.

An interval number $[\underline{a},\overline{a}]$ with $a\leq a$ is the set of real numbers defined by:

$$[\underline{a},\overline{a}]= \{x \mid a\leq x\leq a\} \tag{5}$$

Degenerate into intervals of the form [a, a] are equivalent to real numbers.

One can perform arithmetic operations on interval numbers by lifting arithmetic operations on real numbers. Let A =$[\underline{a},\overline{a}]$ and B =$[\underline{b},\overline{b}]$ be two interval numbers, we have:

$$\begin{aligned} A+B &= \{x + y \mid x\in A, y\in B\} \\ &= [\underline{a}+\underline{b},\ \overline{a}+\overline{b}], \end{aligned} \tag{6}$$

$$\begin{aligned} A\text{-}B &= \{x\text{-}y \mid x\in A, y\in B\} \\ &= [\underline{a}-\overline{b},\ \overline{a}-\underline{b}], \end{aligned} \tag{7}$$

$$\begin{aligned} A \bullet B &= \{x \bullet y \mid x\in A, y\in B\} \\ &= [\ \min(\underline{a}\,\underline{b},\ \underline{a}\,\overline{b},\ \overline{a}\,\underline{b},\ \overline{a}\,\overline{b}), \\ &\quad \max(\underline{a}\,\underline{b},\ \underline{a}\,\overline{b},\ \overline{a}\,\underline{b},\ \overline{a}\,\overline{b})] \end{aligned} \tag{8}$$

$$\begin{aligned} A/B &= \{x/y \mid x\in A, y\in B\} \\ &=[\underline{a},\overline{a}] \bullet [1/\overline{b},\ 1/\underline{b}],\ 0\in[\underline{b},\overline{b}] \end{aligned} \tag{9}$$

The results of interval number operations are again closed and bounded intervals. When 0 is included in the set B, the expression of A/B is undefined. One may lift any operations on factual numbers, such as minimum and maximum, to power operations on intervals of real numbers.

III. THE MEASURE OF UNCERTAINTY BASED ON GRANULAR COMPUTING'S MODEL

A. Roughness of Rough Set in Approximation Space

Roughness is the simplest method to measure the uncertainty, the concept was originally proposed by Professor Pawlak; and the definition of roughness is based on accuracy of approximation, which can be reflected as the difference between the number 1 and approximation accuracy. Later, Huang has introduced the concept of roughness into approximation space, used to describe the uncertainty of the rough set in approximation space [14]. The roughness in approximation space is defined by:

$$\rho_C(X)=1-\frac{\underline{\underline{C}}(X)}{\overline{C}(X)} \tag{10}$$

B. Rough Entropy of Rough Set in Approximation Space

As the shortcomings of roughness, Liang presented the roughness in rough set should also have the size of strict relationship in two approximation spaces with the strict partial order. He defined the rough entropy of the rough set can be described by the product of knowledge rough entropy and roughness. Huang has introduced the concept of rough entropy into approximation space, used to describe the uncertainty of the rough set in approximation space, and it can measure the uncertainty about approximation space bitterly. The rough entropy in approximation space is defined by:

$$E_c(X) = \rho_C(X)E(C) \tag{11}$$

In which, $E_c(X) = \sum_{i=1}^{m} \frac{|C_i|}{m} \log_2 |C_i|$.

At a certain degree, the rough entropy is more precise to the roughness. In the paper [15], Wang has proposed that the uncertainty of rough sets should keep unchanged along with the segmentation of knowledge granularity in the positive domain or the negative domain, but the definition of rough entropy in formula is strictly decreasing.

IV. Dealing Uncertainty Knowledge with Methods of Set-theoretic Granular Computing

The precise mathematical method can be used to describe the deterministic rule for the uncertainty of the phenomenon. People also get other description methods. One of them is probability theory which to reveal the statistical regularity of the stochastic phenomenon. The other is fuzzy sets and rough sets, which is also to reveal the regularity of the fuzzy phenomenon. These methods are all the breakthrough and development of exact mathematical. In probability theory, the events which its probability is 1 can be known as the inevitable events, and these events are occurred determinately. In the fuzzy set theory, the object which its membership function is 1 must belong to a set. In rough set theory, the elements in lower-approximate set are the determinable elements. Therefore, the uncertainty phenomenon always contains some certain laws, such as the probability statistical regularity, and these determinable rules can help people to grasp the random phenomenon better. However, fuzzy or uncertain contains certain law which is unknown. If such a rule exists, how to use precise mathematical describe is one of the important tasks about the problem of uncertainty.

A. Rough Observable Set

The concept of indiscernible relation provides a common approach for the description of relations between elements on the universe. Any two elements are indiscernible or equivalent if we have no present knowledge about them. As the equivalence relation has been put forward before, the equivalence classes can be regarded as the basic observable object rather than the elements when we observe a rough set.

Based on the definition of rough membership function, some basic concepts are given as follows:

Let $A=(U, R)$ be an approximate space, R is an equivalence relation. For $X \subseteq U$, $x \in U$, we have

(1) If $0< \mu_X^R(x) = \frac{|X \cap [x]_R|}{|[x]_R|} < \beta$, then $[x]_R \cap X$ is called rough $\tilde{\beta}$lower observable, and the part $[x]_R \cap X$ can be neglected;

(2) If $\beta< \mu_X^R(x) = \frac{|X \cap [x]_R|}{|[x]_R|} \leqslant 1$, then $[x]_R \cap X$ is called rough $\tilde{\beta}$upper observable, and we can use the part $[x]_R \cap X$ instead of X;

(3) If $\mu_X^R(x) = \frac{|X \cap [x]_R|}{|[x]_R|} = \beta$, then $[x]_R \cap X$ is called rough β–unobservable, and we can neglect the part $[x]_R \cap X$ or we can use the part $[x]_R \cap X$ instead of X. In this paper we use the part $[x]_R$ instead of the set which intersects with X and the rough membership function x is β.

In which β is a coefficient and obviously ® ∈ (0, 1]. Of course, the coefficient ® can get the value 0, but if ®=0, then it indicates that the part $[x]_R \cap X$ is an empty set. That is the element x does not belong to X certainly. It is no meaning to discuss this situation for our paper. So we let the value of ® be in the range of (0, 1]. Usually, we let β =0.5, but we can also set the value of ® according to the importance of attributes in the information system.

Definition 5 Let $A=(U, R)$ be an approximate space, X is a subset of U. The approximation of X can be defined as

$$appr_\beta(X) = \bigcup \{[x]_R \mid x \in U \wedge \mu_X^R(x) \geq \beta\} \tag{12}$$

The set $appr_\beta(X)$ is called rough β-observable set of X, referred to as the rough observable set, or the approximation of X [16].

It can be shown that the rough β-observable set of X has the following properties:

(1) for any $x \in X$, if $0< {}_X^R(x) \leq 0.5$, then $appr_\beta(X) = R_*(X)$;

(2) for any $x \in X$, if $0.5 \leq {}_X^R(x) <1$, then $appr_\beta(X) = R^*(X)$;

(3) for any $x \in X$, if ${}_X^R(x)=1$, then $appr_\beta(X)=X$;

The above properties express that $appr_\beta(X)$ is a set which between the lower and upper approximation of X, denote as $R_*(X) \subseteq appr_\beta(X) \subseteq R^*(X)$. Therefore, the set $appr_\beta(X)$ can be more approximately to describe the rough set X than the lower and upper approximations of X.

Let U be the universe and let R be an equivalence relation on U. X is a rough set of U, and U/R is the partition of U, the lower and upper approximations of X as shown in fig 1.

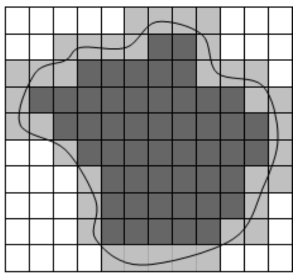

Figure 1. Lower and upper approximations of X.

Rough β-observable set $appr_\beta(X)$ of X can be gained with the defined concept of the rough observable set, as shown in Figure 2.

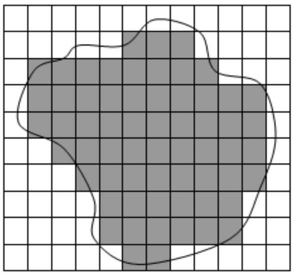

Figure 2. Rough β-observable set of X.

As shown in Figure 2, the shape of $appr_\beta(X)$ is closer to the shape of X than to the shape of upper and lower approximation of X. So the set $appr_\beta(X)$ can reflect the properties of the rough set X approximately. If the partition of the universe is more detailed, $appr_\beta(X)$ is closer to X, and it is more probable to replace set X with $appr_\beta(X)$. Because $appr_\beta(X)$ is definable, the knowledge of X can be mastered approximately by $appr_\beta(X)$.

Rough set theory has an overlap with many other theories such as fuzzy set, evidence theory, and probability logic. All these imprecision theories can be interpreted with lower and upper approximation of rough set and its properties in rough set theory. However, the knowledge of a rough set can be described between lower and upper approximation, which has not the precise definition. In this paper, we can use the concept of rough observable set to describe a rough set on the given universe approximately.

One of the reasons is that fuzzy boundary region which can cause the knowledge has uncertainty. The fuzzy boundary region can be regarded as an incomplete domain that is people may not fully understand and master of all information or knowledge about. It is even difficult to get knowledge of unknown sets with the limit information on incomplete domain, such as in n-dimensional space. However, as we have the information about some parts of incomplete domain, we can use the information to get more information or knowledge about it. With the concepts of rough membership function and rough inclusion, a method was proposed to process the uncertainty knowledge that it can get some part knowledge of unknown sets with knowledge of known sets on the incomplete domain.

B. Rough Inclusion

Definition 6 Let X and Y are the limit sets and let x be the element of U. If any element of X is belonged to Y, then the set of X is the subset of Y, which can be denoted as $X \subseteq Y$. It can be defined as

$$X \subseteq Y \Leftrightarrow \forall x (x \in X \rightarrow x \in Y) \tag{13}$$

This is the definition of $\subseteq$ in set theory. Obviously, X and Y are the classic sets. The definition implies that we can use the knowledge of Y to describe the set of X, if X and Y satisfied the relation $\subseteq$. In fact, it is special that the sets of X and Y are all the classic sets. At the most of the time, the sets of X and Y are all the rough sets or one of them is rough set. And there is a certain relation between them. For example, they contain the same elements, which can be described as $X \cap Y \neq \Phi$. So the question is that if we can use the knowledge of X or Y to infer another set Y or X' knowledge, which is unknown. This is what we are going to discuss.

The definition of rough inclusion [17] which proposed by Polkowski has been modified in order to fit this paper's requirement.

Definition 7 Let U be the universe, R be an equivalence relation on U, X and Y are the limit sets. It has:

$$\text{if } \frac{|X \cup Y|}{|X|} = r \ (0<r<1), \text{ then } X \subseteq_r Y \tag{14}$$

Where $\subseteq_r$ is called rough r-inclusion, and $X \subseteq_r Y$ denote that the set X is included in the set Y by the degree of r.

From the definition of rough inclusion above, we can know that:

(1) if r=0, then the sets of X, Y have no intersection.

(2) if r=1, then the sets of X, Y have the feature of inclusion in classic set theory.

(3) if $0< r <1$, it can be viewed as a real sense of rough inclusion.

And we only discuss on the situation of (3).

If X depends totally on Y, we can also have:

$$\alpha_X(Z) \leq \alpha_Y(Z) \tag{15}$$

for every $Z \subseteq U$.

Where $\alpha_X(Z)$ is the accuracy of Z which is the sub set of X on U. The justification that the higher of the degree of dependency of attributes implies the more accuracy for selecting partitioning attributes [18].

In the real world, people cannot grasp all the information or knowledge of the domain. So it is difficult to describe the specific object using the whole knowledge from the domain. We can use the method of rough inclusion to infer information of some unknown objects by the knowledge of known objects, which is associated with the unknown objects.

S is an incomplete information system S= (U, A), where U is a non-empty finite set of elements and A is a non-empty finite set of attributes. Let S'= (U', A') is a sub-information system that we have already mastered it. So it can be known that $U' \subseteq U$, $A' \subseteq A$. X, Y are the sets on S.

Let $Y \subseteq U'$, $X \cap (U - U') \neq \Phi$, $X \subseteq_r Y$, then we can get the knowledge of set Y with the information of system S. As for the set of X, we do not know or even a little. Because the set X contains the elements out of U, then the set X is called the uncertainty set.

The set X is included in set Y with the degree of r, which is denoted as $X \subseteq_r Y$. The relation between sets X and Y will be studied by the concept of rough inclusion. And this relation can be seen as the particular relation which determined by one or more attribute on information system S'.

(1) If X is an uncertainty set, Y is a classic set on sub-universe U about A', that is Y is a definable set, then the set of X is known by the degree: $\frac{|X \cup Y|}{|X|}=r$ ($0<r<1$);

The set of Y is a classic set on U'. So it can be described by attribute A_Y in set A'. With the concept of rough inclusion, it can be known that the attribute A_Y can describe set X by the degree of r, which is the set X also has the property that the set Y has by the degree of r.

(2) If X is an uncertainty set, Y is a rough set on on sub-universe U about A', $X \subseteq_r Y$ and $X \cap appr(Y) \neq \Phi$, then the set of X is known by the degree: $\frac{|X \cup appr(Y)|}{|X|}=r'$ ($0<r'<1$), where the set $appr(Y)$ is a rough observable set of Y.

With the concept of rough observable set, it can be known that $appr(Y)$ is a definable set on U' about A. So it can be used the sub-set $A'_{Y'}$ of attributes set A' to describe $appr(Y)$. And with the concept of rough inclusion, it can be known that the attribute $A'_{Y'}$ can describe set X by the degree of r', which is the set X also has the property that the set $appr(Y)$ has by the degree of r'.

According to the property of rough observable set: $R_*(X) \subseteq appr(X) \subseteq R^*(X)$, there are two cases of $|appr(Y)| \leq |Y|$ and $|appr(Y)| \geq |Y|$, so there are two cases of $r \leq r'$ and $r \geq r'$. But if $r'=0$, then $X \cap appr(Y) = \Phi$. It indicates that the sets X and Y have intersection but only on the fuzzy boundary region. The knowledge of set $appr(Y)$ can not been used to infer the knowable degree of knowledge about X.

From above, we can find some attributes of an information system to describe a rough set X on the universe U of A. And we also can get some properties or knowledge of rough set X with the set which has the intersection with it. Because the knowledge of the rough set is known to a certain degree, rough set Y or other sets are also true.

V. EXPERIMENT

A. The Rough Observable Method

Let $U=\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}\}$, and let R_1, R_2 be the equivalence relation on U. The partition $U/R_1=\{\{e_2, e_3, e_6, e_7, e_{10}, e_{14}\}, \{e_1, e_5, e_{11}, e_{12}, e_{15}\}, \{e_4, e_8\}, \{e_9, e_{13}, e_{16}\}\}$. The partition $U/R_2=\{\{e_1, e_5\}, \{e_2, e_6\}, \{e_3, e_7\}, \{e_4, e_8\}, \{e_9, e_{13}\}, \{e_{10}, e_{14}\}, \{e_{11}, e_{12}, e_{15}\}, \{e_{16}\}\}$. The set of X is a subset of the universe U.

It is easy to see that $R_2 \prec R_1$.

Suppose $X=\{e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_{10}, e_{11}, e_{14}\}$ and the coefficient $\beta=0.5$. Then it is obvious that

$R_{1*}(X)= R_{2*}(X)=\{e_2, e_3, e_4, e_6, e_7, e_8, e_{10}, e_{14}\}$

and

$R_1^*(X)= R_2^*(X)=\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_{10}, e_{11}, e_{12}, e_{14}, e_{15}\}$.

Thus

$\alpha_{R1}(X) = \alpha_{R2}(X) = 61.5\%$.

Note that, in this example, there is a partial relation in the two knowledge representation systems called as R_1 and R_2. The same accuracy or roughness is obtained for each of the two rough sets of X induced by the two knowledge representation systems, respectively. Therefore, it is necessary to use new effective measure deal with this situation which is the rough observable method.

$appr_{R1}(X) = \{e_2, e_3, e_4, e_6, e_7, e_8, e_{10}, e_{14}\}$

and

$appr_{R2}(X) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_{10}, e_{14}\}$.

Hence, the accuracy approximation of X with respect to R_1 and R_2 are given by

$\alpha'_{R1}(X) = 61.5\%$,

and

$\alpha'_{R2}(X) = 76.9\%$,

Respectively. Obviously, $\alpha'_{R1}(X) < \alpha'_{R2}(X)$. It is clear that the accuracy approximation of X with respect to R increases as R becoming finer.

Thus, it indicates that the degree of people mastering the knowledge with X can be improved from 61.5% to 76.9%, and then uncertainty of knowledge about X is decreased. So from this example, it can be seen that while the set X is replaced with approximation set $appr(X)$, that is making it express as the union of elementary sets on the universe, the uncertainty of knowledge about X is certain to a large extent.

B. The Rough Inclusion Method

Let $S=(U, A)$ is an information system, $S'=(U', A')$ is a sub-information system of S, that is $U' \subseteq U$, $A' \subseteq A$. The information of S' is already known by people. U' is a set of all known elements, $A'=\{a, b, c, d\}$ is a attributes set on U. X and Y are two sets of information system S. Let $X=\{e_{32}, e_{42}, e_{43}, e_{52}, e_{53}, e_{62}\}$; $Y=\{e_{24}, e_{32}, e_{33}, e_{34}, e_{42}, e_{43}, e_{44}, e_{45}, e_{53}, e_{54}\}$, and $S'=(U', A')$ can be shown as in table

TABLE I.
INFORMATION TABLE OF ELEMENTS ON U

U'	a	b	c	d
e_{14}	0	1	0	0
e_{15}	0	1	0	0
e_{24}	0	1	0	0
e_{25}	0	1	0	0
e_{31}	0	1	1	0
e_{32}	0	1	1	0
e_{41}	0	1	1	0
e_{42}	0	1	1	0
e_{33}	1	0	0	0
e_{34}	1	0	0	0
e_{43}	1	1	0	0
e_{53}	1	1	0	0
e_{44}	1	0	0	1
e_{45}	1	0	0	1
e_{54}	1	0	0	1
e_{55}	1	0	0	1

In this case, U is an incomplete universe, and people cannot know all the information about it. But the information of sub-universe U' can be known fully. And for A, one partition of U' can be known: $U'/A=\{\{e_{14}, e_{15}, e_{24}, e_{25}\}, \{e_{31}, e_{32}, e_{41}, e_{42}\}, \{e_{33}, e_{34}\}, \{e_{43}, e_{53}\}, \{e_{44}, e_{45}, e_{54}, e_{55}\}\}$. We also known that $Y\subseteq U'$. So the information of set Y can be grasping well.

$R_*(Y)=\{e_{33}, e_{34}, e_{43}, e_{53}\}$

$R^*(Y)=\{e_{14}, e_{15}, e_{24}, e_{25}, e_{31}, e_{32}, e_{41}, e_{42}, e_{33}, e_{34}, e_{43}, e_{53}, e_{44}, e_{45}, e_{54}, e_{55}\}$

$appr(Y)=\{e_{31}, e_{32}, e_{41}, e_{42}, e_{33}, e_{34}, e_{43}, e_{53}, e_{44}, e_{45}, e_{54}, e_{55}\}$;

Y is a rough set on U' about A. X is a set that has intersection with Y, and it has the elements out of U'. It can be known that:

$$r=\frac{\left|\{e_{32}, e_{42}, e_{43}, e_{53}\}\right|}{\left|\{e_{32}, e_{42}, e_{43}, e_{52}, e_{53}, e_{62}\}\right|}=67\%,\ \text{then } X\subseteq_{2/3}Y;$$

We have $X\cap appr(Y)=\{e_{32}, e_{42}, e_{43}, e_{53}\}\neq\Phi$, It can be known that:

$$r'=\frac{\left|\{e_{32}, e_{42}, e_{43}, e_{53}\}\right|}{\left|\{e_{32}, e_{42}, e_{43}, e_{52}, e_{53}, e_{62}\}\right|}=67\%;$$

From above analysis, we can get the elements of set X are known to people by the degree 2/3. Since the information of elements can be described by attributes and values of attributes in an information system. Similarly, we can see that the elements of e_{32} and e_{42} have the attributes of $a_0b_1c_1d_0$, and the elements of e_{43} and e_{53} have the attributes of $a_1b_1c_0d_0$. So the knowledge of set X can be inferred by the set of Y on incomplete domain, and also the knowable degree of knowledge about X is increased.

Granular computing model of uncertainty measurement research based on diverse ways has different measurement principles, but with the uncertainty of the deepening of the research, people have to face the uncertainty measure of axiomatization problems. Different researchers from different angles of different uncertainties of measurement standards, these standards have the same, some different, although are based on their proposed metric formulas are given for the special standards, but has not formed the uncertainty measure axiom. Therefore, granular computing theory and model uncertainty measure axiom is an urgent problem. If we use the way to study, which is similar to the axiomatic definition of probability theory, it will contribute to the uncertainty measure of standardization and standardization. The results obtained in the paper have some significance for promoting further studies on the granular computing theory and the unified frame.

VI. Conclusion

There are several other methods to deal with the uncertainty of knowledge about rough set. But whether an effective method that can be applied to deal with the uncertainty of knowledge is still remains to be an important direction in rough set theory. On the one hand, this paper has put forward the concept of rough observable set and used the rough observable method to deal with the uncertainty knowledge on complete domain based on the concept of rough membership function. This is, in fact, one application of rough membership function in rough set theory, and also a new idea is offered for dealing with the uncertainty knowledge in this paper. On the other hand, people always explore some unknown areas to obtain knowledge. So this paper also has proposed a way of processing uncertainty knowledge. But there still has been some distance from establishing a comprehensive treatment of the uncertainty knowledge. Therefore, how to use the knowledge that people have to infer some unfamiliar areas of knowledge will be one of the future directions.

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Yanli Zhu (1976-) received her B.S degree from Computer Henan normal university in 1999 and her M.S degree from Sichuan Normal University in 2007. She is a faculty member in the School of Information Engineer, Henan Institute of Science and Technology. Her current research interests are data mining and image processing. She is a membership of China Computer Federation.

Quanrui Wang (1981-) received B.S degree in 2004 from Computer and Applications, Henan normal university in Xinxiang, and M.S degree in 2007 from School of Computer Technology, Henan normal university in Xinxiang. At the year of 2009, she enrolls in Wuhan University of Technology in 2009, work hard at D.S degree. The main research direction of she is: the rough set and granular computing.

Shiyong Li (1983-) received his BS degree from HeNan Financial University and his MS degree from GuangXi University in 2004 and 2007, respectively. His research interests include Data Mining and Artificial Intelligence