Novel Methods for Intuitionistic Fuzzy Multiple Attribute Decision Making

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***Abstract*— Decision makers sometimes tend to express their preferences in intuitionistic fuzzy form. Based on evidence theory and grey relational analysis (GRA) technique, this paper aims at investigating multiple attribute decision making problems, in which both attribute values and weight values take the form of intuitionistic fuzzy numbers (IFNs). To this end, each intuitionistic fuzzy evaluation rating is transformed into a corresponding belief structure. In light of the distance metric between two basic probability assignments, we derive a novel method to measure the distance between intuitionistic fuzzy evaluation ratings and develop two GRA based decision methods. In the process, intuitionitic fuzzy weight information is converted into a group of linear constraints. The point and the interval-valued grey relational degrees are respectively derived as the basis to order potential alternatives. Finally, an example regarding air-condition system selection is adopted to illustrate the proposed decision procedures.**

***Index Terms*— Multiple attribute decision making, grey relational analysis, evidence theory, intuitionistic fuzzy number**

I. Introduction

Multiple attribute decision making (MADM) problems have wide application background in real world. Project bidding, software or equipment selection, and performance evaluation all fall into this scope. The common feature of such a kind of problems is that, alternatives need to be evaluated and sorted in term of several predefined attributes to generate the most desirable alternative(s). In practical application, attribute ratings and weight values are usually characterized by imprecision and vagueness. Therefore, it is difficult, even impossible for decision makers to provide an exact value for a specified decision parameter for some MADM problems. Thus, fuzzy set theory was employed to handle the uncertainty in multiple attribute decision making analysis and various algorithms were developed for effective decision aid [1-4].

However, owing to the lack of knowledge, and also the complexity of decision making problem itself, decision makers sometimes cannot reach a complete assignment on the membership degree and the non-membership degree regarding a specified fuzzy evaluation rating. Under such a situation, intuitionistic fuzzy sets (IFSs) proposed by Atanassov [5-6] is appropriate to represent the decision makers' preferences. Within the context of IFSs, the sum of the membership degree and the non-membership degree assigned to each element in the universe of discourse is less than or equal to one. Since it was presented in 1986, intuitionistic fuzzy set has received ever increasing attention and also been applied to the field of multiple attribute decision making. In [7-8], on the basis of linear programming technique, Li developed some methods for solving multiple attribute decision making problems in intuitionistic fuzzy setting. In [9], the combined maximum and the combined minimum average weighted intuitionistic index methods were proposed for intuitionistic fuzzy MADM problems. Xu and Yager [10] defined the dynamic intuitionistic fuzzy weighted averaging (DIFWA) operator and the uncertain dynamic intuitionistic fuzzy weighted averaging (UDIFWA) operator. Moreover, the defined operators were utilized to develop dynamic multiple attribute decision making decision procedures with intuitionistic fuzzy information. Recently, with the employment of cosine similarity metric, Xu and Hu [11] developed two projection models for MADM problems with intuitionistic fuzzy information. Wei [12] utilized grey relational analysis technique to develop a multiple attribute decision making procedure with incomplete weight information in intuitionistic fuzzy environment.

Dempster-Shafer (DS) evidence theory [13], another feasible option to handle uncertainty in decision analysis, has also been introduced into the field of MADM analysis by some pioneers [14-16]. Unlike Bayesian theory, DS evidence theory allows assigning probability degree to the collection of individual hypothesis which implies the insufficiency of knowledge to realize the complete probability allocation among individual hypothesis. From this perspective, both evidence theory and intuitionistic fuzzy theory have the ability to represent incomplete

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information. In [16], the inherent relationship between evidence theory and intuitionistic fuzzy set was elaborated on, and the proposed interpretations were applied to solving a category of intuitionistic fuzzy MADM problems. In the proposed procedure, the Dempster's rule of combination is employed to fuse decision information.

In this paper, based on the distance metric between two pieces of evidence, we proposed a novel method to measure the disparity of two intuitionistic fuzzy evaluation ratings, and developed two grey relational analysis based procedures to solve MADM problems in intuitionistic fuzzy environment. The rest of this paper is organized as follows. In section 2, we briefly introduce the basic concepts and theories on intuitionistic fuzzy set and evidence theory. In section 3, we introduce intuitionistic fuzzy MADM problems discussed in this paper. In section 4, associated with evidence theory, we develop two GRA based decision methods for the concerned problem. In section 5, we employ an example regarding air-condition system selection to illustrate the proposed decision methods. Finally, in section 6, we summarize our study and point out the further research direction.

II. Preliminaries

In this section, we introduce some fundamental concepts and theories on intuitionistic fuzzy set and evidence theory for the further development.

A. Intuitionistic Fuzzy Set

As a generalization of ordinary fuzzy set, the concept of IFS was presented by Atanassov [5-6] in 1986. The relevant definitions are given below.

Definition 1 [17]. Provided that X is a universe of discourse, an ordinary fuzzy set denoted by $A=\left\{\left\langle x,\mu_A\left(x\right)\right\rangle \middle| x\in X\right\}$, is characterized by a membership function $\mu_A\left(x\right)$ such that $\forall x\in X$, $0\le \mu_A\left(x\right)\le 1$.

Definition 2 [5]. Provided that X is a universe of discourse, an intuitionistic fuzzy set denoted by $A=\left\{\left\langle x,\mu_A\left(x\right),v_A\left(x\right)\right\rangle \middle| x\in X\right\}$, is characterized by a membership function $\mu_A\left(x\right)$ and a non-membership function $v_A\left(x\right)$. Moreover, for $\forall x\in X$, the following conditions hold:

(1) $0\le \mu_A\left(x\right)\le 1$;

(2) $0\le v_A\left(x\right)\le 1$;

(3) $0\le \mu_A\left(x\right)+v_A\left(x\right)\le 1$.

Definition 3 [5]. Let A be an intuitionistic fuzzy set in X. For $\forall x\in X$, if $\pi_A\left(x\right)=1-\mu_A\left(x\right)-v_A\left(x\right)$, then $\pi_{\tilde{A}}\left(x\right)$ is the indeterminacy degree of element x to A.

The indeterminacy degree implies the incomplete designation of membership degree and non-membership degree for the element contained in the universe of discourse.

When $\pi_A\left(x\right)=0$, the intuitionistic fuzzy set is reduced to an ordinary fuzzy set. For the convenience of narrative in the sequel, we abbreviate the notation $A=\left\{\left\langle x,\mu_A\left(x\right),v_A\left(x\right)\right\rangle \middle| x\in X\right\}$ to $\left\langle \mu_A\left(x\right),v_A\left(x\right)\right\rangle$.

B. Dempster-Shafer Evidence Theory

Let $\Theta=\left\{h_1,h_2,\cdots,h_N\right\}$ be the frame of discernment, in which the hypothesis contained should be exhaustive and mutually exclusive. The power set of the frame of discernment Θ is denoted by $P\left(\Theta\right)$ consisting of 2^N elements.

$$P\left(\Theta\right)=\left\{\varnothing,\{h_1\},\cdots,\{h_N\},\right. \\ \left.\{h_1,h_2\},\cdots,\{h_{N-1},h_N\},\cdots,\Theta\right\}. \tag{1}$$

in which $\varnothing$ denotes the null set.

Definition 4 [13]. The basic probability assignment (BPA) of a belief structure is a projection $m:P\left(\Theta\right)\to\left[0,1\right]$ such that

(1) $\sum_{X\in P(\Theta)} m\left(X\right)=1$;

(2) $m\left(\varnothing\right)=1$.

The probability function defined above indicates the belief degree to the element contained in the power set. And the element contained in $P\left(\Theta\right)$ with a positive probability value is called focal element.

Definition 5 [18]. Let $m_1\left(Y\right)$ and $m_2\left(Y\right)$ be two basic probability assignments on the frame of discernment $\Theta=\left\{h_1,h_2,\cdots,h_N\right\}$, the distance between $m_1\left(Y\right)$ and $m_2\left(Y\right)$ is defined as

$$d\left(m_1,m_2\right)=\sqrt{\frac{1}{2}\left(\mathbf{m}_1-\mathbf{m}_2\right)^T\mathbf{D}\left(\mathbf{m}_1-\mathbf{m}_2\right)}. \tag{2}$$

in which $\mathbf{m}_1$ and $\mathbf{m}_2$ respectively denote the BPA vectors of $m_1\left(Y\right)$ and $m_2\left(Y\right)$. Moreover, the matrix $\mathbf{D}=\left[d_{ij}\right]_{2^N\times 2^N}$ is defined as

$$d_{ij}=\frac{\left|Y_i\cap Y_j\right|}{\left|Y_i\cup Y_j\right|}. \tag{3}$$

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III. PROBLEM STATEMENT

Let $A=\left\{a_i \middle| i=1,2,...,m\right\}$ be a discrete collection of m potential alternatives to be ranked, and $U=\left\{u_j \middle| j=1,2,...,n\right\}$ be a set of n attributes. The attribute value of alternative $a_i \in A$ with respect to attribute $u_j \in U$ is denoted by an IFN which is described as $\left\langle x_{ij1}, x_{ij2} \right\rangle$.

Thus, the intuitionistic fuzzy decision matrix is formulated as follows.

$$\mathbf{X}=\begin{bmatrix} \langle x_{111},x_{112}\rangle & \langle x_{121},x_{122}\rangle & \cdots & \langle x_{1n1},x_{1n2}\rangle \\ \langle x_{211},x_{212}\rangle & \langle x_{221},x_{222}\rangle & \cdots & \langle x_{2n1},x_{2n2}\rangle \\ \vdots & \vdots & \vdots & \vdots \\ \langle x_{m11},x_{m12}\rangle & \langle x_{m21},x_{m22}\rangle & \cdots & \langle x_{mn1},x_{mn2}\rangle \end{bmatrix}. \tag{4}$$

For a MADM problem, attribute weight plays an important role in electing the most desirable alternative(s). In some real situation, decision makers are prone to expressing incomplete preference information on attribute weight. In view of this point, herein, we also assume that attribute weights take the form of intuitionistic fuzzy numbers. Additionally, the weight vector is denoted by $\mathbf{w}=\left[\langle w_{11},w_{12}\rangle \quad \langle w_{21},w_{22}\rangle \quad \cdots \quad \langle w_{n1},w_{n2}\rangle\right]^T$.

IV. PROBLEM STATEMENT

As a part of grey system theory which was proposed by Deng [19], grey relational analysis is a useful tool for handling incomplete information. When GRA is employed, the entire data of sequences are taken into account. Owing to its practicability, GRA has achieved many successful applications in decision making field [20-23].

In this paper, associated with evidence theory, we employ globalized grey relational analysis technique to solve the MADM problem with intuitionistic fuzzy information.

A.Transformation of Intuitionistic Fuzzy Assessment Information

In the sprit of [16], an intuitionistic fuzzy number can be converted into a corresponding belief structure, in which the frame of discernment is identified with two elements as $\Theta=\left\{h^+,h^-\right\}$. Moreover, the former element of collection Θ is related to the positive evaluation, while the later is related to the negative evaluation. Then, the power set of Θ is formulated as $P(\Theta)=\left\{\varnothing,h^+,h^-,\Theta\right\}$.

Subsequently, the values of basic probability assignment regarding elements h^+ and h^- respectively correspond to the membership degree and the non-membership degree of a given evaluation rating, while the value of basic probability assignment regarding the collectivity Θ corresponds to the indeterminacy degree of the given rating. Therefore, we can obtain a group of belief degrees for rating x_{ij}.

$$\begin{cases} m_{ij}(\varnothing)=0 \\ m_{ij}(h^+)=x_{ij1} \\ m_{ij}(h^-)=x_{ij2} \\ m_{ij}(\Theta)=1-x_{ij1}-x_{ij2} \end{cases}. \tag{5}$$

B. The Definition of Referential Sequence

GRA makes use of grey relational grade (GRD) to measure similarity between the defined referential sequence and the compared sequences. The referential sequence is an ideal frame of reference. Therefore, we need to define referential sequence which is also expressed in belief structure with the following BPA.

$$\begin{cases} m_j^+(\varnothing)=0 \\ m_j^+(h^+)=\max\limits_{1\le i\le m} x_{ij1} \\ m_j^+(h^-)=\min\limits_{1\le i\le m} x_{ij2} \\ m_j^+(\Theta)=1-\max\limits_{1\le i\le m} x_{ij1}-\min\limits_{1\le i\le m} x_{ij2} \end{cases}. \tag{6}$$

C. The Computation of Grey Relational Coefficient

Based on the definition of referential sequence, the grey relational coefficient of alternative $a_i \in A$ with respect to attribute $u_j \in U$ is calculated by the following formula.

$$\gamma_{ij}=\frac{d_{\min}+\xi d_{\max}}{d_{ij}+\xi d_{\max}}. \tag{7}$$

where

$d_{\min}=\min\limits_{1\le i\le m}\min\limits_{1\le j\le n} d_{ij}$,

$d_{\min}=\max\limits_{1\le i\le m}\max\limits_{1\le j\le n} d_{ij}$,

d_{ij} is the distance between the basic probability assignments m_{ij} and m_j^+,

ξ is the distinguished coefficient and could be set as $\xi=0.5$ in computation.

From formula (7), it is easily seen that the precondition of computing grey relation coefficient is to determine the distance between two pieces of evidence. To reach the goal, we employ the BPA based distance metric.

Proposition 1 The distance between the basic probability assignment m_{ij} and m_j^+ is described as:

$$d_{ij}=\frac{1}{2}\sqrt{2\left(\Delta m_{ij1}^2+\Delta m_{ij2}^2\right)+\Delta m_{ij3}^2} \tag{8}$$

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where

$\Delta m_{ij1} = m_{ij}\left(h^{+}\right) - m_i^{+}\left(h^{+}\right)$,

$\Delta m_{ij2} = m_{ij}\left(h^{-}\right) - m_i^{+}\left(h^{-}\right)$,

$\Delta m_{ij3} = m_{ij}\left(\Theta\right) - m_i^{+}\left(\Theta\right)$.

Proof. In light of Definition 5, we have:

$$d_{ij} = \sqrt{\frac{1}{2}\mathbf{\Delta m}_{ij}^{T}\mathbf{D}\mathbf{\Delta m}_{ij}} . \tag{9}$$

in which

$\mathbf{\Delta m}_{ij} = \left(0, \Delta m_{ij1}, \Delta m_{ij2}, \Delta m_{ij3}\right)^{T}$,

$$\mathbf{D} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix}.$$

Therefore, we have

$$\begin{aligned} &\mathbf{\Delta m}_{ij}^{T}\mathbf{D}\mathbf{\Delta m}_{ij} \\ &= \Delta m_{ij1}^2 + \Delta m_{ij2}^2 + \Delta m_{ij3}^2 \\ &+ \frac{1}{2}\Delta m_{ij3}\left(\Delta m_{ij1} + \Delta m_{ij2}\right) \\ &= \Delta m_{ij1}^2 + \Delta m_{ij2}^2 \\ &+ \frac{1}{2}\Delta m_{ij3}\left(\Delta m_{ij1} + \Delta m_{ij2} + 2\Delta m_{ij3}\right) \\ &= \frac{1}{2}\left[2\left(\Delta m_{ij1}^2 + \Delta m_{ij2}^2\right) + \Delta m_{ij3}^2\right]. \end{aligned} \tag{10}$$

Substitute formula (10) into (9), and the result contained in Proposition 1 is straightforward.

D. The Computation of Grey Relational Degree

If the weight vector is known with exact values, we can calculate the grey relational degree of alternative $a_i \in A$ directly through the following formula:

$$\gamma_i = \sum_{j=1}^{n} w_j \gamma_{ij} . \tag{11}$$

However, the weight values are difficult to be obtained exactly. Therefore, in the paper, we consider the intuitionistic fuzzy weight information. Referring to the method in [7-8], we can derive a group of linear constraints from each intuitionistic fuzzy weight value, namely, the weight $w_j = \left\langle w_{j1}, w_{j2} \right\rangle$ corresponds to the linear constraints $w_{j1} \le w_j \le 1 - w_{j2}$.

For the incomplete weight information which is expressed in linear constraint, there are usually two disposition methods in the existing literature. One is to identify a desirable weight vector under a certain criterion, and thus derive the corresponding overall evaluation value for each alternative. As indicated in [11], an ideal weight vector should be identified to make the grey relational degree as large as possible.

Thus, we can formulation the following programming model:

$$\begin{aligned} &\max_{\mathbf{w}} \gamma = \left(\gamma_1, \gamma_2, \cdots, \gamma_m\right) \\ &\text{s. t. } \; w_{j1} \le w_j \le 1 - w_{j2}, \; j = 1, 2, \cdots, n \\ &\qquad \sum_{j=1}^{n} w_j = 1. \end{aligned} \tag{12}$$

With the disposition method proposed in [24], model (12) is equivalent to:

$$\begin{aligned} &\max_{\mathbf{w}} z \\ &\text{s. t. } \; \sum_{j=1}^{n} w_j \gamma_{ij} \ge z, i = 1, 2, \cdots, m \\ &\qquad w_{j1} \le w_j \le 1 - w_{j2}, \; j = 1, 2, \cdots, n \\ &\qquad \sum_{j=1}^{n} w_j = 1. \end{aligned} \tag{13}$$

Model (13) could be solved by standard software. After getting the optimal solution of model (13), the GRD could be determined using formula (11).

Another method is to derive the upper bound and the lower bound of overall value for each alternative and form the interval-valued overall evaluation value. Then, the feasible candidates are ranked through the ranking method of interval numbers, such as in [7-8].

In the setting of interval-valued overall evaluation value, we can construct the following two linear programming models to compute the value range of grey relation degree for alternative $a_i \in A$ as follows.

$$\begin{aligned} &\max_{\mathbf{w}}/\min \gamma_i = \sum_{j=1}^{n} w_j \gamma_{ij} \\ &\text{s. t. } \; w_{j1} \le w_j \le 1 - w_{j2}, \; j = 1, 2, \cdots, n \\ &\qquad \sum_{j=1}^{n} w_j = 1. \end{aligned} \tag{14}$$

Likewise, the two linear programming models contained in (14) are also easy to be resolved.

Denote the optimal solutions of the maximum and the minimum problems in (14) by $\mathbf{w}^{u*} = \left[w_j^{u*}\right]_{n\times 1}$ and $\mathbf{w}^{l*} = \left[w_j^{l*}\right]_{n\times 1}$. Furthermore, let $\gamma_i^{u} = \sum_{j=1}^{n} w_j^{u*}\gamma_{ij}$ and $\gamma_i^{l} = \sum_{j=1}^{n} w_j^{l*}\gamma_{ij}$. Then, the interval-valued grey relational degree of alternative $a_i \in A$ is formed as $\gamma_i = \left[\gamma_i^{l}, \gamma_i^{u}\right]$.

Thus, we can rank the alternatives according to the ranking method of interval numbers and identify the optimal solution. Regarding the ranking method of interval numbers, the details could be found from [25] which provided an excellent survey.

In the above, combining evidence theory and grey relational analysis technique, we describe two decision methods for MADM problems with intuitionistic fuzzy information. One is based on point grey relational degree, and another is based interval-valued grey relational degree.

As a whole, the decision procedures can be summarized as follows.

Algorithm1
Point GRD based procedure

Step 1-1 Build alternative set $A=\left\{a_i \,\middle|\, i=1,2,...,m\right\}$ and attribute set $U=\left\{u_j \,\middle|\, j=1,2,...,n\right\}$. Collect decision information to form decision matrix $\mathbf{X}=\left[x_{ij}\right]_{m\times n}$ and weight vector $\mathbf{w}=\left[w_j\right]_{n\times 1}$.

Step 1-2 Transform the intuitionistic fuzzy assessment information into corresponding belief structures in light of (5).

Step 1-3 Identify the referential sequence according to formula (6), and calculate grey relational coefficients to form a matrix $\mathbf{Y}=\left[\gamma_{ij}\right]_{m\times n}$.

Step 1-4 Construct the linear programming model to solve the desirable weight vector $\mathbf{w}^*=\left[w_j^*\right]_{n\times 1}$.

Step 1-5 Compute GRD using $\gamma_i=\sum_{j=1}^{n} w_j^*\gamma_{ij}$. Identify the optimal solution in light of the values of GRD.

Algorithm2
Interval-valued GRD based procedure

Step 2-1 to 2-3 is same as Step 1-1 to 1-3.

Step 2-4 In light of (14), construct linear programming models to solve the value range of GRD for each potential alternative.

Step 2-5 Rank the alternatives by means of ranking method to interval number and identify the optimal solution.

V. ILLUSTRATIVE EXAMPLE

We make use of an illustrative example regarding an air-condition system selection problem in [16] to explain the decision procedures in this paper. Three potential candidates are taken into consideration and respectively denoted by a_1, a_2 and a_3. The evaluation attributes are as follows.

u_1: economical,

u_2: function,

u_3: being operative.

The detailed intuitionistic fuzzy evaluation information is listed in Table 1.

TABLE I.
THE INTUITIONISTIC FUZZY EVALUATION INFORMATION

	u_1	u_2	u_3
a_1	<0.75,0.10>	<0.80,0.15>	<0.40,0.45>
a_2	<0.60,0.25>	<0.68,0.20>	<0.75,0.05>
a_3	<0.80,0.20>	<0.45,0.50>	<0.60,0.30>

In light of (5), the basic probability assignments are derived and shown in Table 2. In Table 2, we denote the BPA of each rating using a collection with three elements respectively represent the belief degrees of h^+, h^- and Θ. Because the belief degree of the null set is zero, we omit it in Table 2.

TABLE II.
THE BASIC PROBABILITY ASSIGNMENT OF EVALUATION RATING

	BPA
x_{11}	{0.75,0.10,0.15}
x_{21}	{0.60,0.25,0.15}
x_{31}	{0.80,0.20,0}
x_{12}	{0.80,0.15,0.05}
x_{22}	{0.68,0.20,0.12}
x_{32}	{0.45,0.50,0.05}
x_{13}	{0.40,0.45,0.15}
x_{23}	{0.75,0.05,0.20}
x_{33}	{0.60,0.30,0.10}

According to (6), the basic probability assignments of referential sequence are identified and shown in Table 3.

TABLE III.
THE BASIC PROBABILITY ASSIGNMENT OF REFERENTIAL SEQUENCE

u_1	u_2	u_3
{0.80,0.10,0.10}	{0.80,0.15,0.05}	{0.75,0.05,0.20}

Furthermore, according to formula (7), we calculate the grey relational coefficient and get the following matrix.

$$\mathbf{Y}=\begin{bmatrix} 0.8131 & 1.0000 & 0.3333 \\ 0.5134 & 0.6569 & 1.0000 \\ 0.6850 & 0.3498 & 0.4703 \end{bmatrix}.$$

In the present problem, the intuitionistic fuzzy weight information is described as follows.

$$\mathbf{w}=\left[\langle 0.25,0.25\rangle \quad \langle 0.35,0.40\rangle \quad \langle 0.30,0.65\rangle\right]^T.$$

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Subsequently, we can convert the intuitionistic fuzzy weight information into the following linear constraints.

$$0.25 \le w_1 \le 0.75$$
$$0.35 \le w_2 \le 0.60$$
$$0.30 \le w_3 \le 0.35$$
$$w_1 + w_2 + w_3 = 1.$$

Above all, in light of (13), we establish the linear programming model to solve weight vector is established as follows.

$$\begin{aligned} &\max_{\mathbf{w}} z \\ &\text{s. t. } 0.8131w_1 + w_2 + 0.3333w_3 \ge z \\ &\quad 0.5134w_1 + 0.6569w_2 + w_3 \ge z \\ &\quad 0.6850w_1 + 0.3498w_2 + 0.4703w_3 \ge z \\ &\quad 0.25 \le w_1 \le 0.75 \\ &\quad 0.35 \le w_2 \le 0.60 \\ &\quad 0.30 \le w_3 \le 0.35 \\ &\quad w_1 + w_2 + w_3 = 1. \end{aligned}$$

Solving the above linear programming model, we get the ideal weight vector $\mathbf{w}^* = \begin{bmatrix} 0.35 & 0.35 & 0.30 \end{bmatrix}^T$. Accordingly, we can derive the grey relational degrees for the three air-condition systems as follows.

$$\gamma_1 = 0.7346,$$
$$\gamma_2 = 0.7096,$$
$$\gamma_3 = 0.5033.$$

From the values of grey relational grade, we can conclude that $a_1 \succ a_2 \succ a_3$ and thus a_1 is the most desirable air-condition system. The result is same as in literature [16].

In the following, we adopt the interval-valued GRD based method to solve the present problem. The following two linear programming models are established to solve the bounds of GRD of alternative a_1 as follows.

$$\begin{aligned} &\max_{\mathbf{w}}/\min\ 0.8131w_1 + w_2 + 0.3333w_3 \\ &\text{s. t. } 0.25 \le w_1 \le 0.75 \\ &\quad 0.35 \le w_2 \le 0.60 \\ &\quad 0.30 \le w_3 \le 0.35 \\ &\quad w_1 + w_2 + w_3 = 1. \end{aligned}$$

By solving the above programming models, we get:

$$\gamma_1 = [0.7118, 0.7543].$$

Using the similar method, we can derive:

$$\gamma_2 = [0.7096, 0.7411],$$
$$\gamma_3 = [0.4698, 0.5033].$$

With the possibility based ranking method of interval numbers, we calculate the possibility degree between grey relational degrees and get the following relations.

$$\gamma_1 \underset{0.6041}{\succ} \gamma_2 \underset{1.0000}{\succ} \gamma_3.$$

Thus, we have:

$$a_1 \succ a_2 \succ a_3.$$

Hence, a_1 is the most desirable air-condition system. Again, we get the same result as in literature [16].

VI. CONCLUSIONS AND FUTURE WORK

Combining evidence theory and grey relational analysis technique, in this paper, we investigate multiple attribute decision making problems, in which both attribute ratings and weight values are expressed in intuitionistic fuzzy numbers. A novel method is provided to scale the disparity between two intuitionistic fuzzy numbers. Moreover, the point and the interval-valued grey relational degrees are respectively derived for each alternative on the basis of linear programming technique. An illustrative example regarding air-condition system selection is employed to interpret the proposed decision procedures and demonstrate their feasibility.

In the following, we plan to adopt other distance or similarity metrics within the framework of evidence theory to extend the present research and compare the decision results of different metrics.

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